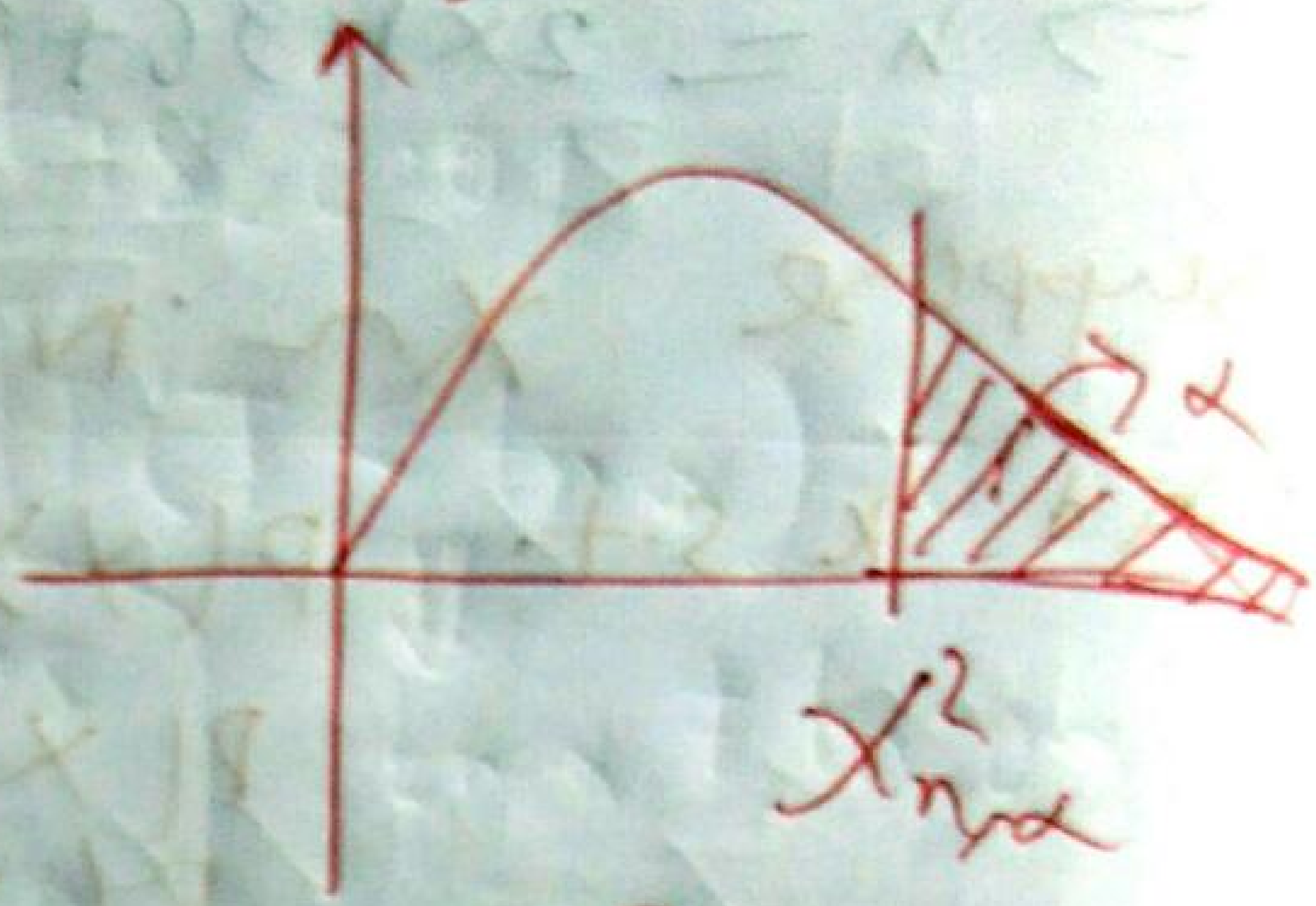
Exercise on T-dist^n and χ^2-dist^n

① Suppose $P(\chi^2 > \chi^2_{n,\alpha}) = \alpha$ find $\chi^2_{n,\alpha}$.



(i) ~~$P(\chi^2_n > \chi^2_{n,}$~~

$P(\chi^2 > \chi^2_{10,0.5}) = 0.05$

$\Rightarrow \chi^2_{10,0.5} = 0.9345$

(ii) $P(\chi^2 > \chi^2_{11,0.05}) = 0.05$

(iii) $\chi^2_{12,0.01}$ (iv) $\chi^2_{0.02}$ (v) $\chi^2_{0.5}$

② Suppose (i) $P(\chi^2 < \chi^2_{10,\alpha}) = 0.05$ what is $\chi^2_{10,\alpha}$?

$\Rightarrow P(\chi^2 < \chi^2_{10,\alpha}) = 0.05$

$\Rightarrow 1 - P(\chi^2 > \chi^2_{10,\alpha}) = 0.05$

$\Rightarrow P(\chi^2 > \chi^2_{10,\alpha}) = 0.95$

$\Rightarrow \chi^2_{10,\alpha} = \chi^2_{10,0.95} = 3.940$.

$\Downarrow$

$\chi^2_{10,\alpha} \neq \chi^2_{10,0.05}$

if $P(\chi^2 > \chi^2_{10,\alpha}) = 0.05$ then $\chi^2_{10,\alpha} = \chi^2_{10,0.05}$

(ii) $P(\chi^2 < \chi^2_{7,\alpha}) = 0.1$

(iii) ~~$P(\chi^2 < \chi^2_{0.98}$~~

$P(\chi^2 < \chi^2_{18,\alpha}) = 0.5$

(iv) $P(\chi^2 < \chi^2_{24,\alpha}) = 0.70$.

③ ~~Find the probabil~~

Let $W \sim \chi^2_7$. Then find the following probabilities?

(i) $P(W > 2.833)$ (ii) $P(W > 3.8)$

(iii) $P(W > 12)$, (iv) $P(W > 16.622)$

(v) $P(W < 1.564)$ (vi) $P(W < 2.83)$

(vii) $P(W < 6.346)$

④ Let $X_1, X_2, X_3, X_4 \sim N(\mu, \sigma^2)$.

Find (i) $P\left(\frac{(n-1)S^2}{\sigma^2} < 0.35\right)$ (iv) $P\left(\frac{S^2}{\sigma^2} < 3.78\right)$

(ii) $P\left(\frac{(n-1)}{n}\frac{S^2}{\sigma^2} < 0.5915\right)$ (v) $P\left(\frac{S^2}{\sigma^2} > 3.78\right)$

$$\Downarrow$$

$$= P\left(\frac{(n-1)S^2}{\sigma^2} < 0.5915 \times n\right)$$

$$= P\left(\frac{(n-1)S^2}{\sigma^2} < 0.5915 \times 4\right)$$

$$= P\left(\underbrace{\frac{(n-1)S^2}{\sigma^2}}_{\chi^2_{n-1}} < 2.366\right)$$

$$= \cancel{P}\ 1 - P(\chi^2_3 > 2.366)$$

$$= 1 - 0.5 = 0.5$$

~~(iii) P(~~ (iii) $P\left(\frac{S^2}{\sigma^2} < 1.547\right)$

$$= P\left(\frac{(n-1)S^2}{\sigma^2} < \cancel{(n-1)}\ 3 \times 1.547\right)$$

$$= P\left(\cancel{\frac{(n-1)S^2}{}}\ \chi^2_3 < 4.642\right) = 1 - 0.2$$

$$= 0.8$$

⑤ Find $t_{n,\alpha}$ s.t. $P(T>t_{n,\alpha})=\alpha$.

(i) $t_{4,0.1}$, (ii) $t_{12,0.025}$, (iii) $t_{18,0.01}$

(iv) $t_{25,0.005}$. (v) $t_{18,0.01}$

⑥ Find $t_{n,\alpha}$ s.t. ~~$P(T<t_{n,\alpha})=0.95$~~

$\Downarrow$

$t_{n,\alpha} \neq t_{n,0.95}$

(i) $P(T<t_{20,\alpha})=0.95$

(ii) $P(T<t_{9,\alpha})=0.90$

(iv) $P(T<t_{16,\alpha})=0.99$

⑦ Find

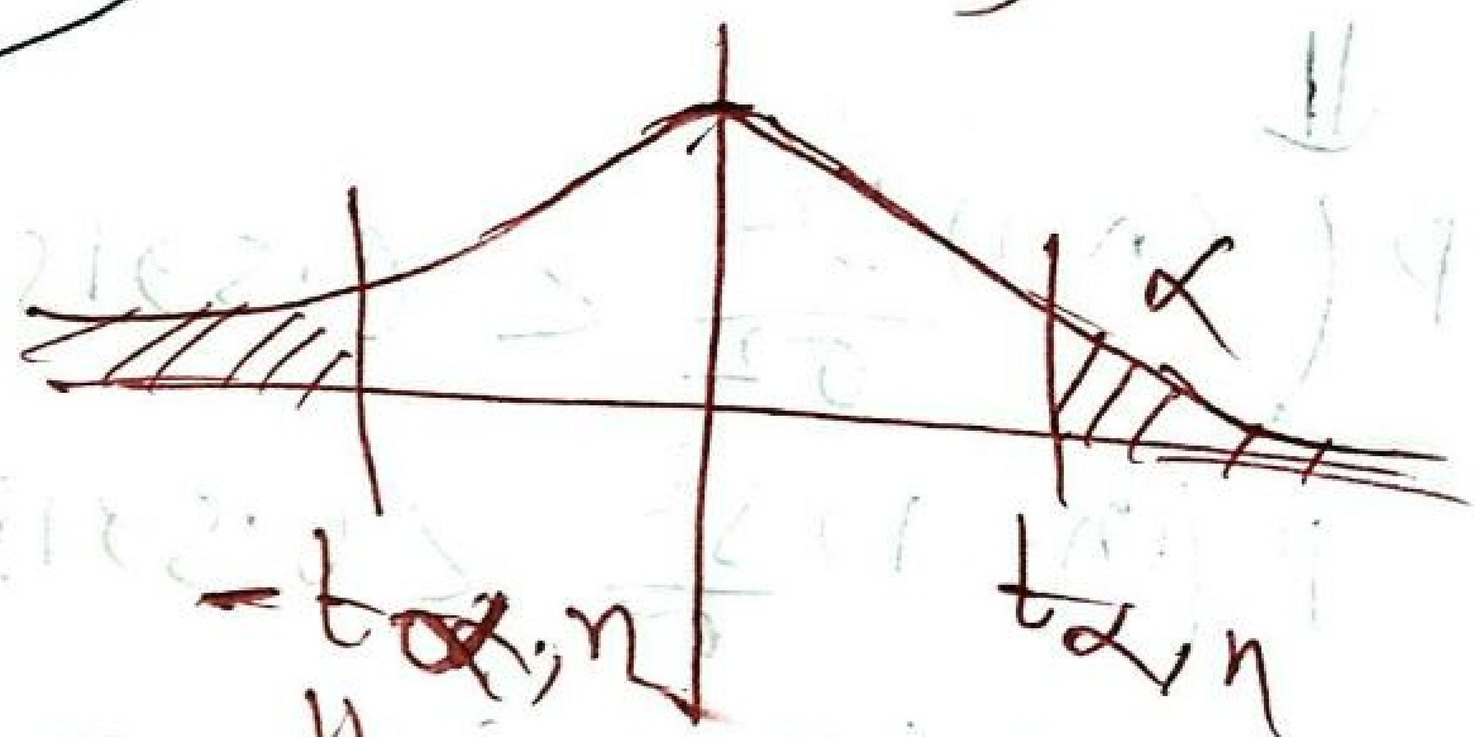
→(i) $t_{0.995,10}$ s.t. $P(T>t_{10,0.995})=0.995$

$P(\cancel{T>}\ t_{10,0.995}$

$=t_{10,1-0.005}$

$=-t_{0.005,10}$

$=-3.169$



$t_{1-\alpha,n}$.

$P(T>t_{\alpha,n})=\alpha$

$P(T<-t_{\alpha,n})=\alpha$

(ii) $t_{9,0.99}$ (iii) $t_{22,0.99}$

(iv) $t_{9,0.95}$ (v) $t_{16,0.90}$

⑧ ~~Find~~ Let $T\sim t_{17}$

$P(T>1.333)$, $P(T<1.734)$,

$P(T>2.567)$, $P(T<2.567)$

$P(T>-2.567)$ $P(T<-2.898)$

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⑨ Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

Find $P(|\bar{X}-\mu| \le 1.028S)$, $n = 10$.

$\Rightarrow P(\cancel{1.028 \le \bar{X}-\mu}$

$\Rightarrow P(|\bar{X}-\mu| \le 1.028S)$

$= P(-1.028S \le \bar{X}-\mu \le 1.028S)$

$= P\left(-\cancel{\frac{1.028S}{\sqrt{10}}} \le \cancel{\bar{X}}\right.$

$= P\left(-1.028 \le \frac{\bar{X}-\mu}{S} \le 1.028\right)$

$= P\left(-\sqrt{10}\times 1.028 \le \underbrace{\frac{\bar{X}-\mu}{S/\sqrt{10}}}_{\downarrow\, t_{10-1}} \le 1.028\times\sqrt{10}\right)$

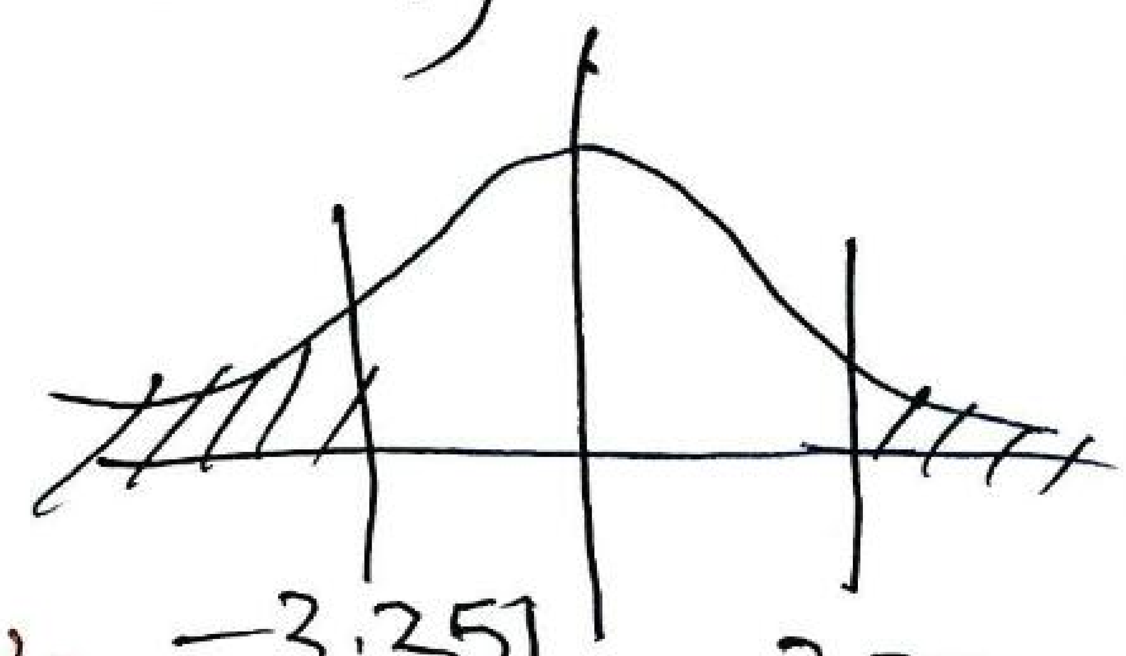
$= P(-3.251 \le T_9 \le 3.251)$ where $T = \frac{\bar{X}-\mu}{S/\sqrt{10}} \sim t_{10-1} = t_9$

$= P(T_9 \le 3.251) - P(T_9 \le -3.251)$

$= 1 - P(T_9 > 3.251) - P(T_9 \le -3.251)$

$= 1 - 0.005 - 0.005$

$= \cancel{1-2\times 0} = 0.99$



-3.251 $\quad$ 3.251

$P(T > 3.251)$

$= P(T < -3.251)$

⑩ Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

Let $S = 9$, $n = 12$.

(i) Find $P(\bar{X}-\mu \le \cancel{15.1753}\ 7.937)$

(ii) $P(|\bar{X}-\mu| \le 3.523)$

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