বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2021

(Under CBCS Pattern)

Semester - V

Subject : MATHEMATICS

Paper : SEC 3 - T

Full Marks : 40

Time : 2 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[NUMBER THEORY]

Group-A

1. Answer any ***three*** questions : 12×3=36

(a) (i) Prove that the product of any m consecutive integers is divisible by m.

(ii) Prove that the number of primes are infinite.

(iii) Show that $\frac{n^7}{7}+\frac{n^3}{3}+\frac{11n}{21}$ is an integer for all $n \in N$. 4+4+4

(b) (i) Solve the system of linear congruences $x \equiv 1$ (mod 3), $x \equiv 2$ (mod 5), $x \equiv 3$ (mod 7).

(ii) If n be a positive integer and a is prime to n, then $a^{\phi(n)} \equiv 1$ (mod n) where $\phi(n)$ denotes the Euler's phi-unction.

(iii) Show that there are ininitely many primes of the form $4n-1$. 4+4+4

(c) (i) State and prove the fundamental theorem of arithmatic.

(ii) Show that $n^4 + 4^n$ is a composite number for all $n > 1$, where $n \in N$. 8+4

(d) (i) If a and b are integers, not both zero, then there exist integers u and v, such that g.c.d $(a, b) = au + bv$.

(ii) Find the general solution in integers of the equation $7x + 11y = 1$.

(iii) Find the sum of all even positive divisors of 2700. 4+4+4

(e) (i) Define Mobius function. Show also that this function is multiplicative.

(ii) If $n = p_1^{\alpha_1} p_2^{\alpha_2} ......p_1^{\alpha_r}$, where $p_1, p_2, ..., p_r$ are distinct primes and $\alpha_1, \alpha_2, ..., \alpha_r$ are postive integers, then show that

$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right).....\left(1-\frac{1}{p_r}\right)$$

(iii) Show that g.c.d $(a, a+2) = 1$ *or* 2 for every integer a. (2+4)+4+2

(f) (i) Define Dirichlet product of two arithmatical functions. Let $g(n) = 1, f(n) = n, \forall n \cup N$, then their dirichlet product is equal to sum of divisors of n.

(ii) If $2^n + 1$ is an odd prime for some integer n, prove that n is a power of n.

(iii) The total number of positive divisors of a positive integer n is odd if and only if n is a perfect square. (1+3)+4+4

Group-B

2. Answer any ***two*** questions : 2×2=4

   (a) Find the number of zeros of the right end of the integer 140!

   (b) If p be a prime number and p/ab then either p/a or p/b.

   (c) Deine Fuler's phi-function (ϕ function).

   (d) Prove that for $n > 3$, the integers n, $n + 2$, $n + 4$ cannot be all primes.

OR

[BIO-MATHEMATICS]

Group-A

1. Answer any ***three*** questions : 12×3=36

(a) Distinguish between continuous time and discrete term modal in ecology. Give examples of continuous and discrete time prey-predator models. Explain the stability analysis of any of them.

(b) Describe economic model of harvesting of renewable resource. Explain in short economic overfishing.

(c) Write the diference between food chain and food web with examples. Formulate a three specks ood chain continuous time model.

(d) Explain the comment "a small change in classical Lotka-Voltena model shows that there is no periodic orbit wich is a marked contrast to the classical Lotka-Volterra system."

(e) Explain any two types of biurcation with example.

(f) The prodotor prey equations with additional deaths by DDT one $\frac{dx}{dt} = \beta_1 x - C_1 xy - p_1 x$, $\frac{dy}{dt} = -\alpha_2 y + c_2 xy - p_2 y$, where all parameters are positive constants x(t) denotes the prey population and Y(t) denotes the predator populations at any time.

(i) Find all the equillibrium points. (ii) What effect does the DDT have on the non-zero equillibrium populations compared with the case when there is no DDT? (iii) Show that the predator fraction of the total average prey predator population is given by $f = \frac{1}{1 + \left(\frac{a_1(\alpha_2 + p_2)}{c_2(B_1 - p_1)} \right)}$

Group-B

2. Answer any ***two*** questions : 2×2=4

(a) What do you mean by mathematical modelling of ecological system.

(b) Explain the term functional response.

(c) What do you mean by bifuncation of a system?

(d) Write in breif about Michaelis-Mentien Kinetics.

OR

[MATHEMATICAL MODELLING]

Group-A

1. Answer any ***three*** questions : 12×3=36

(a) An 8-lb weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equillibrium position, thereby stretching the spring 6 inch. The weight is then pulled down 3 inch below its equillibrium position and released at t = 0 with an initial velocity of 1 ft/sec directed downward. Neglecting the resistence of the medium and assuming that no external forces are present, determine the (i) amplitude (ii) period and (iii) frequency of the resulting motion.

(b) A 16-lp weight is attached to the lower end of a coil spring suspended from the celling, the spring constant of the spring being 10 lb/ft. The weight comes to rest in its equillibrium position. Beginning at t = 0 an external force given by F(t)=5 cos 2t is applied to the system. Determine the resulting motion if the damping force in pounds is numerically equal to $2\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instertancous velocity in feet per second.

(c) The differential equation for the motion of a unit mass on a certain coil spring under the action of an external force of the form F(t)=30 *cos wt* is $\frac{d^2x}{dt^2}+a\frac{dx}{dt}+24x=30\,cos\,wt$. where $a \geq 0$ is the domping coefficient.

(i) Graph the resonance curves of the system for a = 0, 2, 4, 6 and $4\sqrt{3}$.

(ii) If a = 4, find the resonance frequency and determine the amplitude of the steady-state vibration when the forcing function is in resonance with the system.

(iii) Proceed as in port (ii) if a = 2.

(d) A circult has m series an electromotive force by E=100sin40t V, a resistor of 10Ω and an inductor of 0.5H. If the initial current is 0, find the current at time t > 0. Also interpret the result graphically.

(e) Consider the population of a country. Assume constent per capita birth and death rates and that the population follows an exponential growth (or decay) process.

Assume there to be signiicant immigration and emigration of people into and out of the country. Give clear explanations of how the differential equations are obtained from the word equations :

(i) Assuming the overall immigration and emigration rates are constant, formulate a single differential equation to describe the population size over time.

(ii) Suppose instead that all immigration and emigration occurs with a neighbouring country, such that the net movement from one country to the other is proportional to the population difference between the two countries and such that people move to the country with the larger population. Formulate a coupled system of equations as a model for this situation.

(f) A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subscequent time.

Group-B

2. Answer any ***two*** questions : 2×2=4

(a) State Hooke's law.

(b) What do you mean by damping force?

(c) Briefly describe the resonance phenomena of a system.

(d) Write Kirchhoff's Voltage law.