বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2021

(Under CBCS Pattern)

Semester - III

Subject : MATHEMATICS

Paper : DSC 1C / 2C / 3C - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[REAL ANALYSIS]

(Theory)

Group-A

Answer any ***four*** of the following questions : 12×4=48

1. (a) Prove that Union of two countable sets is countable. 6

(b) If a power series $\sum_{n=0}^{\infty} a_n x_0^n$ converges for $x = x_0 \neq 0$ then prove that the power series is absolutely convergent for all x when $|x| < |x_0|$. 6

2. (a) Let $S=\left\{m+\frac{1}{n}: m\in N,\ n\in N\right\}$. Find the derivative set of *S*.

(b) Prove that $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+.....+\frac{1}{\sqrt{n^2+n}}\right)=1$.

(c) Prove that a Canchy sequence of real number is convergent. 4+4+4

3. (a) Prove that an absolutely convergent series is convergent.

(b) Test the convergent of the series $1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1.3}{2.4}\cdot\frac{1}{5}+\frac{1.3.5}{2.4.6}\cdot\frac{1}{7}+.......$

(c) Test the convergence of the series $\sum_{1}^{\infty}\frac{1}{n}$. 4+4+4

4. (a) Let $D\subset R$ and for each $n\in N, f_n: D\to R$ is continuous on *D*. If the sequence $\{f_n\}$ be uniformly convergent on *D* to a function *f*, then *f* is continuous on *D*.

(b) If $f_n = x^2e^{-nx}, x\in [0,\infty)$. Then show that the sequence $\{f_n\}$ is uniformly convergent on [0, 1]. 6+6

5. (a) State Abel's theorem. Assume the expansion $\log(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-....$ for $-1\le x\le 1$. Prove that $\int_0^1\log(1-x)\,dx=-1$. 2+4

(b) Find the radius of convergence of the power series $\sum_{n=0}^{\infty}a_nx^n$, where

$$a_n=\begin{cases}\left(\frac{1}{3}\right)^n, & \text{if } n \text{ be odd}\\ \left(\frac{1}{2}\right)^n, & \text{if } n \text{ be even}.\end{cases}$$ 6

6. (a) Show that a finite set has no limit point.

(b) Prove that $\lim n^{\frac{1}{n}} = 1$.

(c) Show that the sequence $\{f_n\}$ converge to a function f for all real x where $f_n(x) = \frac{1}{n^3 + n^4 x^2}$. 4+4+4

7. (a) Find Sup A and in fA where $A = \{x \in R : 3x^2 + 8x - 3\}$.

(b) Show that there does not exist a rational number r such that $r^2 = 5$.

(c) Prove that the union of two closed sets in R is a closed set. 4+4+4

8. (a) Show that the sequence of functions $\{\tan^{-1} nx\}_n, x \geq 0$ is uniformly convergent in any interval [a, b] but is only pointwise convergent in [0, b].

(b) State and prove Cauchy's General Principle of convergence. 6+6

Group-B

Answer any ***six*** of the following questions : 2×6=12

1. Prove that for all $n \geq 2, (n+1)! > 2^n$.

2. Find the value/values of x for which the power series $\sum_{n=0}^{\infty} n!\, x^n$ converges.

3. Determine the domain of the real function $f(x) = \sqrt{2 + x - x^2}$.

4. If $x_n = 2 + (-1)^n 2^{-n}$, then show by definition that the sequence $\{x_n\}$ is convergent.

5. Show that the function defined by $f(x) = \frac{1}{x}, x \in [1, \infty)$ is uniformly continuous on $[1, \infty)$.

6. Give an example of the open cover of the set (0, 1] which does not have a finite sub cover.

7. Examine if the set S is closed in R, where $S=\left\{x\in R:\sin\frac{1}{x}=0\right\}$.

8. Examine the convergence of the series $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\left(\frac{4}{9}\right)^4+\ldots\ldots+\ldots\ldots$

9. Find the radius of convergence of the power series $\sum\left(\sqrt[n]{n}+1\right)^n.x^n$

10. Find $\lim_{x\to 0}\sum_{n=1}^{\infty}\frac{\cos nx}{n(n+1)}$.