~~(1) Let $X \sim Bin(10, p)$~~

MME and MLE

(1) Find the MMEs for

(i) $X_1, X_2, \cdots, X_n \sim Gamma(\alpha, \lambda)$

(ii) $X_1, X_2, \cdots \cdots X_n \sim Geo(p)$

(2) Let $X \sim Bin(10, p)$, $0 \leq p \leq 1$. Find the MLE and MME of p and check if they are unbiased.

(3) Consider a random sample $X_1, X_2, \cdots, X_n$ from a double exponential distribution with ~~distribution~~ density

$$f(x, \theta, \sigma) = \frac{1}{\sigma} \exp\left(-\frac{|x-\theta|}{\sigma}\right), \quad x > \theta, \ \theta \in \mathbb{R}, \ \sigma > 0.$$

Find MME's and MLE's of θ and σ.

If a random sample has observations 36.2, 41.3, 31.6, 38.4, 44, 33.9, 37.1, 43.6, 36.8, 37.2, 31.7 and 36, evaluate MMEs and MLEs of θ and σ

(4) Let $X_1, X_2, \cdots, X_n$ be a random sample from a $N(14, \sigma^2)$ population. Find the MME and the MLE of σ^2. Show that they are unbiased and consistent.

A random sample from this population is:

14.3, 12.5, 14.7, 13.9, 14.4, 14.3, 14.5, 13.7, 13.6, 14.1.

Evaluate the MME and the MLE of σ^2.

(5) Let $X \sim U(0, \theta)$. The data obtained on 10 test rods are:

10, 7, 11, 12, 8, 8, 9, 10, 9, 13.

Find MME and MLE of θ.

(6) Assume that X is a a gamma r.v. with b and α unknown. On the basis of 15 observations on X given below evaluate the MME's of b, α:

1.2, 2.0, 1.6, 1.8, 1.1, 2.5, 2.1, 2.6, 2.2, 1.7, 1.5, 2.0, 3.0, 1.8.

Suppose $b=4$, then find the MLE of α.

(7) Let $x_1, x_2, \ldots x_n$ be a random sample from a continuous distribution with the probability density function

$$f(x, \alpha, \beta) = \begin{cases} \frac{\alpha x^{\alpha-1}}{\beta^\alpha} ; & 0 \le x \le \beta \\ 0 ; & \text{otherwise} \end{cases}$$

Find the MMEs and MLEs of α and β.

(8) Lengths of pins (mm) produced by a machine follow a $N(\mu, \sigma^2)$ distribution. Find the maximum likelihood estimators of μ and σ^2 based on a random sample of size 10 with observations:

7.12, 7.13, 7.01, 6.95, 6.89, 6.97, 6.99, 6.93, 7.05, 7.02.

(9) $X_1, X_2, \cdots, X_n \sim U(a, b)$. Find MLE's of a & b

(10) The combined weights of passengers and their luggage (in kg) are uniformly distributed on the interval (a, b). The weights observed for a random sample of 9 passengers were: 90, 135, 120, 127, 115, 108, 96, 112. Find the method of moments estimators of a and b, Find MLEs of and b.