Snecdor's F-distribution

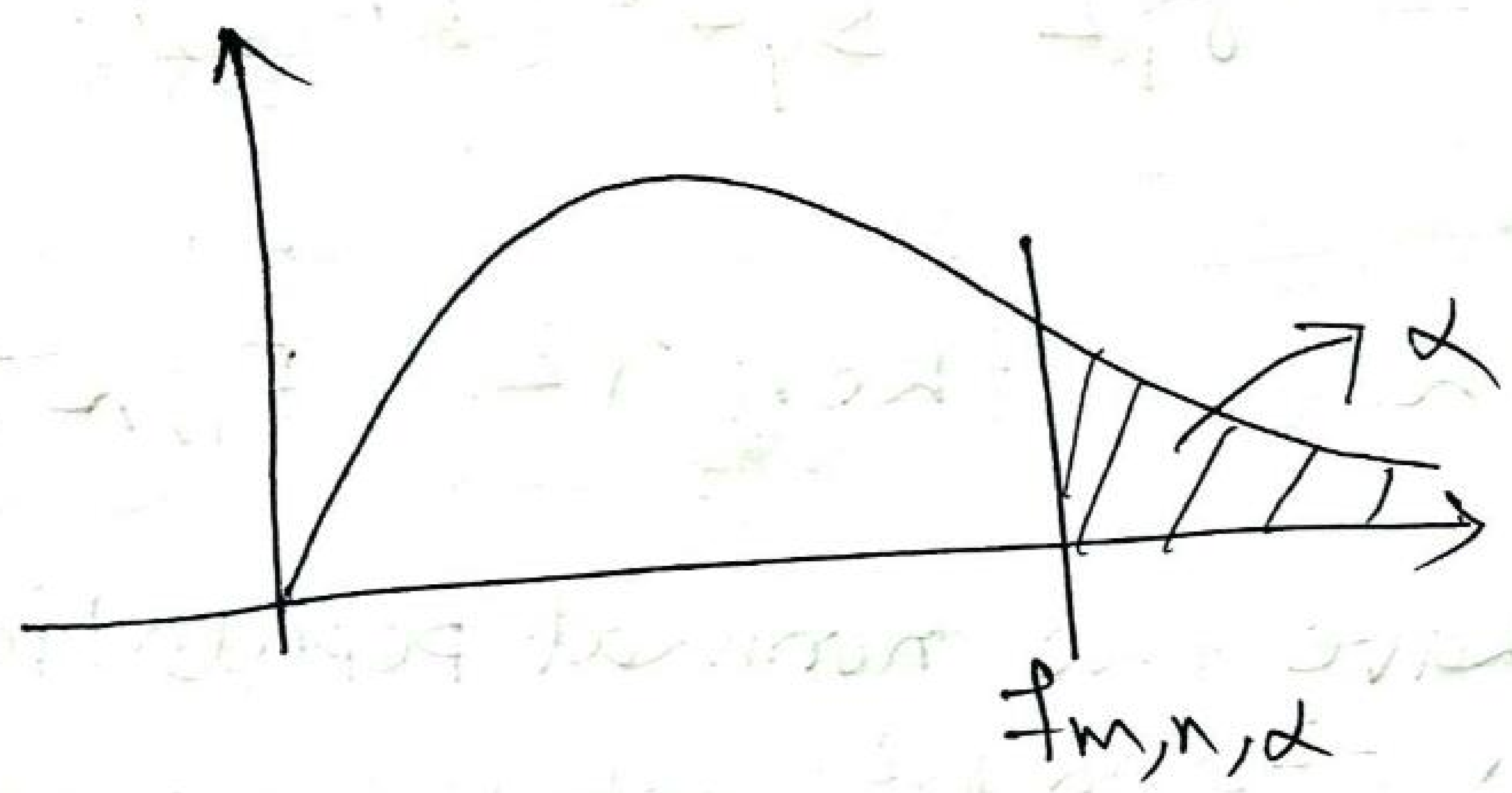
Let Y_1 and Y_2 be independent χ^2_m and χ^2_n r.v. Then ~~W=~~ Y_1

$$W = \frac{Y_1/m}{Y_2/n} = \frac{nY_1}{mY_2}$$ follows an $F_{m,n}$ distⁿ.

$$f_W(w) = \left(\frac{m}{n}\right)^{m/2} \frac{1}{B(\frac{m}{2},\frac{n}{2})} \frac{w^{m/2-1}}{(1+\frac{m}{n}w)^{\frac{m+n}{2}}} ; w>0$$

$$E(w) = \frac{n}{n-2}, n>2$$

$$V(w) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}, n>4$$



$$P(W \geq f_{m,n,\alpha}) = \alpha.$$

- If $U \sim F_{m,n}$ then $\frac{1}{U} \sim F_{n,m}$.

$$\text{so } \alpha = P(U \geq f_{\alpha,m,n}) = P\left(\frac{1}{U} \leq \frac{1}{f_{\alpha,m,n}}\right)$$

$$= P\left(V \leq \frac{1}{f_{\alpha,m,n}}\right) \quad V \sim F_{n,m}$$

$$= 1 - P\left(V \geq \frac{1}{f_{\alpha,m,n}}\right)$$

So, $P\left(V \geq \frac{1}{f_{\alpha,m,n}}\right) = 1-\alpha$

$$\Rightarrow \boxed{f_{1-\alpha,n,m} = \frac{1}{f_{\alpha,m,n}}}$$

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Let $X_1, \ldots, X_m \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

independent $Y_1, \ldots, Y_n \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$

$$S_x^2 = \frac{1}{m-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad S_y^2 = \frac{1}{n-1} \sum (Y_j - \bar{Y})^2$$

$$U = \frac{(m-1) S_x^2}{\sigma_1^2} \sim \chi^2_{m-1}, \quad V = \frac{(n-1) S_y^2}{\sigma_2^2} \sim \chi^2_{n-1}$$

$$\frac{U/(m-1)}{V/(n-1)} =$$

$$\frac{U/(m-1)}{V/(n-1)} = \frac{\sigma_2^2}{\sigma_1^2} \frac{S_x^2}{S_y^2} \sim F_{m-1, n-1}.$$

<u>Thm</u>:- If $T \sim t_n$, then $T^2 \sim F_{1,n}$.

Suppose we have two normal populations with variances σ_1^2 and σ_2^2. Let independent random samples of sizes n_1 and n_2 be taken from populations 1 and 2, respectively and let S_1^2 and S_2^2 be the sample variances. Then the ratio $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$ has an F-dist^n with $n_1 - 1$ numerator d.f. and $n_2 - 1$ denominator degrees of freedom.

* ~~X~~

Assignment - 1

① Using statistical table, find the following ~~point~~ values:

(a) $\chi^2_{0.95,8}$ (c) $\chi^2_{0.025,20}$

(b) $\chi^2_{0.50,12}$ (d) $\chi^2_{\alpha,10}$ such that ~~$P(\chi^2$~~ $P(\chi^2_{10} \le \chi^2_{\alpha,10}) = \cancel{0.975}\ 0.95$.

(e) $t_{0.25,10}$ (g) $t_{\alpha,10}$ such that $P\{t_{10} \le t_{\alpha,10}\} = 0.95$

(f) $t_{0.25,20}$

(h) $F_{0.25,4,9}$ (c) $F_{0.95,6,8}$

(l) $F_{0.05,15,10}$ (d) $F_{0.90,24,24}$

② Let $X_1, \cdots, X_n$ be i.i.d. $N(\mu,\sigma^2)$, find $P(|\bar{X}-\mu| \le 1.028S)$.

③ A population of power supplies for a personal computer has an output voltage that is normally distributed with a mean of 5.00 V and a standard deviation of 0.10 V. A random sample of eight power supplies is selected. Specify the sampling distn of $\bar{X}$.

④ A procurement specialist has purchased 25 resistors from vendor 1 and 30 resistors from ~~vo~~ vendor 2.

Let $X_{11}, X_{12}, \cdots, X_{125}$ represent the vendor 1 observed resistances assumed normally and independently distributed with a mean of 100 Ω and a standard deviation of 1.5 Ω.

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Similarly, let $X_{21}, X_{22}, \ldots X_{230}$ represent the vendor 2 observed resistances assumed normally and independently distributed with a mean of $105\,\Omega$ and a standard deviation of $2.0\,\Omega$. What is the sampling distn of $\overline{X}_1 - \overline{X}_2$?

(5) Consider the resistor problem in (4). If we could not assume that resistance is normally distributed, what could be said about the sampling distribution of $\overline{X}_1 - \overline{X}_2$?

(6) Suppose that independent random samples of sizes n_1 and n_2 are taken from two normal populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. If $\overline{X}_1$ and $\overline{X}_2$ are the sample means, find the sampling distn of the statistic

$$\frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

(7) Sylvania's 40-watt light bulbs will burn a random time X before failing. Let X have mean μ and s.d. 100 hours. If n of these bulbs are placed on test till they burn out, resulting in observations $X_1, \ldots, X_n$, how large n should be so that the probability that $\overline{X}$ differs by μ by less

than 50 hours is at least 0.95?

(8) Let $X_1, \ldots, X_n, X_{n+1}$ be i.i.d $N(\mu, \sigma^2)$ and let $\bar{X}$ and S^2 denote the sample mean and sample variance based on $X_1, \ldots, X_n$. Find the dist^n of $\sqrt{\frac{n}{n+1}}\left(\frac{X_{n+1}-\bar{X}}{S}\right)$

(9) Let $X_1, \ldots, X_m$ be a random sample from $N(\mu_1, \sigma^2)$ pop^n and $Y_1, \ldots, Y_n$ be another independent random sample from $N(\mu_2, \sigma^2)$ pop^n. Let $\bar{X}, \bar{Y}, S_1^2, S_2^2$ be the sample means and sample variances based on X and Y-samples respectively. Determine the dist^n of

$$U = \frac{\alpha(\bar{X}-\mu_1)+\beta(\bar{Y}-\mu_2)}{S\sqrt{\alpha^2/m+\beta^2/n}}, \text{ where } S^2 = \frac{(m-1)S_x^2+(n-1)S_y^2}{(m+n-2)},$$

$\alpha \neq 0, \beta \neq 0$

(10) Consider two independent samples — the first of size 10 from a normal pop^n with variance 4 and the 2nd of size 5 from a normal pop^n with variance 2. Compute the probability that the sample variance from the 2nd sample exceeds the one from the first.

(11) The temperature at which certain thermostat are set to go on is normally distributed with variance σ^2. A random sample is to be drawn and the sample variance S^2 computed. How many observations are required

to ensure that $P(S^2/\sigma^2 \le 1.8) \ge 0.95$?

(11) Let X_1 and X_2 be independent random variables. Find $P(X_1^2+X_2^2 \le \sigma^2)$.

(12) Let $X_1, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ popn.

For $1<k<n$, define

$$U = \frac{1}{k}\sum_{i=1}^{k} X_i, \quad V = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i,$$

$$S_k^2 = \frac{1}{k-1}\sum_{i=1}^{k}(X_i-U)^2, \quad T^2 = \frac{1}{n-k-1}\sum_{i=k+1}^{n}(X_i-V)^2$$

Find the distn of

$$W_1 = \frac{U+V}{2}, \quad W_2 = \frac{(k-1)S^2+(n-k-1)T^2}{\sigma^2},$$

$$W_3 = \frac{S^2}{T^2}, \quad W_4 = \frac{\sqrt{k}\,(U-\mu)}{T}, \text{ and}$$

$$W_5 = \frac{\sqrt{(n-k)}\,(V-\mu)}{T}.$$

(13) Let $X \sim N(100, 4)$

Find $P(X \le 104)$.

$$= P\left(\frac{X-100}{2} \le \frac{104-100}{2}\right)$$

$$= P(Z \le 2) = \Phi(2)$$

$$= 1 - P(Z>2)$$

$$= 1 - 0.0228$$

$$= 0.9772.$$