Prove that P.E = S.E + I.E.

$$L = f(q_1, q_2) + \lambda(y^0 - P_1 q_1 - P_2 q_2) \quad \text{——— (1)}$$

$$\frac{\delta L}{\delta q_1} = f_1 - \lambda P_1 = 0 \quad \text{——— (2)}$$

$$\frac{\delta L}{\delta q_2} = f_2 - \lambda P_2 = 0 \quad \text{——— (3)}$$

$$\frac{\delta L}{\delta \lambda} = y^0 - P_1 q_1 - P_2 q_2 = 0 \quad \text{——— (4)}$$

Total diff. eqn (2), (3) and (4), we get,

$$f_{11} dq_1 + f_{12} dq_2 - P_1 d\lambda - \lambda dP_1 = 0$$

$$\Rightarrow \cancel{f_{11} dq_1 + f_{12} dq_2} - P_1 d\lambda = \lambda dP_1 \quad \text{——— (5)}$$

$$f_{21} dq_1 + f_{22} dq_2 - P_2 d\lambda - \lambda dP_2 = 0$$

$$\Rightarrow f_{21} dq_1 + f_{22} dq_2 - P_2 d\lambda = \lambda dP_2 \quad \text{——— (6)}$$

$$dy^0 - P_1 dq_1 - q_1 dP_1 - P_2 dq_2 - q_2 dP_2 = 0$$

$$\Rightarrow -P_1 dq_1 - P_2 dq_2 = -dy^0 + q_1 dP_1 + q_2 dP_2 \quad \text{— (7)}$$

from (5), (6) and (7), we get.

$$D = \begin{vmatrix} f_{11} & f_{12} & -P_1 \\ f_{21} & f_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix} > 0.$$

dq_1
dq_2
$d\lambda$

$$D_1 = \begin{vmatrix} \lambda dP_1 & f_{12} & -P_1 \\ \lambda dP_2 & f_{22} & -P_2 \\ -dy^0 + q_1 dP_1 + q_2 dP_2 & -P_2 & 0 \end{vmatrix}$$

$$|D_1| = \lambda dP_1 \begin{vmatrix} f_{22} & -P_2 \\ -P_2 & 0 \end{vmatrix} - \lambda dP_2 \begin{vmatrix} f_{12} & -P_1 \\ -P_2 & 0 \end{vmatrix} + (-dy^0 + q_1 dP_1 + q_2 dP_2) \begin{vmatrix} f_{12} & -P_1 \\ f_{22} & -P_2 \end{vmatrix}$$

$$= \lambda dP_1 D_{11} - \lambda dP_2 D_{21} + (-dy^0 + q_1 dP_1 + q_2 dP_2) D_{31}$$

$$dq_1 = \frac{D_1}{D} = \frac{\lambda dP_1 D_{11} + \lambda dP_2 D_{21} + (-dy^0 + q_1 dP_1 + q_2 dP_2) D_{31}}{D} \quad \text{——— (8)}$$

Dividing both side by dP_1 and assume, $dy^\circ = 0$, $dP_2 = 0$

$$\frac{\delta q_1}{\delta P_1} = \frac{\lambda D_{11} + 0 + D_{31}(0 + q_1 + 0)}{D}$$

$$\frac{\delta q_1}{\delta P_1} = \frac{\lambda D_{11}}{D} + \frac{q_1 D_{31}}{D} \quad \text{—— (9)}$$

$P.E = S.E + I.E$.

Now we know that $\frac{\delta q_1}{\delta P_1} = P.E$. Now we will prove that -

$\frac{\lambda D_{11}}{D}$ is S.E and $\frac{q_1 D_{31}}{D}$ is I.E.

Now, to find the I.E we assume that -

assume $dP_1 = 0$, $dP_2 = 0$

from eqn (8)

$$dq_1 = \frac{0 + 0 + D_{31}(-dy + 0 + 0)}{D}$$

$$= \frac{-D_{31}\,Dy}{D}$$

$$\therefore I.E = \frac{dq_1}{dy} = -\frac{D_{31}}{D} \quad \text{—— (10)}$$

$\therefore$ I.E is (−) (with respect to price change) for normal goods.

Consider a price change that is compensated by an income change such that, $dU = 0$ (constant utility on same IC)

Since, $\frac{f_1}{f_2} = \frac{P_1}{P_2}$

we can write,

$$P_1 dq_1 + P_2 dq_2 = 0$$

So from eqn (7)

$$-dy + q_1 dP_1 + q_2 dP_2 = 0$$

From eqn (8),

$$S.E = \left(\frac{\delta q_1}{\delta P_1}\right)_{u = const.} = \frac{\lambda D_{11}}{D} \cdot = (-) \text{ (always } (-))$$

$$\left(\frac{\delta q_1}{\delta P_1}\right)_{\substack{P_2,\, y \\ \text{const.}}} = \left(\frac{\delta q_1}{\delta P_1}\right)_{U=\text{const.}} - q_1\left(\frac{\delta q_1}{\delta y}\right)_{P_1 P_2 = \text{const.}}$$

$$P.E = S.E + (-q_1) I.E$$

Since $D > 0$, $\lambda > 0$ and $D_{11} = -P_2^2 < 0$

$\therefore$ Substitution effect is always $-$ve. —— (i)

For normal goods I.E is $+$ve

$\therefore$ P.E is $-$ve for normal goods.

~~For inferior goods ...~~

For inferior goods I.E is $+$ve (With respect to price change)

$\therefore$ P.E may be $+$ve or $-$ve, if $S.E > I.E$ then $P.E < 0$ ($-$ve)

if $S.E < I.E$ then $P.E > 0$ ($+$ve) for griffen goods.

So, all ~~goods~~ Griffen goods are inferior goods but all inferior goods are not Griffen goods.

Given $U = (x+2)(y+1)$, $P_x = 4$, $P_y = 6$, $B = 130$.

Write the Lagrangean function for utility maximisation

Find the optimal levels of x and y

Is the 2nd order sufficient condition for maximum

... give any comparat

(Sai)

Date

$$\frac{\delta V}{\delta L} = f_L - \lambda w - 0 = 0 \rightarrow \lambda = \frac{f_L}{w} \quad — (2)$$

$$\frac{\delta V}{\delta K} = f_K - \lambda r = 0 \rightarrow \lambda = \frac{f_K}{r} \quad — (3)$$

$$\frac{\delta V}{\delta L} = C - wL - rK = 0 \quad — (4)$$

Solving eq^n (2) & (3)

$$\frac{f_L}{w} = \frac{f_K}{r}$$

$$\frac{f_L}{f_K} = \frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow MRTS_{LK} = \frac{w}{r}$$

Slope of isoquant = slope of isocost line.

$\therefore$ The 1st order $cond^n$ for output maximisation.

Total differentiation eq^n (2), (3) & (4)

$$f_{LL}d_L + f_{LK}dK - \lambda dw - w d\lambda = 0$$

$$\Rightarrow f_{LL}d_L + f_{LK}dK - w d\lambda = \lambda dw \quad — (5)$$

$$f_{KL}d_L + f_{KK}dK - \lambda dr - r d\lambda = 0$$

$$\Rightarrow f_{KL}d_L + f_{KK}d_K - r d\lambda = \lambda dr \quad — (6)$$

$$dC - w dL - L dw - r dK - K dr = 0$$

$$\Rightarrow -w dL - r dK = -dC + L dw + K dr = 0 \quad — (7)$$

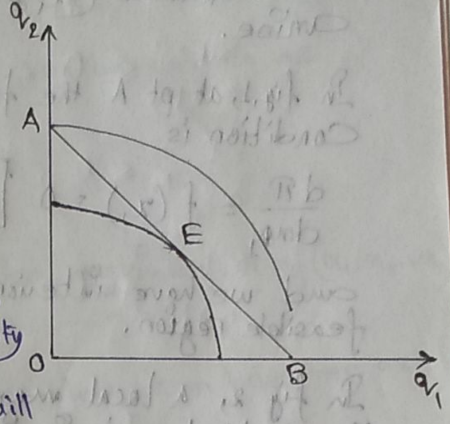
Solving 5, 6 & 7

$$\begin{vmatrix} f_{LL} & f_{LK} & -w \\ f_{KL} & f_{KK} & -r \\ -w & -r & 0 \end{vmatrix} > 0,$$ this means isoquant is convex to origin.

$\therefore$ 2nd order $cond^n$ for output max^m.

* ~~Contrainet~~ Optimisation with Unequal Constant

The first order condition for constrained optimisation with equal constant are not always necessary for maximum. If the indifference curves are concave rather than convex (assumptions of the quasi-concave utility function is violated) the indifference curves are bowed away from the origin. In this case the MRS is decreasing.



The first order condition for maximum is satisfied at pt. E, but the 2nd order condition is not satisfied, therefore this pt. represent a local utility minimum, and the consumer can increase its utility by moving from point E to point A (from the pt. of tangency towards eighter axis). Consumer consume only one commodity at the optimum. If he spends all his income on one commodity he will buy ~~one~~ only q_1 or only q_2 depending upon wheather, $f\left(\frac{y_0}{P_1}, 0\right) > f\left(0, \frac{y_0}{P_2}\right)$ or $f\left(\frac{y_0}{P_1}, 0\right) < f\left(0, \frac{y_0}{P_2}\right)$

In this example he will buy one q_2.

// Non-Linear Programming and Kuhn-Tucker condition.

In the classical optimisation problem, with no explicit restrictions on the signs of the choice variables, and with no inequalities in the contraint. In non-Linear programming there exists a first order condition (like Lagrangean function), known as Kuhn-Tucker conditions. The kuhn-Tucker conditions cannot be accorded to the status of necessary conditions unless a certain proviso is satisfied. On the other hand under certain specific ~~substances~~ circumstances, the condition turn out to the sufficient conditions or even necessary and sufficient conditions as well

~~Constrained Optimisation with~~

Optimisation with single variable case.

Let we have to maximise, $\pi = f(x_1)$

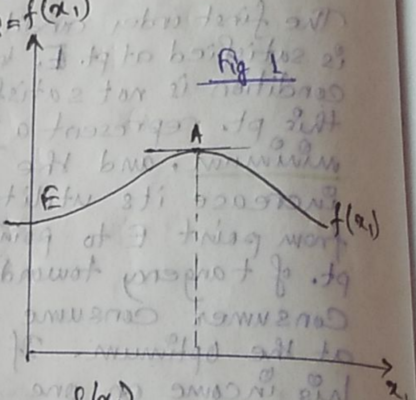
Subject to, $x_1 \geqslant 0$

The function f is assumed to be differenciable, in view of the restriction $(x_1 \geqslant 0)$. Now 3 possible situations may arise.

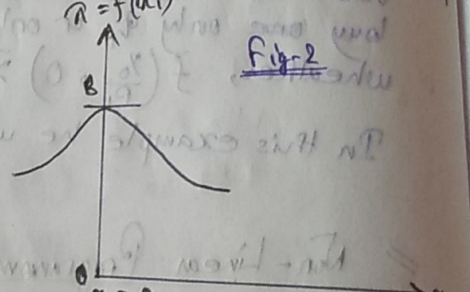
In fig 1, at pt A the first order condition is

$$\frac{d\pi}{dx_1} = f'(x_1) = 0 \quad \left[\text{where, } x_1 > 0\right]$$

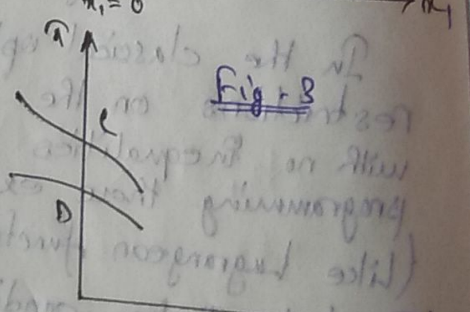
and we have interior solution in feasible region.



In fig 2, a local maximum pt. on the vertical axis is B. [where, $x_1 = 0$]



In fig 3, at pt C or D, the local maximum may present but the first order condition in that case is $f'(x_1) < 0$ [where is $x_1 = 0$].



→ Mathematical

In order to find the Local maximum of π for a value of x_1, it must satisfy one of the 3 possible condition –

i) $f'(x_1) = 0$, for $x_1 > 0$

ii) $f'(x_1) = 0$, for $x_1 = 0$

iii) $f'(x_1) < 0$, for $x_1 = 0$

In general, the optimum solution can be written as

$$f'(x_1) \leq 0 \quad \text{for } x_1 \geq 0.$$

and $x_1 f'(x_1) = 0$

⇒ When the problem contains n- variables, maximise $\pi = f(x_1, x_2 \ldots x_n)$
subject to. $x_j \geq 0 \quad (j = 1, 2, \ldots n)$

The first order condition for this problem can be written as
$f_j \leq 0$ and $x_j f_j = 0$, where $f_j = \frac{\Delta\pi}{\Delta x_j}$

Problem: Problems with 3 choice variables and 2 constant

Q. Maximise, $\pi = f(x_1, x_2, x_3)$
subject to, $g^1(x_1, x_2, x_3) \leq r_1$
$g^2(x_1, x_2, x_3) \leq r_2$
and, $x_1, x_2, x_3 \geq 0$

Sol. With 2 dummy variables, S_1 & S_2, we can tranfer the inequality into equan form.

$$g^1(x_1, x_2, x_3) + S_1 = r_1$$
$$g^2(x_1, x_2, x_2) + S_2 = r_2$$
and, $x_1, x_2, x_3, S_1, S_2 \geq 0$

Now, using Lagrangean function,

$$L = f(x_1, x_2, x_3) + \lambda_1 \{r_1 - g^1(x_1, x_2, x_3) - S_1\}$$
$$+ \lambda_2 \{r_2 - g^2(x_1, x_2, x_3) - S_2\}$$

1st order condition

$$\frac{\delta L}{\delta x_1} = \frac{\delta L}{\delta x_2} = \frac{\delta L}{\delta x_3} = \frac{\delta L}{\delta S_1} = \frac{\delta L}{\delta S_2} = \frac{\delta L}{\delta \lambda_1} = \frac{\delta L}{\delta \lambda_2} = 0$$

Since, the x_j $(j = 1, 2, 3)$ and S_i $(i = 1, 2)$ ~~and~~ ~~non-negative~~ are non-negative. The 1st order condition for this inequality problem can be written as

$$\frac{\delta L}{\delta x_j} \leq 0 \text{, and } x_j \frac{\delta L}{\delta x_j} = 0$$

$$\frac{\delta L}{\delta S_i} \leq 0 \text{, and } S_i \frac{\delta L}{\delta S_i} = 0$$

$$\frac{\delta L}{\delta \lambda_i} = 0$$

$$\frac{\delta L}{\delta x_j} = f_j - \lambda_1 g_j^1 - \lambda_2 g_j^2 \leq 0 \text{ and } x_j \frac{\delta L}{\delta x_j} = 0$$

$$\frac{\delta L}{\delta S_i} = -\lambda_1 - \lambda_2 \leq 0 \text{, and } S_i \frac{\delta L}{\delta S_i} = 0$$

Now,

$$\frac{\delta L}{\delta S_1} = -\lambda_1 \leq 0$$

$$\therefore \lambda_1 \geq 0$$

Similarly, $\lambda_2 \geq 0$

Now,

$$\frac{\delta L}{\delta \lambda_1} = r_1 - g^1(x_1, x_2, x_3) - S_1 = 0$$

$$= r_1 - g^1(x_1, x_2, x_3) \geq 0$$

or $\frac{\delta L}{\delta \lambda_2} = r_2 - g^2(x_1, x_2, x_3) \geq 0$

This enables us to express the 1st order condition without dummy variable. So we now write 1st order condition as —

Saathi

Date 1/4/19

Interpretation of Kuhn-Tucker condⁿ:

profit maximisation

In the above problem we have

$f_j \equiv$ Marginal gross profit of the j^{th} product

$\lambda_i \equiv$ shadow price of the i^{th} resources (the opportunity cost of using a unit i^{th} resource)

g_j^i = amt. of i^{th} resource used up in producing the marginal unit of j^{th} product.

$\lambda_i g_j^i \equiv$ Marginal imputed cost of i^{th} resources for production of a unit of j^{th} product.

$\sum_i \lambda_i g_j^i \equiv$ Aggregate marginal imputed cost of j^{th} product

$$\frac{\delta L}{\delta x_j} = f_j - \sum_i \lambda_i g_j^i \leq 0 \quad \forall j$$

Thus, the marginal condition requires the marginal gross profit of the j^{th} product be no greater than its aggregate marginal imputed cost.

The marginal condⁿ $\frac{\delta L}{\delta \lambda_i} \geq 0$ requires the firm to stay within the capacity limitation of every resource in the plant.

The complementary ~~slag~~ slackness condⁿ is that, if the i^{th} resource is not fully used in the optimum solⁿ ($\delta L/\delta \lambda_i > 0$), the shadow price of that resource must be equal to zero ($\lambda_i = 0$). On the other hand if a resource has a +ve

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