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Theories of Investment: A Theoretical Review with Empirical Applications

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Theories of Investment: A Theoretical Review with Empirical Applications

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Abstract

This paper review theories of investment and their empirical applications. Starting with the basic profit maximization problem of the firm, the neoclassical, accelerator, Tobin's q theories are derived with the use of dynamic optimization. This illustrates how the various theories of investments differ, and in particular the underlying differences in assumptions are illuminated. Moreover, empirical applications are reviewed and a particular emphasis is put on how to measure Tobin's marginal q.

Keywords: investment theory, accelerator principle, marginal q, Tobin's Q, allocation of capital, dynamic optimization.

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1 Introduction[1]

This paper review theories of investment and their empirical applications. Starting with the basic profit maximization problem of the firm, the neoclassical, accelerator, Tobin's q theories are derived with the use of dynamic optimization. This illustrates how the various theories of investments differ, and in particular the underlying differences in assumptions are illuminated. Moreover, empirical applications are reviewed and a particular emphasis is put on how to measure Tobin's marginal q.

2 Theories of investment

John M. Keynes and Irving Fisher, both argued that investments are made until the present value of expected future revenues, at the margin, is equal to the opportunity cost of capital. This means that investments are made until the net present value is equal to zero. An investment is expected to generate a stream of future cash flows, $C(t)$. Since investment, I, represents an outlay at time 0, this can be expressed as a negative cash flow, $-C_0$. The net present value can then be written as:

$$NPV = -C_0 + \int_0^{\infty} C(t)e^{(g-r)t}dt \tag{1}$$

where g denotes growth rate and r the opportunity cost of capital (discount rate). As long as the expected return on investment, i, is above the opportunity cost of capital, r, investment will be worthwhile. When $r = i$ the $NPV = 0$. The return on investment, i, is equivalent to Keynes' *marginal efficiency of capital* and Fisher's *internal rate of return*. From equation (1) the PV of an investment, I, can be written as $C_1/(r-g)$, implying that $PV/I = 1$.

Fisher referred to the discount rate as the *rate of return over costs* or the *internal rate of return*. Keynes, on the other hand, called it the *marginal efficiency of capital*, (Baddeley, 2003, and Alchian, 1955). Keynes (1936) argued that investments are made until *"there is no longer any class of capital assets of which the marginal efficiency exceeds the current rate of interest"* (as quoted in Baddeley, 2003, p. 34). The fundamental difference between the "Keynesian view" and Fisher ("Hayekian view") lies in the perception of risk and uncertainty, and how expectations are formed. Keynes did not regard investment as an adjustment process toward equilibrium. Hayek (1941) and Fisher (1930), on the other hand, regarded investment as an optimal adjustment path towards an optimal capital stock. In the Keynesian theory investment are not determined by some underlying optimal capital stock.[2] Instead genuine or *radical* uncertainty takes a central position. Keynes believed that humans were *"animal spirited"* and that this, combined with irrational and volatile expectations, made the thought of investment as an adjustment process toward equilibrium futile.

[1] Financial Support from Marianne and Marcus Wallenberg Foundation is greatfully acknowledged. Valuable comments on this manuscrips has been given by Åke E. Andersson, Per-Olof Bjuggren and Börje Johansson.

[2] Keynes (1936) and many economists after him argue that the crucial issue is how individuals form expectations. In a world of "Knightsian" uncertainty probabilities of alternative outcomes cannot be calculated. According to some economists this leads to erratic shifts in expectations which render the notion of an optimal capital stock meaningless. For a discussion of expectations, the efficient market hypothesis, and its implications for investment theory, see section 4.4 and in particular note 21.

From Keynes and Fisher modern investment theories have emerged, incorporating various aspects of Keynes and Fisher. The net present value rule for investment has become a standard component of corporate finance. Jorgenson's (1963) neoclassical theory of investment basically formalizes ideas put forward by Fisher. Keynes' work on subjective probabilities foreshadowed modern probabilistic approaches, such as Markowitz (1952), which has led to the emergence of a very large literature on portfolio choice. Arguably, Keynes has also influenced the so-called accelerator theory of investment, known for its applications to business cycles by Samuelsson (1939a and b). Clearly, Keynes also inspired Tobin and Brainard in their development of Tobin's *Q* (Brainard and Tobin, 1968, and Tobin, 1969) to incorporate expectations. The methodology to measure marginal *q* developed by Mueller and Reardon (1993) also belongs to this line of thought.

2.1 NEOCLASSICAL THEORY OF INVESTMENT

In this section we derive the relationship between the *neoclassical theory*, *accelerator principle* and *Tobin's Q-theory* of investment. All three theories assume optimization behavior on behalf of the decision maker (investor). The *neoclassical* and *Tobin's* theory of investment explicitly assumes profit/value maximization. The *accelerator* theory of investment assumes this implicitly, by assuming that investment is determined by an optimal capital stock.

The starting point for Jorgenson's (1963, 1967 and 1971) neoclassical investment theory is the optimization problem of a firm. Maximizing profits in each period will yield an optimal capital stock. Assuming that the production function can be written as a conventional Cobb-Douglas function[3]:

$$Y(t) = f\big(K(t), L(t)\big) = AK^{\alpha}L^{1-\alpha} \qquad (2)$$

where $Y(t)$ is firm output, K is capital and L denotes labor, all in period t. The profit function for a representative firm can then be expressed as follows:

$$\pi(t) = p(t)Y(t) - s(t)I(t) - w(t)L(t) \qquad (3)$$

$\pi(t)$ denotes profit, $p(t)$ is the price of output, $s(t)$ is the price of capital and $w(t)$ is the wage. Assuming profit maximization, the current value of a firm, V(0), can be written as:

$$\begin{aligned} V(0) &= \max E_{\Phi_0}\int_0^{\infty}\pi(t)e^{-rt}dt = \\ &E_{\Phi_0}\int_0^{\infty}\big[p(t)Y(t) - s(t)I(t) - w(t)L(t)\big]e^{-rt}dt \end{aligned} \qquad (4)$$

[3] This assumes so-called putty-putty technology which means that the substitutability between capital and labor is complete. For a discussion on these so-called of putty-clay and clay-clay models where the substation between the production factors are allowed to vary between zero and one, see Baddeley (2003) and Precious (1987).

s.t. $dK/dt = I(t) - \delta K(t) = \dot{K}(t)$
and $K(0)$ is given.

The term *E* is an expectations operator conditional on the information set, Φ, available for the firm in each period. We leave this aside for now and return to the role of expectations and the efficient market assumption in section 4.4. To avoid clutter and simplify, the time notations are dropped from now on.

To maximize *V*(0) the first step is to set up a Lagrangian:

$$L = V(0) + \int_0^\infty \lambda\left[(I - \delta K) - \dot{K}\right]e^{-rt}\,dt \qquad (5)$$

which gives:

$$L = \int_0^\infty \left[pY - sI - wL + \lambda(I - \delta K) - \lambda\dot{K}\right]e^{-rt}\,dt \qquad (6)$$

From this we obtain the familiar *current value* Hamiltonian[4]:

$$H = pf(K,L) - sI - wL + \lambda(I - \delta K) \qquad (7)$$

where the Lagrangian multiplier $\lambda(t)$ is our costate variable. It should be noted that $\lambda(t)$ represents the shadow price of capital. Differentiating the Hamiltonian, we obtain the following first order conditions:

$$\frac{\partial H}{\partial I} = -s + \lambda = 0 \qquad (8)$$

This condition holds that the opportunity cost of capital shall be equal to the shadow price of capital.

$$\frac{\partial H}{\partial L} = pf_L' - w = 0 \qquad (9)$$

This condition simply says that the labor should be employed until the marginal revenue of labor equates with the wage. Recalling the maximum principle (Intriligator, 1971) we get:

[4] For more details on dynamic optimization and the Hamiltonian, see Intriligator (1971) and Chiang (2000).

$$\frac{\partial H}{\partial \lambda} = \frac{\partial K}{\partial t} = I - \delta K = 0 \qquad (10)$$

which says that in equilibrium net investment should be zero and gross investment equal to the depreciation of *K*. Finally, the marginal condition for capital is:

$$\frac{\partial H}{\partial K} = pf_K' - \lambda\delta = 0 \qquad (11)$$

The *canononical* equation (Intriligator, 1971) requires that $\dot{y} = -\partial H/\partial K$, where *y* is the control variable such that $y = \lambda e^{-rt}$ at time *t*. Thus:

$$-\frac{\partial H}{\partial K} = \frac{d}{dt}\left[e^{-rt}\lambda(t)\right] = \frac{\partial \lambda}{\partial t} - r\lambda \qquad (12)$$

This means that equation (11) can be written as:

$$-pf_K' + \lambda\delta = \frac{\partial \lambda}{\partial t} - r\lambda \qquad (13)$$

From equation (8) we know that $s = \lambda$, which implies that $\partial s/\partial t = \partial \lambda/\partial t$. This also means that $\partial H/\partial K$ can be stated in the following way:

$$pf_K' + s\delta = \frac{\partial s}{\partial t} - rs \qquad (14)$$

Rearranging this we obtain:

$$pf_K' = s\left[\delta + r - (\partial s/\partial t)/s\right] \qquad (15)$$

Since pf_K' is the marginal rate of return on capital, mrr_k, equation (11) can be rewritten as the marginal product of capital:

$$f_K' = s\left[\delta + r - (\partial s/\partial t)/s\right]/p \qquad (16)$$

Note that $f_K' = \partial Y/\partial K$. Jorgenson's (1963) user cost of capital, *c*, is defined as: $s\left[\delta + r - (\partial s/\partial t)/s\right]$, which means that:

$$pf'_K = c \tag{17}$$

This can now be used to derive the optimal capital stock, K^*, and the investment function. Using Cobb-Douglas technology the marginal product of capital becomes:

$$\frac{\partial Y}{\partial K} = f'_K = \alpha K^{\alpha-1} L^{1-\alpha} \tag{18}$$

which in turn can be expressed as:

$$\frac{\partial Y}{\partial K} = \frac{\alpha Y}{K} \tag{19}$$

Multiplying by p_t and recalling equation (17) we get:

$$\frac{\partial H}{\partial K} = p\frac{\alpha Y}{K} = c \tag{20}$$

Solving for *K* we obtain an expression for the optimal capital stock:

$$K^* = \frac{p\alpha Y}{c} \tag{21}$$

It is now easy to see that K^* depends on output, price of output and the user cost of capital, *c*. Thus, investment become the change in capital between two periods:

$$I = \frac{p\alpha Y}{c} - K^*(t-\tau) \tag{22}$$

Note, that this assumes that *K*(*t*) adjusts instantaneously and fully to $K^*(t)$. Assuming that the adjustment to the optimal capital stock is only partial each period this can be incorporated into equation (22) by introducing an adjustment parameter that depends on the difference between actual and desired capital, see e.g. Mueller (2003). Since the neoclassical theory assumes that the capital adjusts immediately and completely to the desired capital stock the investment function is essentially eliminated[5]. It has therefore been suggested that Jorgenson's theory is in fact a capital theory and not an investment theory.

[5] Investments are only defined as a flow over time; in this case it means that investments are the flow between *t*-τ and *t*.

2.2 ACCELERATOR THEORY

The accelerator approach is often associated with a Keynesian approach which is primarily due to its assumption of fixed prices[6]. The acceleration principle was however first suggested by Clark (1917) and is well known for its applications by Samuelsson (1939a and b) to business cycles. The accelerator is, in fact, merely a special case of the neoclassical theory of investment where the price variables have been reduced to constants. If the price of output is assumed to be constant and the price variables *s* and *r* in Jorgenson's (1963) user cost of capital, $\left(c = s\left[\delta + r - (\partial s / \partial t)/s\right]\right)$, are fixed, equation (21) reduces to following:

$$K^* = \alpha Y \tag{23}$$

This is simply the well-known accelerator principle where the desired capital stock is assumed to be proportional to output. Investment in any period will therefore depend on the growth in output:

$$I = \alpha \dot{Y} \tag{24}$$

Given flexible prices and partial adjustment toward the desired capital stock each period investment depend on prices of output and input and interest rates (cost of capital). Vernon Smith (1961) demonstrates what he calls the: *"logical inseparability" of 'marginal efficiency' and the 'accelerator' determinant of investment expenditures".* Smith (1961) used calculus of variation to derive his results[7].

Again, this version of the accelerator assumes a complete and instantaneous adjustment of the capital stock. An alternative is the so-called flexible accelerator that includes lags in the capital stock. Eisner and Strotz (1963) suggest that these lags are because the unit price of capital, *s*(*t*), increases with the adjustment speed, (see also Lucas, 1967). Allowing for lags in the adjustment of the capital stock, however, make the neoclassical theory virtually indistinguishable from the accelerator theory (see Eisner and Strotz, 1963, and Eisner and Nadiri, 1968)[8].

Furthermore, it is worthwhile to note that even though the accelerator principle is often coupled with a Keynesian approach, Keynes himself, as noted above, was very skeptical towards approaches like this. First,

[6] Assuming fixed prices means that the factor substitution elasticity becomes zero, whereas in Jorgenson's neoclassical theory the factor substitution elasticity is one. This issue is addressed in so-called putty-clay models where in the short run the elasticity is zero, but in the long run the elasticity is one. These models are beyond the scope of this dissertation.

[7] It should be noted that Smith (1961) derived this relationship prior to Jorgenson (1963).

[8] The error terms will also become autocorrelated.

Keynes was very critical towards formal models of economic behavior. Second, and more fundamentally, Keynes did not believe that investment is determined as adjustment towards equilibrium[9].

2.3 Q-THEORY OF INVESTMENT

There are two fundamental problems with both the accelerator theory and the neoclassical theory of investment. First, by implication, both theories hold that $K_t^* = K_t$ in each period meaning that the adjustment of the capital stock, to its desired level, is instantaneous and complete each period. The solution to this is to add an adjustment cost function to the optimization problem, (see Gould, 1968, Lucas 1967 and Treadway, 1969). The second problem is that expectations play no role in the neoclassical and accelerator theories. A solution to this problem was offered by Brainard and Tobin (1968) and Tobin (1969): investment is made until the market value of assets is equal to the replacement cost of assets. Furthermore, by adding a marginal adjustment cost function to the profit function the neoclassical theory becomes logically equivalent to the *Q*-theory. The *Q*-theory of investment as suggested by Brainard and Tobin (1968) and Tobin (1969) was, in some ways, foreshadowed by Keynes (1936). Keynes (1936), for example, argued that stock markets will provide guidance to investors and that: *"There is no sense in building up new enterprise at a cost greater than at which an existing one can be purchased,"* (Keynes, 1936, as quoted in Baddeley, 2003, p. 39).

Adding an adjustment cost function to the profit function, the firm value (equation (4)) can be written as:

$$\begin{aligned} V(0) &= \max E_{\Phi_0}\int_0^\infty \pi(t)e^{-rt}dt \\ &= E_{\Phi_0}\int_0^\infty \left[p(t)Y(t) - s(t)I(t) - \vartheta(I(t))s(t)I(t) - w(t)L(t)\right]e^{-rt}dt \end{aligned} \tag{25}$$

where $\vartheta(I(t))$ is the marginal adjustment cost function. Setting up the Hamiltonian and differentiating yield the same marginal conditions for *K*, *L*, and *λ* as before. *Mutatis mutandis*, the *current value* Hamiltonian is written as:

$$H = pf(K,L) - sI - \vartheta(I)sI - wL + \lambda(I - \delta K) \tag{26}$$

As can be easily seen the marginal conditions are all the same as under neoclassical theory with the exception for investment. This condition now reflects the adjustment cost:

$$\frac{\partial H}{\partial I} = -s - \vartheta(I)s - \vartheta'(I)sI + \lambda = 0 \tag{27}$$

[9] If expectations are volatile and humans "animal spirited" this leads to constant shifts in the Keynesian investment demand schedule, which makes the notion of investments as determined by an desired capital stock meaningless since the desired stock of capital keeps shifting before equilibrium is reached.

This can be written:

$$\lambda = s\left[\vartheta(I) + \vartheta'(I)I + 1)\right] \qquad (28)$$

Since λ is the shadow price of capital and s is the cost of one additional unit of capital the quotient λ/s is, in other words, the marginal return on capital relative to the cost of capital. Therefore, dividing by s and defining marginal q as $q_m = \lambda/s$, equation (27) can be written as:

$$q_m = \vartheta(I) + \vartheta'(I)I + 1 \qquad (29)$$

This allows us to define investment as an implicit function of q_m:

$$I = \varphi(q_m) \qquad (30)$$

Differentiating with respect to capital and investment yields a differential equations system.[10] Solving for the optimal capital stock will give the same optimum as under neoclassical theory of investment. The difference is that investment is determined as the optimal adjusted path to the optimal capital stock. In short, the *Q*-theory incorporates all the assumption of the neoclassical theory of investments but puts a restriction on the speed of capital stock adjustment by adding an adjustment cost function. Solving for the optimal capital stock under *Q*-theory of investment will yield the same optimal capital stock as the neoclassical. More interestingly, investment is worthwhile as long as $\lambda/s = q_m > 1$. When $q_m = 1$ there are no more profitable investment opportunities and $K_t = K_t^*$.

Note, the q_m should be interpreted as the *marginal return on capital relative to the opportunity cost of capital*. Marginal *q*, in other words, measures the return on investment relative to the opportunity cost of capital; the quotient λ/s is a *marginal* version of Tobin's *Q*. Typically, Tobin's *Q* is measured as the market-to-book ration, this, however, translates to a measure of the *average return on capital,* which is different from $\lambda/s = q_m$. Hayashi (1982) demonstrates that average *Q* will be equal to marginal *q* only under very restrictive assumption; the firm must be a price taker and the production and installment functions must be homogenous[11]. The methods to measure marginal *q* and average *Q* are discussed in the next section.

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[10] Differentiating q_m gives: $\frac{\partial q_m}{\partial t} = (r+\delta)q_m - \frac{\partial s_t / \partial t}{s_t} - \frac{p_t f'_K}{s_t}$, and since *I* is a function of q_m we can write:

$$\frac{\partial K}{\partial t} = I_t - \delta K_{t-1} = \varphi(q_m) - \delta K_{t-1}.$$

[11] The elasticity of capital with respect to output, which can be derived from the accelerator principle, in equilibrium will also be one. This point is further developed in chapter 2.

2.4 MEASURING TOBIN'S AVERAGE Q AND MARGINAL Q

Tobin's average *Q,* measured as the market-to-book ratio, has become very popular as a measure of investment opportunities. However, there are a number of measurement problems associated with both Tobin's average *Q* and the marginal *q*.

Tobin's average *Q*, Q_a, is defined as the total market value, M_t, divided by the replacement cost of the firm capital at time *t*, K_t:

$$Q_{a,t} = \frac{M_t}{K_t} \tag{31}$$

$Q_{a,t}$ is measured by the total market value of assets, M_t, over the book value of assets. M_t is the market value of debt and equity. In this the numerator and the denominator may both contain measurement errors. To begin, the market value is essentially the expected net present value of all cash flows.

$$M_t = E\left(\int_0^\infty C(t)e^{-rt}dt\right)\Phi_t \tag{32}$$

For listed firms the market value of equity is usually straightforward to obtain. The market value of debt is however typically not available. If the value of the firm is maximized in equation (4) the market value will be equal to the expected value of the future cash flows. However, the market may at any point in time make errors in their valuation of the firm. This can be incorporated into the analysis by adding an error term, μ_t to M_t. If the efficient market hypothesis holds, this means that Φ contains all historical, public and private information relevant for the value of the firm. If this information is discounted into the market evaluation of the firm then $M_t = V_t$. The efficient market hypothesis also holds that $E(\mu_t) = 0$.[12]

The second potential source for measurement error is how to obtain a correct value for the replacement value of the capital stock. The usual solution is to use the accounted book value of the capital. Since this is typically an incorrect measure of the replacement cost of capital the market-to-book value becomes difficult to interpret. It is for example not possible to evaluate performance of firms that have a market-to-book ratio in close proximity to one. Badrinath and Lewellen (1997) argue that the problems of finding accurate

[12] Note that from the efficient market hypothesis we have: $E(\mu_t) = 0$ and $E(\mu_t, \mu_{t-1}) = 0$, and therefore also $E\left(\sum_{j=0}^{n} \mu_{t+j}\right) = 0$. More recent research suggests that in the short run the efficient market hypothesis fails (i.e. Farmer and Geanakoplos, 2008 and Lo, 2004). Casti (2008) argues that once one recognizes the possibility that investors are forming expectation based on assumptions regarding the behavior of other investors, this leads to a world of induction rather than deduction. In computer models of stock markets taking this behavior (i.e. trading based on technical analysis) into account prices have been found to settle down in random fluctuations around its fundamental value. Within these oscillations very complex patterns overshooting, crashes et cetera are found (see Arthur et al., 1996).

measures of the replacement cost of assets makes conventional market-to-book measures flawed and arbitrary.

Q_a measures the average return on the capital over its cost of capital. However, for adjustments of the capital stock the marginal return on capital is more relevant. Marginal *q* measures the marginal return on capital. Marginal *q*, q_m, can be derived from Tobin's average *Q*, see Mueller and Reardon (1993) for the original derivation. The marginal return on capital is then:

$$q_{m,t} = \frac{\Delta M_t}{\Delta K_t} = \frac{M_t - M_{t-1} - \delta M_{t-1}}{K_t - K_{t-1}} \qquad (33)$$

where $-\delta$ is the depreciation rate. For empirical purposes a *multi-period weighted average* of (33) can also be derived:

$$\overline{q}_m = \frac{M_{t+n} - M_{t-1}}{\sum_{j=0}^{n} I_{t+j}} + \frac{\sum_{j=0}^{n} \delta_{t+j} M_{t+j-1}}{\sum_{j=0}^{n} I_{t+j}} - \frac{\sum_{j=0}^{n} \mu_{t+j}}{\sum_{j=0}^{n} I_{t+j}}$$

(34)

Note that it is necessary to assume a depreciation rate in both equation (32) and (33). However, it is also possible to estimate q_m and $-\delta$ simultaneously. Since the market value in period *t* can be written as:

$$M_t = M_{t-1} + PV_t - \delta M_{t-1} + \mu_t \qquad (35)$$

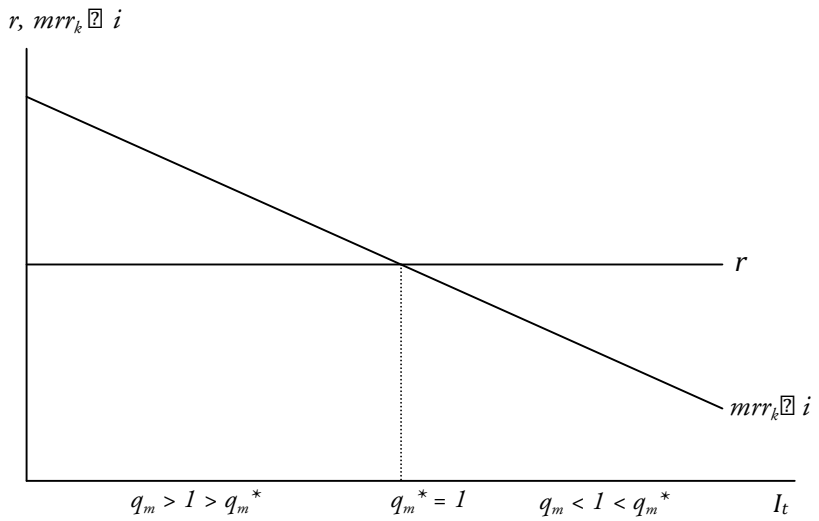
where PV_t is the present value of the cash flows generated by investment in period *t*, and μ_t the standard error term. The net present value rule of investment stipulates that investment should be made up to the point where $PV_t = I_t$. This implies the $PV_t/I_t = 1$, which can be rewritten as $PV_t/I_t = q_m$ (see section 3; equation 1). By dividing both sides of equation (35) by M_{t-1} and rearranging it we get the following equation:

$$\frac{M_t - M_{t-1}}{M_{t-1}} = -\delta + q_m \frac{I_t}{M_{t-1}} + \frac{\mu_t}{M_{t-1}} \qquad (36)$$

This equation can be empirically estimated with actual accounting data and share price information. The equation assumes that the capital market is efficient in the sense that market value is unbiased estimates of future cash flows. As *t* grows larger the term μ_t / M_{t-1} will approach 0. For more details and derivation of marginal *q* from the net present value rule of investment see Eklund (2008).

Marginal q, $q_{m,}$ has also a number of advantages over market-to-book measures of average Q. Above all, a marginal performance measure is more appropriate than an average Tobin's Q, when testing hypotheses about managerial discretion since average measures of performance confuse average and marginal returns (see Gugler and Yurtoglu, 2003). Moreover, q_m has a straightforward interpretation. Not having a correct measure of the replacement cost of assets makes the interpretation of Tobin's Q problematic. In Figure 1, i is the return on investment, r is the cost of capital, I is investment, and $q_m = (i/r)$ is marginal q. If managers invest in a project that yields a return that is less than the cost of capital, $q_m < 1$, which means that managers are over-investing ($q_m < 1$ see Figure 1). That is, the investment has a return less than the cost of capital, which means that the shareholders would have been better off if the firm instead had distributed these funds directly to them. For the firm to maximize shareholder-value, q_m must be equal to one. Conversely, if $q_m > 1$ managers are not investing enough. This means that the marginal investment has a return in excess of the cost of capital and that the firm should have invested more ($q_m > 1$ in Figure 1).

Figure 1 Return on investment, cost of capital and marginal *q*



The drawback of marginal q is that it if the market fails to value correctly a firm in one period this may lead to re-evaluations in subsequent periods. This means that the error component, μ, contains a potentially large re-evaluation factor. However, as the number of annual observations increase one should expect this component to approach zero. This also means that the single period version of q_m may contain relatively large valuation errors which makes it less appropriate as performance measure or control for investment opportunities.

Another advantage of marginal q over Tobin's Q and other performance measures is that it reduces the endogenity problems. For more details see Gugler and Yutrtoglu (2003) Naturally, one problem with both marginal q and average Q is that, in most cases, it is not possible to obtain market values for unlisted firms.

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