বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

B.Sc. Honours Examination 2021

(CBCS)

1st Semester

MATHEMATICS

PAPER—C1T

CALCULUS , GEOMETRY AND DIFFERENTIAL EQUATION

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *four* questions. 4×12

1. (a) Find the equation of the asymptotes of the curve

$$r^n f_n(\theta) + r^{n-1} f_{n-1}(\theta) + ... + f_0(\theta) = 0$$

(b) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \, Sinnx \, dx$ show that

$2(n - 1) I_n = 1 + (n - 2) I_{n-1}$ and hence deduce

$I_n = \frac{1}{n-1}$ 5+5+2

2. (a) Circles are described on the double ordinates of the parabola $y^2 = 4ax$ as diameters. Prove that the envelope is the parabola $y^2 = 4a(x + a)$.

(b) If $y = \sin\left(m\cos^{-1}\sqrt{x}\right)$ then prove that $\lim_{x\to 0}\frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n+2}$.

(c) Find a,b,c such that $\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} \to 2 \; as \; x \to 0$. 4+4+4

3. (a) Show that the arc of the upper half of the cardiode $r = a(1 - \cos\theta)$ is bisected at $\theta = \frac{2}{3}\pi$. Find also the perimeter of the curve.

(b) Show that the curve $re^\theta = a(1+\theta)$ has no point of inflexion.

(c) Find the asymptotes of the parametric curve $x = \frac{t^2+1}{t^2-1}$ and $y = \frac{t^2}{t-1}$.

4. (a) Show that feet of the normals from the point (α, β, ν) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2 - c^2)}{x} + \frac{\beta b^2(c^2 - a^2)}{y} + \frac{\nu c^2\left(a^2 - b^2\right)}{z} = 0.$$

(b) Find the equation of the right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$ and radius is 2. 7+5

5. (a) Prove that cosh(x + y) = coshx coshy + sinhx sinhy.

(b) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of their common circle is $\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}$.

(c) Find the polar equation of the normal to the conic $\frac{1}{r}=1+e\cos\theta, e>0$.

2+5+5

6. (a) Find the equation of the generator of the cone $x^2 + y^2 = z^2$ through the point (3, 4, 5).

(b) Given that the asteroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ is the envelope of the family of ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that a + b = c.

(c) State the existence and uniqueness theorem for the solution of ordinary differential equation. 4+4+4

7. (a) Solve : $x\frac{dy}{dx}-y=x\sqrt{x^2+y^2}$.

(b) If m and n are positive integers, show that

$$\int_a^b (x-a)^m (\mathrm{b}-x)^n\,dx=\frac{m!n!}{(m+n+1)!}(b-a)^{m+n+1}$$

(c) Solve $y = 2px + y^2p^3$ and find the general and singular solutions.

3+4+5

8\. (a) Compute the length of the curve $x = 2\cos\theta, y = \sin 2\theta, 0 \le \theta \le \pi$.

(b) Find the points of inflection on the curve $r\left(\theta^2 - 1\right) = a\theta^2$

(c) If $I_n = \int_0^1 x^n \tan^{-1} x dx, n$ beine positive integer greater than 2, prove that

$$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$

3+3+6

Answer any *six* questions. 6×2

9. Find the value of $\lim_{x\to\infty}\left[a_0x^m + a_1x^{m-1} + \ldots + a_m\right]^{1/x}$, in being a positive integer and $a_0 \ne 0$.

10. Let $I_n = \int_0^1 (lnx)^h \, dx$. Show that $I_n = (-1)^n \lfloor\underline{n}$, n being positive integer.

11. The curves $y = x^n$, $y^m = x$ (m, n > 0) meet at (0, 0) and (1, 1). Find the area between these two curves.

12. Find α if x^α be an integrating factor of $\left(x - y^2\right)dx + 2xy\, dy = 0$.

13. Find the curve for which the curvature is zero at every point and which passes through the point (0, 0) where $\frac{dy}{dx} = 3/2$.

14. Solve the differential equation :

$$4x^3ydx + (x^4 + y^4)dy = 0 \cdot$$

15. Generate a reduction formula for $\int \tan^n x\, dx$, $n \in Z^+$ and n > 1.

16. Find the equations of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$.

17. Find the asymptote (if any) of the curve y = a log $\left[\sec\left(\frac{x}{a}\right)\right]$.

18. On the ellipse $r(5 - 2\cos\theta) = 21$, find the point with the greatest radius vector.

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