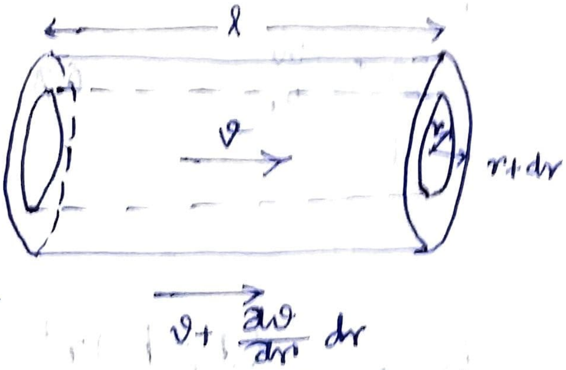
The Poiseuille formula :-



In a circular tube the flow is laminar, so that the cylinderical sheath at the boundary is stationary. As we move to the centre each successive cylinder sheath move with slightly higher velocity. The inner radius 'r' and outer radius r+dr. The velocity of inner sheath is 'v' and outer sheath 'v+dv'.

The volume of fluid flow at unit time will be $2\pi r v dr$ for inner sheath and total volume at any point at any time will be 'V'.

So
$$V = \int_0^a 2\pi r v \, dr.$$
a = radius of the tube.

We know ~~pres~~ force = pressure × Area.

Now if the pressure at left end p_1 and pressure of right end be p_2, Then

$$\text{force} = f_x = (p_1 - p_2)\, 2\pi r\, dr.$$

Now, at one meter² of area viscosity apply opposite to the flow $= -\eta\left(\frac{dv}{dr}\right)$. Here $A = 1\,m^2$. If surface area of inner sheath $S = 2\pi r dr$. ~~and outer sheath~~ Then total force $= \eta s \frac{dv}{dr}$. Now it is for outer sheath be $d\left(\eta s \frac{dv}{dr}\right) + \eta s\left(\frac{dv}{dr}\right)$

∴ Now net viscous force will be

$$\eta s \frac{dv}{dr} + d\left(\eta s \frac{dv}{dr}\right) - \eta s\left(\frac{dv}{dr}\right) = -2\pi r (p_1 - p_2)\, dr.$$

$$\Rightarrow d\left(\eta s \frac{dv}{dr}\right) = -2\pi r (p_1 - p_2)\, dr$$

Intregrating both side -

$$\eta s \frac{dv}{dr} = -2\pi (P_1 - P_2) r^2 + A$$

A = intregation constant.

$S = 2\pi r l$.

or $$\frac{dv}{dr} = \frac{-(P_1-P_2) r^2}{2\eta l} + \frac{A}{2\pi \eta l r}$$

$$dv = \frac{-(P_1-P_2) r}{2\eta l} dr + \frac{A}{2\pi \eta l r} dr.$$

Intregrating both side -

$$v = \frac{-(P_1-P_2) r^2}{4\eta l} + \frac{A}{2\pi l} \ln r + B$$

B = Second intregation constant

Now, $r = 0$; $\ln r$ = undeterminate.

so $$v = -\frac{(P_1-P_2) r^2}{4\eta l} + B$$ [discarding 2nd term]

$r = a$; $v = 0$

$$0 = \frac{-(P_1-P_2) a^2}{4\eta l} + B.$$

Putting the value of 'B' on above eqn

$$v = \frac{(P_1-P_2)(a^2-r^2)}{4\eta l}.$$

Now volume $$V = \int_0^a 2\pi r v \, dr.$$

$$V = \frac{\pi(P_1 - P_2)}{2\eta l} \int_0^a (a^2 - r^2)\, r\, dr$$

$$\boxed{V = \frac{\pi a^4 (P_1 - P_2)}{8\eta l}}$$

This is Poiseuille's formula.

This eqn can be rewrite as:

$$\boxed{V = -\frac{\pi a^2}{8\eta}\frac{dp}{dx}}$$

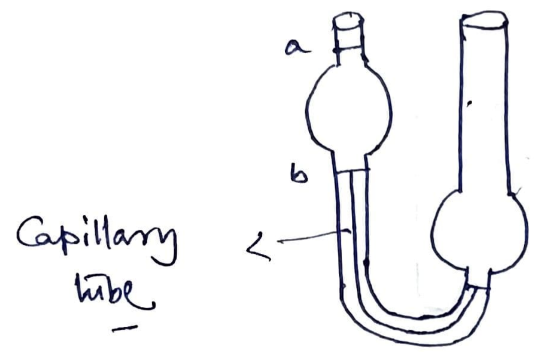
Stoke's Law :-

Another way to find η of a liquid to measure the rate of fall of a spherical solid through the liquid. The layer of fluid in contact with the ball moves along with it (no-slip condition) and a gradient of speed develops in the fluid surrounding the sphere's ~~moti~~. This gradient generates a viscous force F_{fr} resisting the sphere's motion. This viscous force is proportional to the speed of moving ball.

$$\boxed{F_{fr} = 6\pi\eta r v}$$ → This is Stoke's Law.

r = solid sphere radius
v = velocity of the moving sphere

Viscometer :-



The viscometer is an instrument for determining viscosity by measuring the time required for fixed volume of a liquid to flow through a capillary tube.
The pressure difference varies with time is proportional to the density of a liquid.

$$\frac{\eta_1}{\eta_2} = \frac{\rho_1 t_1}{\rho_2 t_2}$$

ρ_1 & ρ_2 is first determined by specific gravity bottle. The a known soln or liquid's t_1 is is determined by sucking the liquid in the bubble and recording it for the volume from 'a' mark to 'b' mark. At certain temp, the known soln or liquid's ρ_1 and η_1 are known. Further for the unknown soln or liquids t_2 & ρ_2 is determined. Then by the eqn η_2 is determined.

for liquid

$$\ln \eta = a + \frac{E}{RT} \quad \left[a \text{ \& } E \text{ are constant} \right]$$

i.e. with increase of 'T', η decrease.