Testing of Hypothesis

A statistical hypothesis is a statement about parameter of a popn(s).

If p denotes the probability of head in toss of a coin, then a hypothesis can be —

$H_0: p = 1/2$, $H_1: p = 2/3$, $H_2: p: 3/4$.

Null and alternative hypothesis :-

suppose in a coin tossing experiment we want to test whether a coin is unbiased or not.
Let p denote the probability of occurance of head. Then we want to test if $\underbrace{p = 1/2}_{\text{null hypothesis}}$ or $\underbrace{p \neq 1/2}_{\text{alternative hypothesis}}$

$H_0: p = 1/2$	$H_0: p = 1/2$
$H_1: p \neq 1/2$	$H_1: p = 1/4$.

The null hypothesis is a hypothesis which is tested for possible rejection under the assumption that it is true.

Two types of errors :-

When we conduct a test of hypothesis, we are likely to make two types of errors :-

Type I error :- Rejecting H_0 when it is true → error of the first kind.

$\alpha = P(\text{type I error})$
$= P(\text{reject } H_0 \text{ when it is true})$

Type II error :- Accepting H_0 when it is false.
→ error of 2nd kind

$\beta = P(\text{type II error})$
$= P(\text{accept } H_0 \text{ when it is false})$.

Power of test :-

Power of a test $= 1-\beta$
$= P(\text{rejecting } H_0 \text{ when it is false})$.

Tests for parameters of normal pop^{n}s

Let $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$

Test for μ

Case I :- σ^2 is known

Problem 1 :- $H_0 : \mu \le \mu_0$, $H_1 : \mu > \mu_0$ | $H_0 : \mu = \mu_0$, $H_1 : \mu > \mu_0$.

The test is; reject H_0 if

$$\underbrace{\frac{\sqrt{n}(\overline{X} - \mu_0)}{\sigma}}_{\text{test statistic}} > z_\alpha$$

$\alpha \to$ level of significance.

Problem 2 :- $H_0 : \mu \ge \mu_0$, $H_1 : \mu < \mu_0$ or $H_0 : \mu = \mu_0$, $H_1 : \mu < \mu_0$

The test is reject H_0 if

$$\frac{\sqrt{n}(\overline{X} - \mu_0)}{\sigma} < -z_\alpha .$$

Problem 3 $H_0 : \mu = \mu_0$, $H_1 : \mu \ne \mu_0$

The test is reject H_0 if

$$\left| \frac{\sqrt{n}(\overline{X} - \mu_0)}{\sigma} \right| \ge z_{\alpha/2} .$$

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Case II : σ^2 is unknown

Problem 1:- $H_0 : \mu \le \mu_0$, $H_1 : \mu > \mu_0$ or $H_0 : \mu = \mu_0$, $H_1 : \mu > \mu_0$

Test is reject H_0 if

$$\frac{\sqrt{n}\,(\bar{x}-\mu_0)}{s} \geq t_{\alpha, n-1}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 .$$

Prob 2:- $\cancel{H_0 : \mu \le \mu_0}$, $\cancel{H_1 : \mu > \mu_0}$ or $\cancel{H_0} : \mu$

$H_0 : \mu \geq \mu_0$, $H_1 : \mu < \mu_0$ or $H_0 : \mu = \mu_0$, $H_1 : \mu < \mu_0$

test : Reject H_0 if $\dfrac{\sqrt{n}\,(\bar{x}-\mu_0)}{s} \le -t_{\alpha, n-1}$

Prob 3:- $\left.\begin{array}{l} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{array}\right\}$ test : Reject H_0 if

$$\left|\frac{\sqrt{n}\,(\bar{x}-\mu_0)}{s}\right| \geq t_{\alpha/2, n-1}$$

Testing for σ^2 :-

Case I :- μ is known

Prob 1 :- $H_0 : \sigma^2 \leq \sigma_0^2$ or $H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 > \sigma_0^2$ $\qquad$ $H_1 : \sigma^2 > \sigma_0^2$

Test is reject H_0 if

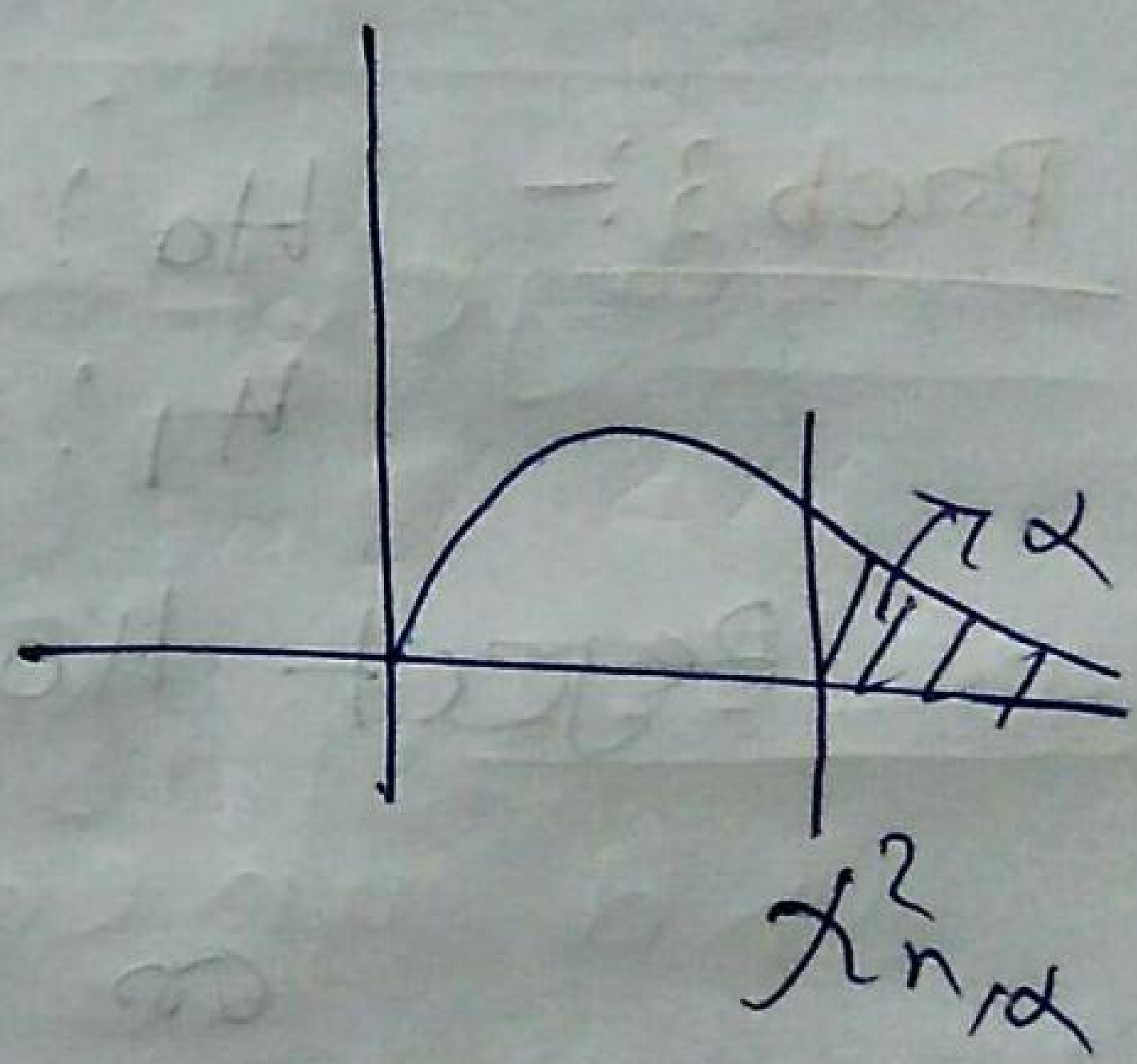
$$\frac{\Sigma (x_i - \mu)^2}{\sigma_0^2} > \chi^2_{n,\alpha}.$$

Prob 2 :- $H_0 : \sigma^2 \geq \sigma_0^2$ or $H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 < \sigma_0^2$ $\qquad$ $H_1 : \sigma^2 < \sigma_0^2$

Test is reject H_0 if

$$\frac{\Sigma (x_i - \mu)^2}{\sigma_0^2} < \chi^2_{n,1-\alpha}$$



Prob 3 :- $H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 \neq \sigma_0^2$

Test is reject H_0 if

$$\frac{\Sigma (x_i - \mu)^2}{\sigma_0^2} < \chi^2_{n,1-\alpha/2} \quad \text{or} \quad \frac{\Sigma (x_i - \mu)^2}{\sigma_0^2} > \chi^2_{n,\alpha/2}$$

Case II :- μ is unknown

Prob 1 :- $H_0 : \sigma^2 \leq \sigma_0^2$ or $H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 > \sigma_0^2$ $\quad H_1 : \sigma^2 > \sigma_0^2$

Reject H_0 if $\frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{\alpha, n-1}$

Prob 2 :- $H_0 : \sigma^2 \geq \sigma_0^2$ or $H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 < \sigma_0^2$ $\quad H_1 : \sigma^2 < \sigma_0^2$

Test is : reject H_0 if

$$\underbrace{\frac{(n-1)s^2}{\sigma_0^2}}_{\text{test statistic}} < \chi^2_{1-\alpha, n-1}$$

Prob 3 :- $H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \neq \sigma_0^2$

Reject H_0 if $\frac{(n-1)s^2}{\sigma_0^2} < \chi^2_{1-\alpha/2, n-1}$

or $\frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{\alpha/2, n-1}$

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Testing for parameters of two normal populations :-

Independent $\left\langle \begin{array}{l} X_1, X_2, \ldots, X_m \sim N(\mu_1, \sigma_1^2) \\ Y_1, Y_2, \ldots, Y_n \sim N(\mu_2, \sigma_2^2) \end{array} \right.$

Testing for comparison of Means :-

Case I :- σ_1^2 & σ_2^2 are known

Prob 1 :- $H_0 : \mu_1 \le \mu_2$ or $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 > \mu_2$ $\quad H_1 : \mu_1 > \mu_2$.

Test is reject H_0 if

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \ge z_\alpha$$

Prob 2 :- $H_0 : \mu_1 \ge \mu_2$ or $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 < \mu_2$ $\quad H_1 : \mu_1 < \mu_2$

Test is reject is H_0 if

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \le -z_\alpha$$

Prob 3 :- $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \ne \mu_2$.

Test is reject H_0 if

$$\left| \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \right| \ge z_{\alpha/2}.$$

Case II :- $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (unknown)

Test statistic: $T = \dfrac{\bar{X} - \bar{Y}}{c S_p}$, $c^2 = \frac{1}{m} + \frac{1}{n}$

Prob 1:- $H_0 : \mu_1 \leq \mu_2$

$H_1 : \mu_1 > \mu_2$

Reject H_0 if $T > t_{\alpha, m+n-2}$

Prob 2:- $H_0 : \mu_1 \geq \mu_2$

$H_1 : \mu_1 < \mu_2$

Reject H_0 if $T < -t_{\alpha, m+n-2}$

Prob 3:- $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Reject H_0 if $|T| > t_{\alpha/2, m+n-2}$.

Case III :- σ_1^2 and σ_2^2 are unknown and unequal.

$T = \dfrac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}$, ν = integer part of $\dfrac{\left(\frac{S_1^2}{m} + \frac{S_2^2}{n}\right)^2}{\left[\frac{S_1^4}{m^2(m-1)} + \frac{S_2^4}{n^2(n-1)}\right]}$

Prob 1:- Reject H_0 if $T > t_{\alpha, \nu}$

$H_0 : \mu_1 \leq \mu_2$

$H_1 : \mu_1 > \mu_2$

Prob 2 :- $H_0 : \mu_1 \geq \mu_2$

$H_1 : \mu_1 < \mu_2$

Reject H_0 if $T < t_{\alpha, \nu}$

Prob 3 :- $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Reject H_0 if $|T| > t_{\alpha/2, \nu}$.

Testing for variance

Case I :- μ_1 & μ_2 are known

Prob 1 :-

$H_0 : \sigma_1^2 \leq \sigma_2^2$

$H_1 : \sigma_1^2 > \sigma_2^2$

Test statistic is given by

$$W = \frac{n \sum_{i=1}^{m} (x_i - \mu_1)^2}{m \sum_{j=1}^{n} (y_j - \mu_2)^2}$$

Test is reject H_0 if $W > f_{\alpha, m, n}$.

Prob 2 :- $H_0 : \sigma_1^2 \geq \sigma_2^2$

$H_1 : \sigma_1^2 < \sigma_2^2$

Reject H_0 if $W < f_{1-\alpha, m, n}$

Prob 3 :- $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Reject H_0 if $W < f_{1-\alpha/2, m, n}$

or $W > f_{\alpha/2, m, n}$.

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Testing for variance

Case II :- When μ_1 and μ_2 are unknown.

$$W = \frac{S_1^2}{S_2^2}$$

Prob 1 :- $H_0 : \sigma_1^2 \leq \sigma_2^2$ or $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 > \sigma_2^2$ $\quad H_1 : \sigma_1^2 > \sigma_2^2$

Reject H_0 if

$W > f_{\alpha, m-1, n-1}$

Prob 2 :- $H_0 : \sigma_1^2 \geq \sigma_2^2$

$H_1 : \sigma_1^2 < \sigma_2^2$

Test is reject H_0 if $W < f_{1-\alpha, m-1, n-1}$.

Prob 3 :- $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Test is reject H_0 if

$W < f_{1-\alpha/2, m-1, n-1}$

or if $W > f_{\alpha/2, m-1, n-1}$.

Paired t-test

$(X_1, Y_1), (X_2, Y_2), \ldots\ldots, (X_n, Y_n)$ is a random sample from a bivariate normal popⁿ with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.

In order to compare means μ_1 & μ_2 we base our test on $D_i = X_i - Y_i$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i, \quad S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2$$

$$T = \frac{\sqrt{n}\,\bar{D}}{S_D}$$

Prob 1:- $H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$

Test is reject H_0 if $T > t_{\alpha, n-1}$

Prob 2:- $H_0: \mu_1 \geq \mu_2$

$H_1: \mu_1 < \mu_2$

Test is: Reject H_0 if $T < -t_{\alpha, n-1}$

Prob 3:- $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Test is reject H_0 if

$|T| > t_{\alpha/2, n-1}$.