

Exercises

① Let $Z \sim N(0,1)$. Find

— (i) $P(Z > 2.73) = 0.0032$

(ii) $P(Z > 2.05)$

(iii) $P(Z > 1.09)$

(iv) $P(Z > 2.87)$

(v) $P(Z > 0.68)$

(vi) $P(Z \leq 1.58) = 1 - P(Z > 1.58)$
 $= 1 - 0.0571$

(vii) $P(Z \leq 1.7)$

(viii) $P(Z \leq 2.55)$

(ix) $P(Z \leq 3.02)$

(x) $P(Z \leq 0.09)$

(2) Find the point z_α s.t.

(i) $P(Z > z_\alpha) = 0.0119$

$\downarrow$

$$z_\alpha = 2.26$$

(ii) $P(Z > z_\alpha) = 0.1271$

(iii) $P(Z > z_\alpha) = 0.2983$

(iv) ~~$P(Z > 0.0294)$~~

$$P(Z > z_\alpha) = 0.0143$$

(v) ~~$P(Z > 0.4880)$~~ $P(Z > z_\alpha) = 0.4880$

~~(vi) $P(Z > 0.03)$~~

(vii) $P(Z > z_\alpha) = 0.0197$

(viii) $P(Z > z_\alpha) = 0.2383$

(ix) ~~$P(Z \leq z_\alpha) = 0.0582$~~

$$\Rightarrow P(Z \leq z_\alpha) = \cancel{0.9932} 0.9418$$

$$\Rightarrow 1 - P(Z > z_\alpha) = 0.9418$$

$$\Rightarrow P(Z > z_\alpha) = 1 - \cancel{0.9932} 0.9418 \\ = 0.0582$$

$$\Rightarrow z_\alpha = 1.57$$

(x) $P(Z \leq z_\alpha) = 0.7703$

(xi) $P(Z \leq z_\alpha) = 0.8925$

(xii) $P(Z \leq z_\alpha) = 0.5557$

(xiii) $P(Z \leq z_\alpha) = 0.95$

(xiv) $P(Z \leq z_\alpha) = 0.90$

③ Let X be a random variable follows $N(800, 144)$.

Find $P(X < 772)$ let

$$\Rightarrow \text{let } Z = \frac{X - 800}{\sqrt{144}} = \frac{X - 800}{12}$$

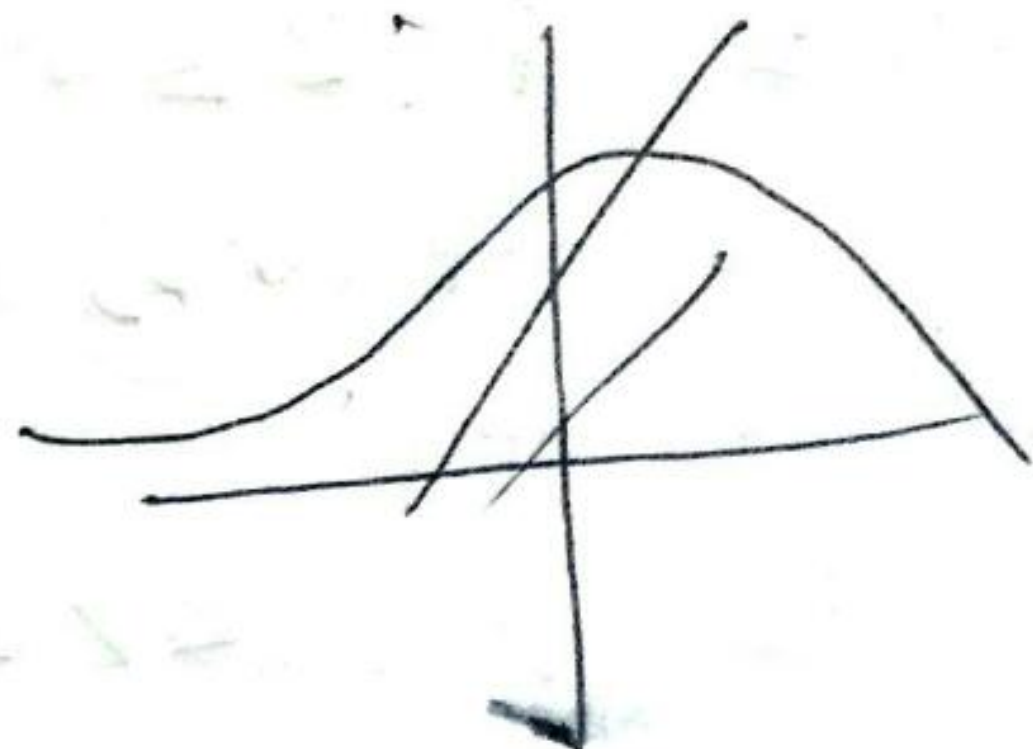
then $Z \sim N(0, 1)$

Now, $P(X < 772)$

$$= P\left(\frac{X - 800}{12} < \frac{772 - 800}{12}\right)$$

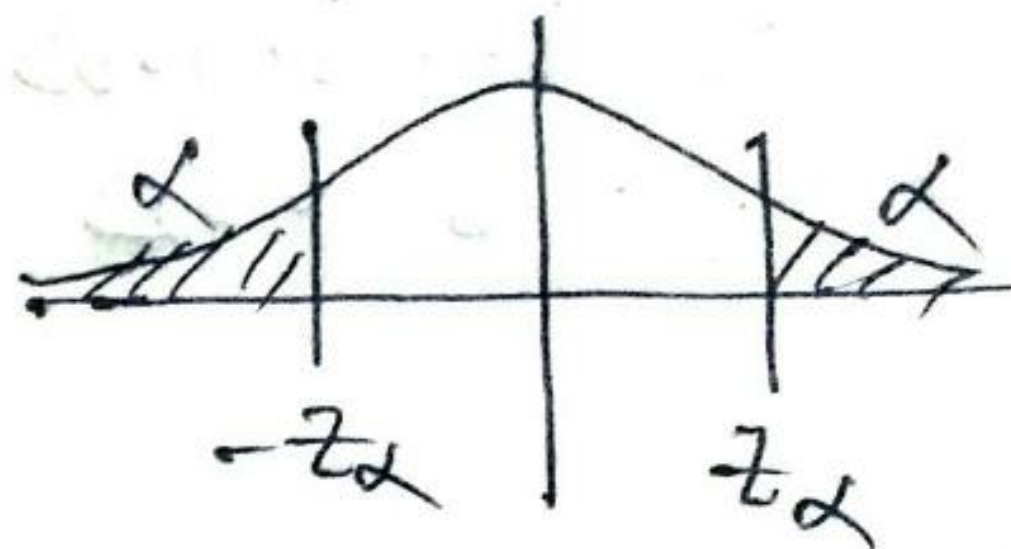
$$= P(Z < -2.33)$$

$$= \Phi(-2.33)$$



As standard normal dist is symmetric about zero

$$P(Z > z_\alpha) = P(Z \leq -z_\alpha)$$



So, $P(Z < -2.33)$

$$= P(Z > 2.33)$$

$$= 0.0099$$

$$1 - P(Z < z_\alpha)$$

$$= P(Z \leq -z_\alpha)$$

$$\Rightarrow 1 - \Phi(z_\alpha) = \Phi(-z_\alpha)$$

$$\Rightarrow \Phi(z_\alpha) + \Phi(-z_\alpha) = 1$$

④ Let $X \sim N(120, 16)$.
Find $P(X > 125)$

⑤ Let $X \sim N(125, 16)$.
Find $P(X > 125)$

⑥ Let $X \sim N(0.4008, 0.0004)$
Find $P(0.399 \leq X \leq 0.401)$

$$= P\left(\frac{0.399 - 0.4008}{\sqrt{0.0004}} \leq Z \leq \frac{0.401 - 0.4008}{\sqrt{0.0004}}\right)$$

$$= P(-4.5 \leq Z \leq 0.5)$$

$$= P(Z \leq 0.5) - P(Z \leq -4.5)$$

$$= \Phi(0.5) - \Phi(-4.5)$$

$$= \Phi(0.5) - (1 - \Phi(4.5))$$

$$= 1 - P(Z > 0.5) - P(Z > 4.5)$$

$$= 1 - 0.3085 - 0$$

$$= 0.6915$$

[suppose
it is given
 $P(Z > 4.5) = 0$]

⑦ X Let $X \sim N(0.4, 0.0004^2)$
Find $P(0.399 \leq X \leq 0.401)$

⑧ Suppose $X \sim N(50, 4)$

Find x s.t. $P(X > x) = 0.025$

$$\Rightarrow P\left(\frac{X - 50}{\sqrt{4}} > \frac{x - 50}{\sqrt{4}}\right) = 0.025$$

$$\Rightarrow P\left(Z > \frac{x - 50}{2}\right) = 0.025$$

$$z_\alpha = 1.96$$

$$\Rightarrow \frac{x-50}{2} = 1.96$$

$$\Rightarrow x = 2 \times 1.96 + 50 = 53.92$$

⑨ Suppose $X \sim N(28, 4)$.

Find x s.t. $P(X > x) = 0.1075$.

$$P(X > x) = 0.1738$$

$$P(X < x) = 0.67$$

⑩ Let $X_1 \sim N(4, 3)$, $X_2 \sim N(4, 4)$, $X_3 \sim N(2, 4)$,
 $X_4 \sim N(3, 2)$

$$Y = X_1 + 2X_2 + X_3 + X_4$$

What is the probability that
 $15 \leq Y \leq 20$,