Equations of the first order but not of the first degree:

Introduction:- An eqn of first order and nth degree may be written as

$$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \cdots + P_{n-1} \cdot p + P_n = 0, \quad \text{---} (1)$$

where $p = \frac{dy}{dx}$ and $P_1, P_2, \ldots P_n$ are given functions of x and y. Here we discuss three special types of such equations:

1. Those solvable for p
2. Those solvable for y
3. Those solvable for x.

(1) Equations solvable for p :-

The eqn (1) is an algebraical equation in p, it has n-roots $p_1, p_2, \ldots, p_n$, these being functions of x and y, we can expressed eqn (1) in the form:

$$(p-p_1)(p-p_2)\cdots(p-p_n) = 0.$$

This can be true only if one or more of the functions on the left-hand side vanishes, and hence any relation between x and y which makes a factor vanish will be a solution of (1), while no relation which does not make some factor vanish can be a solution.

Let $\phi_r(x, y, c_r) = 0$ be the primitives of $p - p_r = 0$

where, $r = 1, 2, \ldots n$.

All possible solutions of the equation (1) will then be included in the relation

$$\phi_1(x,y,c_1)\,\phi_2(x,y,c_2)\cdots\phi_n(x,y,c_n) = 0$$

The general solution of (1) is

$$\phi_1(x,y,c)\,\phi_2(x,y,c)\cdots\phi_n(x,y,c) = 0.$$

where c is arbitrary constant.

Ex – III (A)

Ex. Solve $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$

Solⁿ:

$$x^2p^2 - 2xyp + 2y^2 - x^2 = 0$$

$$(xp - y)^2 = x^2 - y^2$$

$$xp - y = \pm\sqrt{x^2 - y^2}$$

$$p = \frac{y \pm \sqrt{x^2-y^2}}{x}.$$

$$\frac{dy}{dx} = \frac{y \pm \sqrt{x^2-y^2}}{x}, \quad \text{put } y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} \pm \sqrt{1-v^2}$$

Int

$$\int \frac{dv}{\sqrt{1-v^2}} = \pm\int \frac{dx}{x}$$

$$\sin^{-1} v = \pm(\log x + \log c)$$

$$v = \pm \sin(\log cx)$$

$$y = \pm x \sin(\log cx).$$

Ex: $p^2 - 2p\cosh x + 1 = 0$

$$p^2 - \cancel{2}p.\left(\frac{e^x + e^{-x}}{\cancel{2}}\right) + 1 = 0$$

$$p^2 - pe^x - pe^{-x} + 1 = 0$$

$$p(p - e^x) - e^{-x}(p - e^x) = 0$$

$$(p - e^x)(p - e^{-x}) = 0$$

$\Rightarrow$ either $(p - e^x) = 0$ or $(p - e^{-x}) = 0$

$p - e^x = 0$ gives $\frac{dy}{dx} - e^x = 0$

$$\int dy = \int e^x dx$$

$$y = e^x + c_1$$

$$(y - e^x - c_1) = 0$$

$p - e^{-x} = 0$ gives $\frac{dy}{dx} = e^{-x}$

$$\int dy = \int e^{-x} dx$$

$$y = -e^{-x} + c_2$$

$$(y + e^{-x} - c_2) = 0$$

$\therefore$ The general soln is $(y - e^x - c_1)(y + e^{-x} - c) = 0$

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Ex: $y = b + (1+p^2)^{-1/2}$

Sol: $(y-b)^2 = (1+p^2)^{-1}$

$$\frac{1}{1+p^2} = (y-b)^2$$

$$1+p^2 = \frac{1}{(y-b)^2}$$

$$p^2 = \frac{1-(y-b)^2}{(y-b)^2}$$

$$p = \pm \frac{\sqrt{1-(y-b)^2}}{(y-b)}$$

$$\frac{dy}{dx} = \pm \frac{\sqrt{1-(y-b)^2}}{(y-b)}$$

Let, $1-(y-b)^2 = z^2$

$-\cancel{2}(y-b)\,dy = \cancel{2}z\,dz$

$$\frac{(y-b)}{\sqrt{1-(y-b)^2}}\,dy = \pm dx$$

or, $\int \frac{-\cancel{z}\,dz}{\cancel{z}} = \pm\int dx$

$-z = \pm(x+c)$

$z^2 = (x+c)^2$

$-\cancel{\{\sqrt{1-(y-b)^2}\}} = \pm(x+c)$

$1-(y-b)^2 = (x+c)^2$

$\cancel{1-(y-b)^2 = (x+c)^2}$

$(x+c)^2 + (y-b)^2 = 1$

$\cancel{(x+c)^2 + (y-b)^2 = 1}$.

Ans

Ex: $p^3 - p^2(x^2+xy+y^2) + p(x^3y + x^2y^2 + xy^3) - x^3y^3 = 0$.

Sol: $(p^3 - x^3y^3) - (p^2x^2 - px^3y) - (p^2xy - px^2y^2) - (p^2y^2 - pxy^3) = 0$

$(p-xy)(p^2+pxy+x^2y^2) - px^2(p-xy) - pxy(p-xy) - py^2(p-xy) = 0$

$(p-xy)(p^2 + \cancel{pxy} + x^2y^2 - px^2 - \cancel{pxy} - py^2) = 0$

$(p-xy)\{p(p-x^2) - y^2(p-x^2)\} = 0$

$(p-xy)(p-x^2)(p-y^2) = 0$

$\Rightarrow$ either $(p-xy) = 0$ or $(p-x^2) = 0$ or $(p-y^2) = 0$

$p - xy = 0$ gives $\frac{dy}{dx} - xy = 0$

$$\text{or, } \int \frac{dy}{y} = \int x\, dx$$

$$\log y = \frac{x^2}{2} + \log c_1$$

$$y = c_1 e^{x^2/2}$$

$$\left(y - c_1 e^{x^2/2}\right) = 0$$

$p - x^2 = 0$ gives $\frac{dy}{dx} - x^2 = 0$

$$\text{or } \int dy = \int x^2 dx$$

$$y = \frac{x^3}{3} + c_2$$

$$y - \frac{x^3}{3} - c_2 = 0$$

$p - y^2 = 0$ gives $\frac{dy}{dx} - y^2 = 0$

$$\text{or } \int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + c_3$$

$$x + \frac{1}{y} - c_3 = 0$$

$\therefore$ the general sol^n is

$$\left(y - c\, e^{x^2/2}\right) \cdot \left(y - \frac{x^3}{3} - c\right)\left(x + \frac{1}{y} - c\right) = 0$$

(2) Equations solvable for y :-

If the equ^n is solvable for y we diff. the solved form, say $y = F(x, p)$ w.r.to x, —(1) we obtain

$$\frac{dy}{dx} = p = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial p} \cdot \frac{dp}{dx}$$

$$= \phi\left(x, p, \frac{dp}{dx}\right) \quad (\text{say})$$

we solve $p = \phi\left(x, p, \frac{dp}{dx}\right)$ to obtain $\psi(x, p, c) = 0$ —(2)

the primitive (general sol^n) is obtain by eliminating p from (1) and (2).

Ex: $y = x(p+p^3)$

Solⁿ: $y = x(p+p^3)$ —— ①

Diff. ① w.r.to x, we get — , $\left(p = \frac{dy}{dx}\right)$

$$\cancel{p} = \cancel{p}+p^3 + x(1+3p^2)\frac{dp}{dx}$$

$$\frac{1+3p^2}{p^3}\,dp + \frac{dx}{x} = 0$$

$$\int\left(\frac{3}{p} + \frac{1}{p^3}\right)dp + \int\frac{dx}{x} = \log c$$

$$3\log p - \frac{1}{2p^2} + \log x = \log c$$

$$p^3 x = c\,e^{1/2p^2}$$

∴ The general solⁿ is $p^3x = c\,e^{1/2p^2}$ and the given relation.

② Equations solvable for x: —

If the solved form is $x = F(y,p)$,① we diff. it w.r.to y, we obtain $\frac{1}{p} = \phi\left(y, p\,\frac{dp}{dy}\right)$

Solve this eqⁿ and obtain a relation of the form $\psi(y,p,c) = 0$ —— ②

Then eliminate p from ① and ② and obtain the general solⁿ.

Ex: $x = y + a\log p$.

Solⁿ: Diff w.r.t. y we get

$$\frac{1}{p} = p + \frac{a}{p}\frac{dp}{dx}.$$

$$= \frac{p^2 + a\frac{dp}{dx}}{p}$$

$$\frac{dp}{1-p^2} = \frac{dx}{a}$$

$$\frac{1}{2}\log\left|\frac{1+p}{1-p}\right| = \frac{x}{a} + c.$$

∴ The general solⁿ is $\frac{1}{2}\log\left|\frac{1+p}{1-p}\right| = \frac{x}{a} + c$ and the given relation.

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Clairaut's Form :-

A differential equation of the form

$$y = px + f(p) \quad , \quad p = \frac{dy}{dx}.$$

is called Clairaut's equation.

To find general soln :-

The Clairaut's eqn is

$$y = px + f(p) \quad \text{---} (1)$$

Diff. (1) w.r.t 'x' we get

$$p = p + \left(x + f'(p)\right)\frac{dp}{dx}$$

$$\therefore \left\{x + f'(p)\right\}\frac{dp}{dx} = 0$$

$\Rightarrow$ Either $\frac{dp}{dx} = 0$ or $\{x + f'(p)\} = 0$

Now, $\frac{dp}{dx} = 0$ gives $dp = 0$

$$\text{or } \int dp = c$$

$$p = c.$$

$\therefore$ The general soln is $y = cx + f(c)$.

This represents a family of straight lines.

To find singular soln (S.S) :-

$x + f'(p) = 0$ gives $x = -f'(p)$

From given eqn (1) $y = -pf'(p) + f(p)$.

It contains no arbitrary constant and therefore is not a general soln. But it is a solution of (1).

Thus this is a singular soln of (1).

Note :- The singular soln of the Clairaut's equation becomes the envelope of the family of lines given by the general soln.

Ex: Obtain the complete primitive and singular solution of $y = px + \sqrt{1+p^2}$.

Soln: The given eqn is $y = px + \sqrt{1+p^2}$ — (1)

which is in Clairaut's form.

Diff. (1) w.r.t x we get -

$$p = p + x\frac{dp}{dx} + \frac{p}{\sqrt{1+p^2}}\frac{dp}{dx}$$

or, $\left(x + \frac{p}{\sqrt{1+p^2}}\right)\frac{dp}{dx} = 0$

$\Rightarrow$ either $\frac{dp}{dx} = 0$ or $x + \frac{p}{\sqrt{1+p^2}} = 0$.

To find G.S. (general soln) :-

$\frac{dp}{dx} = 0$ gives $dp = 0$

9 int, $\int dp = c$

$p = c$

$\therefore$ The general soln is $y = cx + \sqrt{1+c^2}$.

To find S.S (singular soln) :-

From, $x + \frac{p}{\sqrt{1+p^2}} = 0$

$x = -\frac{p}{\sqrt{1+p^2}}$

From (1),

$y = \frac{1}{\sqrt{1+p^2}}$

Eliminating p by squaring and adding -

$$x^2 + y^2 = \frac{p^2}{1+p^2} + \frac{1}{1+p^2}$$

$$= \frac{p^2+1}{p^2+1}$$

$x^2 + y^2 = 1$.

Another method to find S.S:

The general soln is $y = cx + \sqrt{1+c^2}$

$(y - cx)^2 = 1 + c^2$

$y^2 - 2cxy + c^2x^2 - 1 - c^2 = 0$

$(x^2-1)c^2 - 2cxy + (y^2-1) = 0$ — (

Accepting the existance of its envelope, the singular solⁿ is given by the condition for two equal roots of it. $\boxed{Ax^2+Bx+C=0}$

$\therefore \quad B^2-4AC=0$

$\quad 4x^2y^2-4(x^2-1)(y^2-1)=0$

$\quad x^2y^2-x^2y^2+x^2+y^2-1=0$

$\quad x^2+y^2=1$.

Ex: Solve and find the singular solution of the diff. eqⁿ $\sin\left(x\frac{dy}{dx}\right)\cos y=\cos\left(x\frac{dy}{dx}\right)\sin y+\frac{dy}{dx}$.

Solⁿ: Given equation is

$$\sin\left(x\frac{dy}{dx}\right)\cos y=\cos\left(x\frac{dy}{dx}\right)\sin y+\frac{dy}{dx}$$

or. $\sin(px)\cos y-\cos(px)\sin y=p$

$\sin(px-y)=p$

$\quad y=px-\sin^{-1}p \quad \text{—— (1)}$

which is in clairaut's form.

Diff. (1) w.r.t. x, we get,

$$\cancel{p}=\cancel{p}+x\frac{dp}{dx}-\frac{1}{\sqrt{1-p^2}}\frac{dp}{dx}$$

$$\left(x-\frac{1}{\sqrt{1-p^2}}\right)\frac{dp}{dx}=0$$

$\Rightarrow$ either $\frac{dp}{dx}=0$ or $x-\frac{1}{\sqrt{1-p^2}}=0$

to find G.S:

from $\frac{dp}{dx}=0$

$dp=0$

Int $\int dp=c$

$p=c$.

$\therefore$ The general solⁿ is $y=cx-\sin^{-1}c$.

to find S.S.:

from $x-\frac{1}{\sqrt{1-p^2}}=0$

$p^2=\frac{x^2-1}{x^2} \quad \text{—— (2)}$

Eliminating p from (1) and (2) we have $y=\sqrt{x^2-1}-\sin^{-1}\frac{\sqrt{x^2-1}}{x}$.

which is the required singular solution.

Ex: Reduce $x^2p^2 + y(2x+y)p + y^2 = 0$ to Clairaut's form by the substitution $y = u$, $xy = v$. Hence solve the equation and prove that $y + 4x = 0$ is a singular solution.

Sol^n: Given equation is -

$$\cancel{x^2p^2 + y(2x+y)p + y^2 = 0}$$

$$\cancel{x^2p^2 + 2xyp + y^2 + py^2 = 0}$$

$$\cancel{(xp+y)^2 + py^2 = 0}$$

$$\cancel{p^2q^2 + py^2 = 0}$$

$y = u$, $xy = v$

$\frac{dy}{dx} = \frac{du}{dx}$, $y + x\frac{dy}{dx} = \frac{dv}{dx}$

$p = \frac{du}{dx}$, $y + xp = \frac{dv}{dx}$

Now,

$q = \frac{dv}{du} = \frac{dv/dx}{du/dx} = \frac{y+xp}{p}$

$pq = y + xp$

$p = \frac{y}{q-x}$.

Sol^n: Given equation is -

$$x^2p^2 + y(2x+y)p + y^2 = 0$$

$y = u$, $xy = v$

$\frac{dy}{dx} = \frac{du}{dx}$, $y + x\frac{dy}{dx} = \frac{dv}{dx}$

$p = \frac{du}{dx}$, $y + xp = \frac{dv}{dx}$

$\therefore q = \frac{dv}{du} = \frac{dv/dx}{du/dx} = \frac{y+xp}{p}$, $p = \frac{y}{q-x}$

$$x^2\left(\frac{y}{q-x}\right)^2 + y(2x+y)\left(\frac{y}{q-x}\right) + y^2 = 0$$

$$x^2 + (2x+y)(q-x) + (q-x)^2 = 0$$

$$\cancel{x^2} + \cancel{2xq} - 2x^2 + yq - xy + q^2 \cancel{-2xq} + x^2 = 0$$

$$\cancel{2xq - x^2}$$

$$yq - xy + q^2 = 0$$

$$uq - v + q^2 = 0$$

$$v = uq + q^2 \quad \text{——} (1)$$

Which is in Clairaut's form.

Diff. (1) w.r.t. u, we get

$$\cancel{\frac{dv}{du}} = \cancel{q} + u\frac{dq}{du} + 2q\frac{dq}{du} = 0$$

$$(u + 2q)\frac{dq}{du} = 0$$

$\Rightarrow$ either $\frac{dq}{du} = 0$ or $u + 2q = 0$.

To find G.S

From $\frac{dq}{du} = 0$

$dq = 0$

Int. $\int dq = c$

$q = c.$

$\therefore$ The general soln is $v = uc + c^2$

$xy = yc + c^2.$

To find S.S :-

From $u + 2q = 0$

$q = -\frac{u}{2}$

From ①, $v = -\frac{u^2}{2} + \frac{u^2}{4}$

$v = -\frac{u^2}{4}$

$u^2 + 4v = 0$

$y^2 + 4xy = 0$

$y(y + 4x) = 0$

either $y = 0$ or $y + 4x = 0.$

$\therefore$ The singular soln is $y + 4x = 0.$

Note: Here $y = 0$ is both particular soln and singular soln of the given eqn.

Ex:- Obtain the complete primitive and singular solution of the clairaut's eqn $y = px + \sqrt{a^2p^2 + b^2}$.

Soln: Given eqn is $y = px + \sqrt{a^2p^2+b^2}$ — ①

diff. w.r.t x, $p = p + x\frac{dp}{dx} + \frac{2a^2p \frac{dp}{dx}}{2\sqrt{a^2p^2+b^2}}$

$\left(x + \frac{a^2p}{\sqrt{a^2p^2+b^2}}\right)\frac{dp}{dx} = 0$

$\Rightarrow$ either $\frac{dp}{dx} = 0$ or $x + \frac{a^2p}{\sqrt{a^2p^2+b^2}} = 0$

To find G.S. :-

From $\frac{dp}{dx} = 0$

Ig $\int dp = c$

$p = c.$

$\therefore$ The general soln is

$y = cx + \sqrt{a^2c^2+b^2}.$

To find S.S.:-

From $x + \frac{a^2p}{\sqrt{a^2p^2+b^2}} = 0$

$$\frac{x}{a^2} = -\frac{ap}{\sqrt{a^2p^2+b^2}}.$$

From ① $\quad y = px + \sqrt{a^2p^2+b^2}$

$$= -\frac{a^2p^2}{\sqrt{a^2p^2+b^2}} + \sqrt{a^2p^2+b^2}$$

$$= \frac{-\cancel{a^2p^2} + (\cancel{a^2p^2}+b^2)}{\sqrt{a^2p^2+b^2}}$$

$$\therefore \frac{y}{b} = \frac{b}{\sqrt{a^2p^2+b^2}}$$

Squaring and adding we get -

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \left(-\frac{ap}{\sqrt{a^2p^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2p^2+b^2}}\right)^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2p^2+b^2}{a^2p^2+b^2}$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Ex! $(y-px)(p-1) = p$

Soln:-

$$y - px = \frac{p}{p-1}$$

$$y = px + \frac{p}{p-1} \quad \text{—— ①}$$

diff. w.r.t x, we have

$$\cancel{p} = \cancel{p} + x\frac{dp}{dx} + \frac{\cancel{p}-1-\cancel{p}}{(p-1)^2}\frac{dp}{dx}$$

$$\therefore \left\{x - \frac{1}{(p-1)^2}\right\}\frac{dp}{dx} = 0$$

$\Rightarrow$ either $\frac{dp}{dx} = 0$ or, $x - \frac{1}{(p-1)^2} = 0$

To find G.S.:

From $\frac{dp}{dx} = 0$

$dp = 0$

on int $\int dp = c$

$p = c$

$\therefore$ The general soln is

$$y = cx + \frac{c}{c-1}.$$

To find S.S :-

From general solution

$$y = cx + \frac{c}{c-1}$$

$$\Rightarrow y(c-1) = c(c-1)x + c$$

$$\Rightarrow yc - y = xc^2 - xc + c$$

$$\Rightarrow xc^2 + (1-x-y)c + y = 0. \quad \text{--- (2)}$$

Accepting the existance of its envelope, the singular solution is given by the condition for two equal roots of (2), i.e., $B^2 - 4AC = 0$

$$(1-x-y)^2 - 4xy = 0$$

$$\Rightarrow 1 + x^2 + y^2 - 2x - 2y + 2xy - 4xy = 0$$

$$\Rightarrow x^2 + y^2 - 2xy - 2x - 2y + 1 = 0.$$

Ex: The following equations are deducible to Clairaut's form under suitable substitutions, obtain the complete primitives :

(i) $(x^2+y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$, $[x^2+y^2 = v,\ x+y = u]$

(ii) $(px^2+y^2)(px+y) = (p+1)^2$, $[u = xy,\ v = x+y]$

(iii) $e^{3x}(p-1) + p^3 e^{2y} = 0$, $[e^x = u,\ e^y = v]$

(iv) $y^2(y - xp) = x^4 p^2$, $[x = \frac{1}{u},\ y = \frac{1}{v}]$

(v) $(2x^2+1)p^2 + (x^2+y^2+2xy+2)p + 2y^2+1 = 0$ $[x+y = u,\ xy-1 = v]$

(vi) $y = 3px + 6p^2y^2$ $[y^3 = v]$

(vii) $x^2 - \frac{xy}{p} = f(y^2 - xyp)$ $\frac{y^2}{x^2} = v,\ \frac{1}{x^2} = u.$

(1) Given that $x^2+z^2=v$ and $x+z=u$

$2(x+zp)=\frac{\partial v}{\partial x}$ $\quad (1+p)=\frac{\partial u}{\partial x}$

$\therefore q=\frac{dv}{du}=\frac{dv/\partial x}{du/\partial x}=\frac{2(x+zp)}{(1+p)}$

Now,

$(x^2+z^2)(1+p)^2-2(x+z)(1+p)(x+zp)+(x+zp)^2=0$

or $(x^2+z^2)-2(x+z)\left(\frac{x+zp}{1+p}\right)+\left(\frac{x+zp}{1+p}\right)^2=0$

$v-uq+\left(\frac{q}{2}\right)^2=0$

$v=uq-\frac{q^2}{4}$ ——(1)

diff. (1) w.r.to u, we get

$\cancel{q}=\cancel{q}+u\frac{dq}{du}-\frac{q}{2}\frac{dq}{du}=0$

or, $\left(u-\frac{q}{2}\right)\frac{dq}{du}=0$

$\Rightarrow$ either $\frac{dq}{du}=0$ or $u-\frac{q}{2}=0$

<u>To find G.S.:-</u>

From $\frac{dq}{du}=0$ $\quad$ $\therefore$ The general soln is

$\int dq=c$ $\quad$ $v=uc-\frac{c^2}{4}$

Int $q=c$ $\quad$ $(x^2+z^2)=(x+z)c-\frac{c^2}{4}$

<u>To find S.S.:-</u>

From, $u-\frac{q}{2}=0$

$q=2u$

From (1), $v=uq-\frac{q^2}{4}$

$=2u^2-u^2$

$v-u^2=0$

$(x^2+z^2)-(x+z)^2=0$

$xz=0$,

This is the singular soln of the given eqn.

(IV) $y^2(x-yp)$

$$y^2(y-xp) = x^4p^2$$

Let, $x=\frac{1}{u}$, $y=\frac{1}{v}$

$u=\frac{1}{x}$, $v=\frac{1}{y}$

$\frac{du}{dx} = -\frac{1}{x^2}$, $\frac{dv}{dx} = -\frac{1}{y^2}p$

$\therefore q = \frac{dv}{du} = \frac{dv/dx}{du/dx} = \frac{+p/y^2}{+1/x^2}$

$q = \frac{px^2}{y^2}$

$p = \frac{y^2q}{x^2}$

$$y^2\left(y - \frac{y^2q}{x}\right) = y^4q^2$$

$$\frac{1}{y} - \frac{q}{x} = q^2$$

$$v - uq = q^2$$

$$v = uq + q^2 \quad \text{--- (1)}$$

Diff. (1). w.r.to u, we get

$$\cancel{\frac{dv}{du}} = \cancel{q} + u\frac{dq}{du} + 2q\frac{dq}{du}$$

$$(u+2q)\frac{dq}{du} = 0$$

$\Rightarrow$ either $\frac{dq}{du} = 0$ or $u+2q=0$

__To find GS:__

From $\frac{dq}{du} = 0$

$dq = 0$

or $\int dq = c$

$q = c.$

$\therefore$ The general sol^n is $v = uc + c^2$

$$\frac{1}{y} = \frac{1}{x}c + c^2$$

$$x = yc + xyc^2.$$

__To find SS:__

~~From~~, $u+2q=0$

$q = -\frac{u}{2}$

From (1) $v = uq + q^2$

$$= -\frac{u^2}{2} + \frac{u^2}{4}$$

$$= -\frac{u^2}{4}$$

$$u^2 + 4v = 0$$

$$\frac{1}{x^2} + \frac{4}{y} = 0$$

$$y + 4x^2 = 0.$$

Which is ~~one of the~~ singular sol^n.

OR:-

From G.S. $\Rightarrow$ $xyc^2 + yc - x = 0$

$B^2 - 4AC = 0$

$y^2 + 4x^2y = 0$

$y(y+4x^2) = 0$

Ans