Study Material - Sem. 1 - C2T

- Special Theory of Relativity -

- Dr. T. Kar - Class 3

Addition of Velocities

In the classical or the Galilean mechanics, if a car moves with a velocity $\vec{v}$ with respect to the ground and an insect in the car moves with velocity $\vec{u}'$, then the velocity of the insect w.r.t the ground $\vec{u}$ is given by the vector sum of $\vec{u}'$ and $\vec{v}$. That is,

$$\vec{u} = \vec{u}' + \vec{v} \longrightarrow (1)$$

We shall consider here the relativistic addition of velocities. Let us suppose that the velocities are along the common x- (or x'-) direction of two inertial frames S and S'. Let the S'-frame moves relative to the S-frame with velocity v. Let the velocity of a body (B) be u' in the S'-frame. Its position in the S'-frame is given by $x' = u't'$. In our example, S is the ground frame, and S' is the frame of the car and the body B is the insect. We have from Lorentz transformation equations —

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} = u't' \longrightarrow (2)$$

and $$t' = \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \longrightarrow (3)$$

Then eq. (2) becomes —

$$\frac{x - vt}{\sqrt{1 - v^2/c^2}} = u' \left[\frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \right]$$

or, $x - vt = u' \left[t - \left(\frac{vx}{c^2} \right) \right]$

or, $x = u't + vt - \frac{u' v x}{c^2}$

or, $x + \left(\frac{u'v}{c^2} \right) x = (u' + v) t$

or, $x \left[1 + \frac{u'v}{c^2} \right] = (u' + v) t$

or, $x = \frac{(u' + v) t}{1 + \left(\frac{u'v}{c^2} \right)} \longrightarrow (4)$

The position of the body B in S-frame at time t is given by $x = ut$, where u is the velocity of the body in S-frame. Therefore —

$$ut = \frac{(u' + v) t}{1 + \left(\frac{u'v}{c^2} \right)} \quad \text{or,} \quad u = \frac{u' + v}{1 + \left(\frac{u'v}{c^2} \right)} \longrightarrow (5)$$

This result is known as Einstein's velocity addition theorem.

When u' and v are small compared to c, eq. (5) gives the classical result $u = u' + v$. An important result of the Lorentz transformation is that the speeds of the signals or material objects cannot exceed c, the speed of light. For, if $u > c$, the factor $\sqrt{1-u^2/c^2}$ is imaginary, thus making the time coordinate (t') and the space coordinate (x') imaginary. Also the relativistic velocity addition theorem shows that it is not possible to obtain a velocity greater than that of light by combining two velocities even if each velocity is very close to c. It is obvious from eq. 5 that even u' and v approach c, u approaches c but never exceeds c. The velocity addition theorem shows that the velocity of light is same in all inertial frames, which is in fact, a basic postulate of relativity. For, if $u'=c$, eq. 5 shows that $u=c$, whatever be the value of v.

So far we have discussed the velocity parallel to the direction of relative motion of the two reference frame S and S'. Specifically, the direction was taken to be the x- or x'- direction. To keep it in mind, it is instructive to put a subscript x to u and u'. Equation 5 then gives —

$$u_x = \frac{u'_x + v}{1+(u'_x v/c^2)} \longrightarrow 6$$

We shall now discuss the transformation of velocities that are perpendicular to the direction of the relative motion of S and S' frames. The velocity of a body in the y-direction in the S-frame is

$$u_y = \frac{y_2 - y_1}{t_2 - t_1} = \frac{\Delta y}{\Delta t}$$ where y_2 and y_1 are its positions at t_2 and t_1. The velocity in the S'-frame is —

$$u'_y = \frac{y'_2 - y'_1}{t'_2 - t'_1} = \frac{\Delta y'}{\Delta t'}$$

where y'_2 and y'_1 are the position at the corresponding times t'_2 and t'_1 respectively.

In order to obtain the unprimed coordinates in terms of the primed coordinates, we have to interchange S and S'-frames. Relative to S', S-frame moves in the negative x-direction with velocity v.

Example 1. What will be the apparent length of a meter stick measured by an observer at rest when the stick is moving along its length with a velocity c, $\frac{c}{\sqrt{2}}$, $\frac{\sqrt{3}c}{2}$, $\frac{c}{2}$ and 0·8c where c is the velocity of light in space?

Ans. Let l_0 be the length of the meter stick when at rest. Then its apparent length, when in motion with a velocity v and as measured by the observer at rest, is l and is given by —

$$l = l_0 \sqrt{1 - v^2/c^2}, \quad l_0 = 100 \text{ cm}$$

(i) when $v = c$, $\frac{v^2}{c^2} = 1 \Rightarrow l = l_0 \sqrt{1-1} = 0$

(ii) when $v = \frac{c}{\sqrt{2}}$, $\frac{v^2}{c^2} = \frac{1}{2} \Rightarrow l = 70.7$ cm

(iii) when $v = \frac{\sqrt{3}c}{2}$, $\frac{v^2}{c^2} = \frac{3}{4} \Rightarrow l = 50$ cm

(iv) when $v = \frac{c}{2}$, $\frac{v^2}{c^2} = \frac{1}{4} \Rightarrow l = 86.6$ cm

(v) when $v = 0.8c$, $\frac{v^2}{c^2} = 0.64 \Rightarrow l = 60$ cm

Example 2. A rigid bar of length 1.5 m is at rest relative to frame S'. If it makes an angle $\alpha' = 45°$ with the x'-axis, find the length of the bar and its orientation relative to the frame S when $v = 0.95c$?

Ans. Resolving the length of the bar into components parallel to x' and y' axes, the corresponding lengths in S' are —

$$L_x' = L' \cos\alpha = 1.5 \times \frac{1}{\sqrt{2}} \text{ m} = 1.06 \text{ m}$$

$$L_y' = L' \sin\alpha = 1.5 \times \frac{1}{\sqrt{2}} \text{ m} = 1.06 \text{ m}$$

The vertical components being at right angles to v would not show any contraction when viewed from S.

So, $L_y = L'_y = 1.06\,m$

The horizontal component, being parallel to v, suffers contraction, i.e,

$L_x = L'_x \sqrt{1 - v^2/c^2} = 1.06\left(\sqrt{1-(0.95)^2}\right) m$

$= 0.331\,m$

So, The length of The bar relative to S is —

$L = \sqrt{L_x^2 + L_y^2} = 1.11\,m$

The orientation of The bar relative to S is given by —

$\tan\alpha = \frac{L_y}{L_x} = 3.35 \Rightarrow \alpha = 73.40°$

<u>Example 3.</u> What will be The period of The 'seconds' pendulum measured by a observer moving at a speed of $0.80c$?

<u>Ans.</u> By The formula of time dilation, if T_0 be The time period of a 'seconds' pendu--lum as measured by an observer at rest, The same would be T as measured

by a moving observer and is given by —

$$T = \frac{T_0}{\sqrt{1-v^2/c^2}} = \frac{2 \text{ sec}}{\sqrt{1-(0.8)^2}} = 3.33 \text{ sec.}$$

Example 4. What is the mean life time of π^+ mesons moving with a speed $0.73c$, when the proper life time is 2.5×10^{-8} sec.

Given : $c = 3.0 \times 10^{10}$ cm/sec.

Ans. Let t_0 be the proper life time i.e., life time measured by an observer moving with the particle.

∴ The life time t, measured by an observer at rest, is given by —

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}} = \frac{2.5 \times 10^{-8} \text{ sec}}{\sqrt{1-(0.73)^2}} = 3.66 \times 10^{-8} \text{ sec.}$$

Relativistic Mass

Lorentz transformation equations must introduce modifications in the equations of mechanics so that the forms of these equations do not change on transformation from one inertial frame to another moving with respect to the first. The new mechanics, so developed, is called relativistic mechanics. It is desirable to retain as much of classical mechanics into relativistic mechanics as possible. Also, The laws of relativistic mechanics must reduce to Those of classical mechanics when The speeds are much less Than The speed of light, c.

It has been found That if we define The mass m of a particle moving with a velocity v as a suitable function of v, i.e, $m = m(v)$, Then we can preserve Newton's equation of motion in a form similar to The classical form. Thus, if $\vec{p}$ is The momentum of a particle acted on by a force $\vec{F}$, we can write,

$$\vec{F} = \frac{d\vec{p}}{dt} \longrightarrow (1)$$

The momentum $\vec{p}$ of The particle can be expressed in its classical form i.e,

$$\vec{p} = m\vec{v} \longrightarrow (2)$$

Also, it is possible to retain The form of The classical law for The conservation of momentum for The particles in an isolated system. That is, The sum of The initial momenta of all The interacting particles

$\sum \vec{p}_i$ is equal to the sum of the final momenta of all the particles $\sum \vec{p}_f$: $\sum \vec{p}_i = \sum \vec{p}_f \rightarrow (3)$

The velocity dependent mass in compliance with eq. (1) through eq. (3) is given by -

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \rightarrow (4)$$

When $v \ll c$, $m = m_0$, where m_0 is the classically measured mass of the particle. The relativistic mechanics demands that the mass m of a particle moving with a velocity v be larger than its mass m_0 when it is at rest by the factor $\frac{1}{\sqrt{1-v^2/c^2}}$. The mass '$m$' is called the relativistic mass of the particle and m_0 is called its rest mass. In the new mechanics the mass remains a scalar quantity since its value does not depend on the direction of the velocity of the particle. The rest mass 'm_0' is sometimes referred to as the proper mass since it is the mass of the particle measured, like the proper length and the proper time, in the inertial frame in which the particle is at rest.

Derivation

Let us suppose that in system S' two bodies of equal masses m' travelling with velocities u' and $-u'$ parallel to the x-axis collide and after collision coalesce into one body. This is a simplifying assumption but the formula derived with it can be shown to hold for collisions of any type. The definition of momentum of a body in relativity is the same as in classical dynamics, viz. mass × velocity. Further, the principles of conservation of mass and of momentum hold also. Hence, according to the principles of conservation of mass, the mass of the coalesced bodies after collision is equal to $2m'$. And according to the law of conservation of momentum, the coalesced bodies are at rest in S', since their velocities before collision are equal and opposite. Let us now consider how this impact experiment appears to an observer in system S. The velocities u' and $-u'$ transform, according to the law of addition of velocities, into u_1 and u_2 given by —

$$u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}} \quad \text{and} \quad u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}} \longrightarrow (1)$$

Let the mass of the body travelling with velocity u_1 be m_1 and that of the body with velocity u_2 be m_2.

After collision the coalesced bodies travel with the velocity v with respect to S, since they are at rest in S′.

Using again the principle of conservation of momentum which holds for all frames of reference —

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v \longrightarrow ②$$

Substituting the values of u_1 and u_2 given by eq. ① in eq. ②, we obtain —

$$m_1 \left[\frac{u'+v}{1+\frac{u'v}{c^2}}\right] + m_2 \left[\frac{-u'+v}{1-\frac{u'v}{c^2}}\right] = (m_1+m_2)\, v$$

or, $$\frac{m_1 u' + m_1 v}{1+\frac{u'v}{c^2}} + \frac{m_2 v - m_2 u'}{1-\frac{u'v}{c^2}} = m_1 v + m_2 v$$

or, $$\frac{m_1 u' + m_1 v}{1+\frac{u'v}{c^2}} - m_1 v = m_2 v - \frac{m_2 v - m_2 u'}{1-\frac{u'v}{c^2}}$$

or, $$m_1 \left[\frac{u'+v - v\left(1+\frac{u'v}{c^2}\right)}{1+\frac{u'v}{c^2}}\right] = m_2 \left[\frac{v\left(1-\frac{u'v}{c^2}\right) - v + u'}{1-\frac{u'v}{c^2}}\right]$$

or, $$m_1 \left[\frac{u' + \cancel{v} - \cancel{v} - \frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right] = m_2 \left[\frac{\cancel{v} - \frac{u'v^2}{c^2} - \cancel{v} + u'}{1-\frac{u'v}{c^2}}\right]$$

or, $m_1\left[\dfrac{u'\left(1-\frac{v^2}{c^2}\right)}{1+\frac{u'v}{c^2}}\right] = m_2\left[\dfrac{u'\left(1-\frac{v^2}{c^2}\right)}{1-\frac{u'v}{c^2}}\right]$

or, $\dfrac{m_1}{1+\frac{u'v}{c^2}} = \dfrac{m_2}{1-\frac{u'v}{c^2}}$ or, $\dfrac{m_1}{m_2} = \dfrac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \longrightarrow (3)$

From eq. (1), we have,

$$u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}} \quad \text{and} \quad u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}}$$

Let $u'v = x$, then

$$u_1 = \frac{u'+v}{1+\frac{x}{c^2}} \quad \text{and} \quad u_2 = \frac{-u'+v}{1-\frac{x}{c^2}}$$

or, $u'+v = u_1\left(1+\frac{x}{c^2}\right) \longrightarrow (4)$ and $-u'+v = u_2\left(1-\frac{x}{c^2}\right) \longrightarrow (5)$

(4) + (5)

$$\begin{aligned}
\cancel{u'}+v &= u_1\left(1+\frac{x}{c^2}\right)\\
-\cancel{u'}+v &= u_2\left(1-\frac{x}{c^2}\right)\\
\hline
2v &= u_1\left(1+\frac{x}{c^2}\right) + u_2\left(1-\frac{x}{c^2}\right)
\end{aligned}$$

or, $v = \dfrac{u_1}{2}\left(1+\frac{x}{c^2}\right) + \dfrac{u_2}{2}\left(1-\frac{x}{c^2}\right) \longrightarrow (6)$

(4) − (5)

$$\begin{aligned}
u' + \cancel{v} &= u_1\left(1+\frac{x}{c^2}\right)\\
-u' + \cancel{v} &= u_2\left(1-\frac{x}{c^2}\right)\\
+ \quad\ - \quad & \quad\ -\\
\hline
2u' &= u_1\left(1+\frac{x}{c^2}\right) - u_2\left(1-\frac{x}{c^2}\right)
\end{aligned}$$

or, $u' = \dfrac{u_1}{2}\left(1+\frac{x}{c^2}\right) - \dfrac{u_2}{2}\left(1-\frac{x}{c^2}\right) \longrightarrow (7)$

⑥ × ⑦

$$x'v = \left[\frac{u_1}{2}\left(1+\frac{x}{c^2}\right) - \frac{u_2}{2}\left(1-\frac{x}{c^2}\right)\right]\left[\frac{u_1}{2}\left(1+\frac{x}{c^2}\right) + \frac{u_2}{2}\left(1-\frac{x}{c^2}\right)\right]$$

$$\Rightarrow x = \left[\frac{u_1}{2}\left(1+\frac{x}{c^2}\right)\right]^2 - \left[\frac{u_2}{2}\left(1-\frac{x}{c^2}\right)\right]^2$$

$$= \frac{u_1^2}{4}\left(1+\frac{x}{c^2}\right)^2 - \frac{u_2^2}{4}\left(1-\frac{x}{c^2}\right)^2$$

$$= \frac{u_1^2}{4}\left(1+\frac{2x}{c^2}+\frac{x^2}{c^4}\right) - \frac{u_2^2}{4}\left(1-\frac{2x}{c^2}+\frac{x^2}{c^4}\right)$$

$$= \frac{u_1^2}{4} + \frac{xu_1^2}{2c^2} + \frac{x^2u_1^2}{4c^4} - \frac{u_2^2}{4} + \frac{xu_2^2}{2c^2} - \frac{x^2u_2^2}{4c^4}$$

$$= \frac{1}{4}(u_1^2-u_2^2) + \frac{x}{2c^2}(u_1^2+u_2^2) + \frac{x^2}{4c^4}(u_1^2-u_2^2)$$

$$\Rightarrow \frac{x^2}{4c^4}(u_1^2-u_2^2) + x\left[\frac{u_1^2+u_2^2}{2c^2} - 1\right] + \frac{1}{4}(u_1^2-u_2^2) = 0$$

$$\Rightarrow \frac{x^2}{4c^4}(u_1^2-u_2^2) + x\left[\frac{u_1^2+u_2^2-2c^2}{2c^2}\right] + \frac{1}{4}(u_1^2-u_2^2) = 0$$

$$\Rightarrow \frac{x^2}{4c^4} + \frac{x}{2c^2}\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] + \frac{1}{4} = 0$$

$$\Rightarrow x^2 + 2xc^2\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] + c^4 = 0$$

$$\therefore x = \frac{-2c^2\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] \pm \sqrt{4c^4\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right]^2 - 4c^4}}{2}$$

$$= -c^2\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] \pm c^2\sqrt{\left(\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right)^2 - 1}$$

$$\Rightarrow \frac{x}{c^2} = -\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] \pm \sqrt{\frac{(u_1^2+u_2^2-2c^2)^2 - (u_1^2-u_2^2)^2}{(u_1^2-u_2^2)^2}}$$

$$\therefore \frac{x}{c^2} = -\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] \pm \sqrt{\frac{(u_1^2+u_2^2-2c^2+u_1^2-u_2^2)(u_1^2+u_2^2-2c^2-u_1^2+u_2^2)}{(u_1^2-u_2^2)^2}}$$

$$= -\left[\frac{u_1^2+u_2^2-2c^2}{u_1^2-u_2^2}\right] \pm \frac{1}{(u_1^2-u_2^2)}\sqrt{(2u_1^2-2c^2)(2u_2^2-2c^2)}$$

$$= \frac{-u_1^2-u_2^2+2c^2 \pm 2\sqrt{(u_1^2-c^2)(u_2^2-c^2)}}{(u_1^2-u_2^2)}$$

$$\therefore \frac{u'v}{c^2} = \frac{2c^2-u_1^2-u_2^2 \pm 2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2} \longrightarrow (8)$$

Here, we take —

$$\frac{u'v}{c^2} = \frac{2c^2-u_1^2-u_2^2 - 2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2} \longrightarrow (9)$$

Substituting (9) into (3), we get —

$$\frac{m_1}{m_2} = \frac{1+(u'v/c^2)}{1-(u'v/c^2)} \longrightarrow (3)$$

$$1+\frac{u'v}{c^2} = 1+\frac{2c^2-u_1^2-u_2^2-2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{\cancel{u_1^2}-u_2^2+2c^2-\cancel{u_1^2}-u_2^2-2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{2c^2-2u_2^2-2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{2(c^2-u_2^2)-2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{2\sqrt{c^2-u_2^2}\left[\sqrt{c^2-u_2^2}-\sqrt{c^2-u_1^2}\right]}{u_1^2-u_2^2}$$

$$= \frac{2\sqrt{c^2-u_2^2}\,(\cancel{c^2}-u_2^2-\cancel{c^2}+u_1^2)}{(u_1^2-u_2^2)\left[\sqrt{(c^2-u_2^2)}+\sqrt{c^2-u_1^2}\right]}$$

$$= \frac{2\sqrt{c^2-u_2^2}\,(u_1^2-u_2^2)}{(u_1^2-u_2^2)\left[\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}\right]} = \frac{2\sqrt{c^2-u_2^2}}{\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}}$$

$\longrightarrow$ (10)

$$1-\frac{u'v}{c^2} = 1-\frac{2c^2-u_1^2-u_2^2-2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{u_1^2-\cancel{u_2^2}-2c^2+u_1^2+\cancel{u_2^2}+2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{(u_1^2-u_2^2)}$$

$$= \frac{2u_1^2-2c^2+2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{(u_1^2-u_2^2)}$$

$$= \frac{2(u_1^2-c^2)+2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}}{u_1^2-u_2^2}$$

$$= \frac{2\sqrt{(c^2-u_1^2)(c^2-u_2^2)}-2(c^2-u_1^2)}{u_1^2-u_2^2}$$

$$= \frac{2\sqrt{c^2-u_1^2}\left[\sqrt{c^2-u_2^2}-\sqrt{c^2-u_1^2}\right]}{u_1^2-u_2^2}$$

$$= \frac{2\sqrt{c^2-u_1^2}\left[c^2-u_2^2-c^2+u_1^2\right]}{(u_1^2-u_2^2)\left[\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}\right]}$$

$$= \frac{2\sqrt{c^2-u_1^2}\,(u_1^2-u_2^2)}{(u_1^2-u_2^2)\left[\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}\right]}$$

$$= \frac{2\sqrt{c^2-u_1^2}}{\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}} \longrightarrow (11)$$

∴ from (3) we get,

$$\frac{m_1}{m_2} = \frac{1+(u'v/c^2)}{1-\left(\frac{u'v}{c^2}\right)} = \frac{\dfrac{2\sqrt{c^2-u_2^2}}{\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}}}{\dfrac{2\sqrt{c^2-u_1^2}}{\sqrt{c^2-u_2^2}+\sqrt{c^2-u_1^2}}}$$

$$= \frac{\sqrt{c^2-u_2^2}}{\sqrt{c^2-u_1^2}} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}} \longrightarrow (12)$$

If the body of mass m_2 is moving with zero velocity in system S before collision, so that $u_2 = 0$, then —

$$m_1 = \frac{m_2}{\sqrt{1-u_1^2/c^2}} \longrightarrow (13)$$

Now, both the bodies have the same mass when moving with the same velocity, so that the above equation means that m_1 is the mass of the body when it is moving with velocity u_1 and m_2 its mass when its velocity is zero. In the commonly used notation, $m_1 = m$, $m_2 = m_0$ and $u_1 = v$, so that

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow (14)$$

This is the relativistic formula for the variation of mass with velocity. m_0 is called the rest mass, i.e, when the body whose mass is measured is at rest relative to the observer; m is the effective mass, i.e, the mass of the body when it is moving with a velocity v w.r.t the observer. Hence as far as the observer is concerned, masses on any moving system appear to increase with velocity, becoming infinite when v attains the velocity of light c. This indicates that c is a limiting velocity unattainable by moving material bodies. When v is small compared with c, v^2/c^2 becomes negligible and the mass of the body remains sensibly constant, as postulated in classical mechanics. The relativistic law relating mass and velocity has been directly verified by many scientists.