বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - V

Subject : MATHEMATICS

Paper : C 11 - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS]

Group-A

Answer any *four* questions : 12×4=48

1. (a) Solve $xp + yq = z$. Find a solution representing a surface meeting the parabola $y^2 = 4x$, $z = 1$. 6

   (b) Find the characteristics of the PDE $p^2 + q^2 = 2$ and determine the integral surface which passes through $x = 0$, $y = z$. 6

2. (a) A particle describes a rectangular hyperbola; the acceleration being directed from the centre. Show that the angle θ described about the centre in time t after

leaving the vertex is given by the equation $\tan\theta = \tan h\left(\sqrt{\mu}\, t\right)$, μ is the acc. at distance unity. 6

(b) Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions :

(i) $u(0,t) = u(2,t) = 0$

(ii) $u(x,0) = \sin^3 \frac{\pi x}{2}$

(iii) $u_t(x,0) = 0$ 6

3. (a) Prove that the solution of one-dimensional diffusion equation in the region $0 \le x \le \pi$, subject to the condition :

(i) $u(x,t)$ is finite as $t \to \infty$

(ii) $u(0,t) = 0 = u(\pi, t)$

(iii) $u(x, 0) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$ is $u(x,t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{e^{an^2 t} \sin\left(\frac{n\pi}{2}\right)}{n^2} \sin(nx)$. 10

(b) Define Dirac delta function with a brief explanation. 2

4. (a) Find the canonical form of the PDE $y^2 u_{xx} + 2yu_{xy} + u_{yy} - u_y = 0$. 5

(b) A bead slides down a rough circular wire, which is in a vertical plane, starting from rest at the end of the horizontal diameter. When it has described an angle θ about the centre, show that the square of the angular velocity is $\frac{2g}{a\left(1+4\mu^2\right)}\left\{\left(1-2\mu^2\right)\sin\theta + 3\mu\left(\cos\theta - e^{-2\mu\theta}\right)\right\}$ where μ is the coeficient of friction and a is the radius of the circle. 7

5. (a) A particle of mass M is at rest and begins to move under the action of constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it at a constant rare ρ. Show that its mass will be m when it has travelled a distance $\frac{k}{\rho^2}\left[m - M\left\{1 + \log\frac{m}{M}\right\}\right]$, where K = F-ρV. 7

(b) Find the solution of Laplace's equation $\nabla^2\varphi = 0$ in the semifinite region bounded by $x \geq 0,\ 0 \leq y \leq 1$ subject to the boundary conditions $\left(\frac{\partial\varphi}{\partial x}\right)_{x=0} = 0, \left(\frac{\partial\varphi}{\partial y}\right)_{y=0} = 0$ and $\varphi(x, 1) = f(x)$ where *f(x)* is assumed to be known. 5

6. (a) Solve : $px + qy = z\sqrt{1+pq}$ by charpit's method. 5

(b) A particle describes an ellipse under a force $\frac{u}{(dis\,tan\,ce)^2}$ towards the focus; if it was projected with velocity V from a point distance r from the centre of the force, show that its periodic time is $\frac{2\pi}{\mu}.\left[\frac{2}{r} - \frac{V^2}{\mu}\right]^{-\frac{3}{2}}$. 4

(c) From a partial diferential equation by eliminating the arbitrary function φ from $\varphi\left(x^2 + y^2 + z^2,\ z^2 - 2xy\right) = 0$. 3

7. (a) Discuss the solution of the two dimensional wave equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ by the method of separation of variables. 5

(b) A comet is moving in a parabola about the Sun as focus; when at the end of its latus rectum its velocity suddenly becomes altered in the ratio n : 1, where n<1; show the comet will describe an ellipse whose eccentricity is $\sqrt{1 - 2n^2 + 2n^4}$ and whose major axis $\frac{1}{1-n^2}$, where $2l$ is the latus rectus of the parabolic path. 7

8. (a) A partiocle is acted on by a central repulsive force, which varies as the nth power of the distance; if the velocity at any point of the path be equal to that, which would be acquired in falling from the centre to the point, show that the equation to the path is of the form $r^{\frac{n+3}{2}} cos \frac{n+3}{2}\theta =$ constant. 6

(b) Find the equation of the integral surface of the diferential equation $2y(z-3)p+(2x-z)q = y(2x-3)$ which passes through the circle $z=0,\ x^2+y^2=2x$. 3

(c) Find the general solution of PDE $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$. 3

Group-B

Answer any ***six*** questions : 2×6=12

9. (i) From a PDE when $\varphi(u,\ v)=0$, where $u=x+y+z,\ v=x^2+y^2+z^2$.

(ii) What is the interpretation that $z=f(x,\ y)$ is the integral surface of $pP + qQ = R$.

(iii) Find the characteristics of the equation $u_{xx}+2u_{xy}+sin^2 x\ u_{yy}+u_y=0$.

When it is hyperbolic.

(iv) Define 'Dirichlet boundary condition' and 'Neumann boundary condition'.

(v) If a particle moves on a curve $\sqrt{r}\, cos\frac{\theta}{2}=\sqrt{a}$ with cross-radial velocity constant then show that the velocity of the particle is constant.

(vi) Give the geometrical interpretation of Cauchy IVP $u_t+cu_x=0,\ x\in R, t>0$ where $u(x,0)=f(x), x\in R$.

(vii) Write the diferent types of first order PDE with standar form.

(viii) Verify the equation $z=\sqrt{2x+a}+\sqrt{2y+b}$ is a complete integral of the PDE $\frac{1}{z}=\frac{1}{p}+\frac{1}{q}$.

(ix) Show that if f and g are arbitrary function of their respective arguments then $u=f\left(x-vt+i\alpha y\right)+g\left(x-vt-i\alpha y\right)$ is a solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ where $\alpha^2=1=\frac{v^2}{c^2}$.

(x) A particle describes a curve $s=c\tan\Psi$ with uniform speed v. Find the acceleration indicating its direction.
