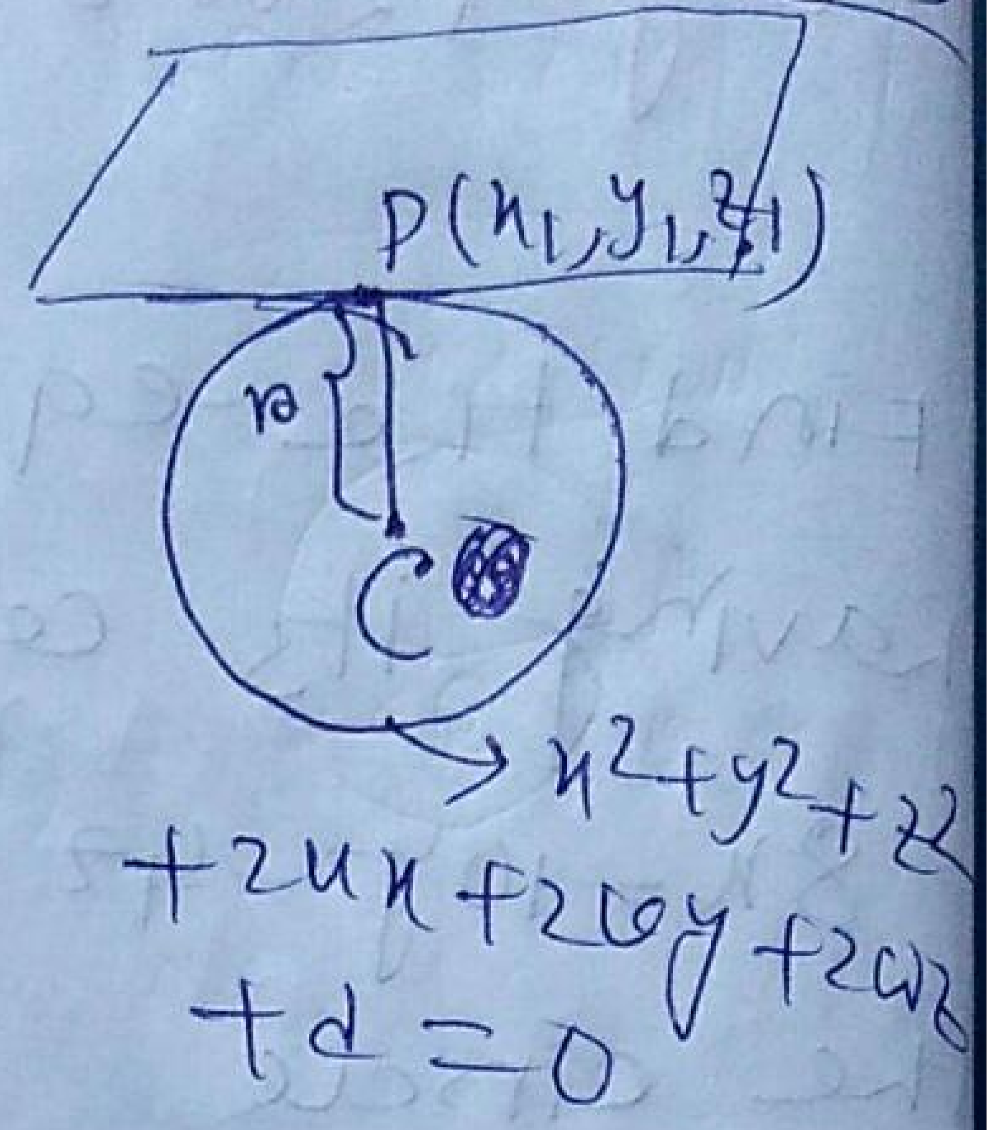
Equation of Tangent plane at pt. (x_1, y_1, z_1)

Find equation of tangent plane (x_1, y_1, z_1) whose equation of sphere is $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ —— (1)



$\Rightarrow$ Centre of the sphere is $C(-u, -v, -w)$.

CP is normal to the tangent plane.

DR's of CP are given by

$x_1+u,\ y_1+v,\ z_1+w$.

Equation of tangent plane is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$a=x_1+u,\ b=y_1+v$
$c=z_1+w$

$\Rightarrow (x_1+u)(x-x_1)+(y_1+v)(y-y_1)+(z_1+w)(z-z_1)=0$

or, $xx_1-x_1^2+ux-ux_1+y_1y-y_1^2+vy-vy_1+z_1z-z_1^2+wz-wz_1=0$

or, $xx_1+yy_1+zz_1+ux+vy+wz+ux_1+vy_1+wz_1$
$=x_1^2+y_1^2+z_1^2+ux_1+vy_1+wz_1+ux_1+vy_1+wz_1$

or, $xx_1+yy_1+zz_1+u(x+x_1)+v(y+y_1)+w(z+z_1)$
$=x_1^2+y_1^2+z_1^2+2ux_1+2vy_1+2wz_1$ —(2)

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Now (x_1, y_1, z_1) lies on the sphere (1) ;

$$\therefore \quad x_1^2 + y_1^2 + z_1^2 + 2ux_1 + 2vy_1 + 2wz_1 = d \quad \text{---(3)}$$

Using (3) in (2) we get,

$$xx_1 + yy_1 + zz_1 + u(x + x_1) + v(y + y_1) + w(z + z_1) + d = 0$$

$\longrightarrow$ Equation of required tangent plane.

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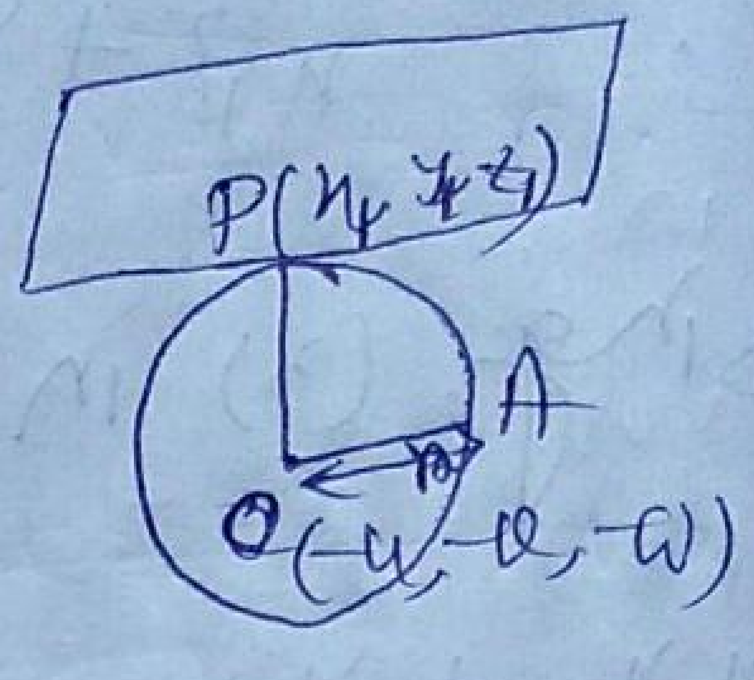
(4) Prove that the given plane is tangential to the given sphere and find point of contact.



⇒ Let the equation of plane is $ax+by+cz+d=0$ —(1)

Equation of sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d'=0$ —(2)

OP = perpendicular distance from centre $C(-u,-v,-w)$ to the plane (1).

OA = radius of the sphere 2.

$OP = OA$ (tangential condition)

i.e., $OP = r$

i.e., $\left|\dfrac{-au-bv-cw+d}{\sqrt{a^2+b^2+c^2}}\right| = \sqrt{u^2+v^2+w^2-d'^2}$

<u>To find point of contact</u>

Let $P(x_1, y_1, z_1)$ is the point of contact

Equation of the line OP is

$\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$ (a, b, c are the DR's of the line OP)

Let (x, y, z) be any point on the line

then

~~for~~ Point of contact

Let $P(x_1, y_1, z_1)$ be the point of contact.

Equation of the line OP is

$$\frac{x-(-u)}{a} = \frac{y-(-v)}{b} = \frac{z-(-w)}{c}$$

i.e., $$\frac{x+u}{a} = \frac{y+v}{b} = \frac{z+w}{c} \quad -(3)$$

Point of contact P is on the line (3)

so, $$\frac{x_1+u}{a} = \frac{y_1+v}{b} = \frac{z_1+w}{c} = k \text{ (say)}$$

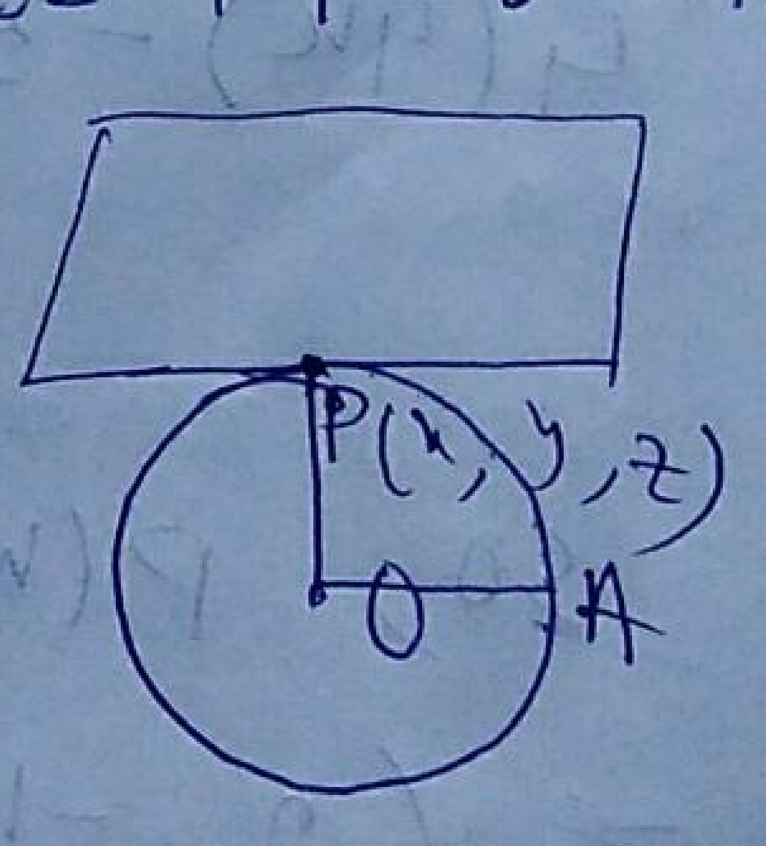
$\Rightarrow$ $x_1 = ak-u$, $y_1 = bk-v$, $z_1 = ck-w$ —(4)

(x_1, y_1, z_1) lies on the plane (1)

so, $a(ak-u)+b(bk-v)+c(ck-w)+d=0$

$\Rightarrow$ find k and put in (4).

① ~~Ex~~ Prove that the plane $4x-3y+6z-35=0$ is tangential to the sphere $x^2+y^2+z^2-y-2z+4=0$ and find point of contact.



$\Rightarrow$ ~~Eq~~ centre of the sphere is $O(0, \frac{1}{2}, 1)$

radius $= \frac{\sqrt{61}}{2}$

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perpendicular distance from the point O to the given plane is

$$= OP = \left| \frac{0\times 4 - \frac{3}{2} + 6 - 35}{\sqrt{16+9+36}} \right| = \frac{\sqrt{61}}{2} = r.$$

Hence the given plane is ~~perpendicular to the~~ the tangent plane to the sphere.

Point of contact

DR's of the line OP are 4, −3, 6.

Equation of the line OP is

$$\frac{x-0}{4} = \frac{y-1/2}{-3} = \frac{z-1}{6} \quad -(1)$$

Now the point of contact P(x, y, z) is on the line (1). so,

$$\frac{x}{4} = \frac{y-1/2}{-3} = \frac{z-1}{6} = k \text{ (say)}$$

$$\Rightarrow x = 4k,\ y = -3k + \frac{1}{2},\ z = 6k+1$$

Since P lies on the plane

$$4(4k) - 3\left(-3k + \frac{1}{2}\right) + 6(6k+1) = 35$$

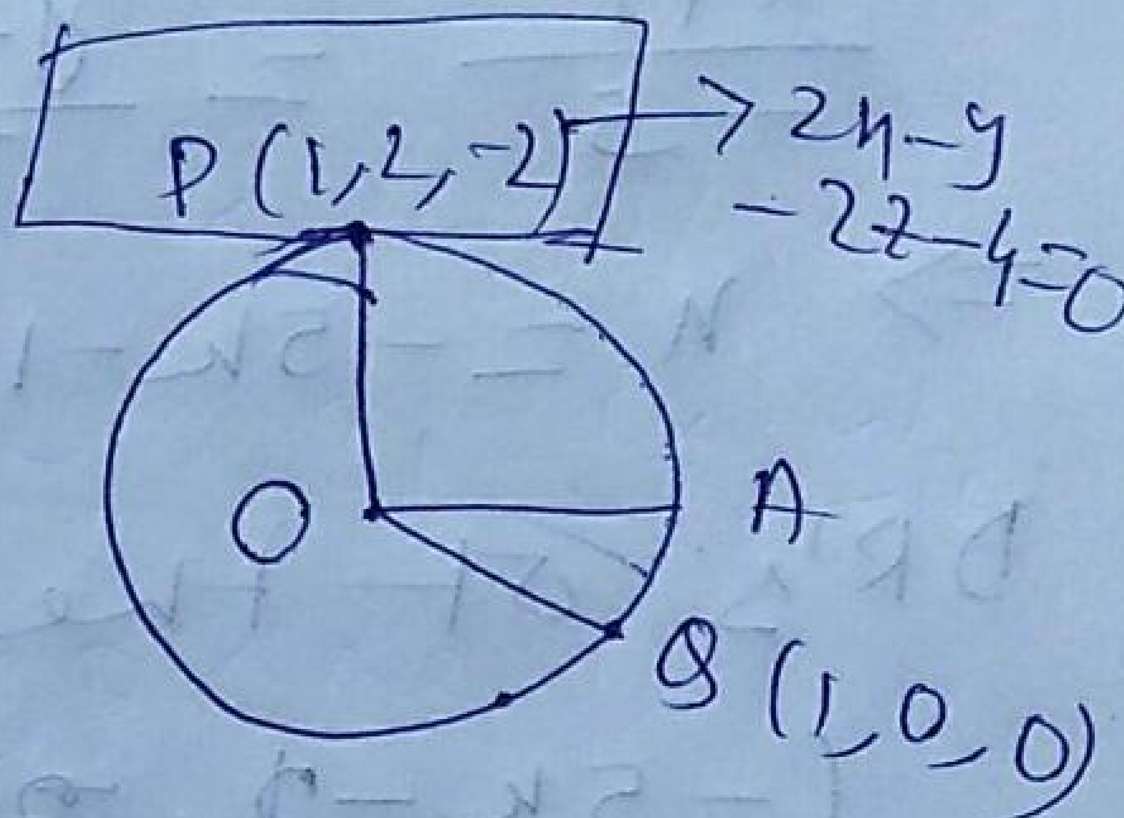
$$\Rightarrow k = 1/2$$

so, P(x, y, z) → pt. of contact

= (2, −1, 4).

(2) Prove that the plane $2x-2y+z+12=0$ ~~x^2+~~ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and also find point of contact.

(3) Find the equation of the sphere which passes through the point $(1,0,0)$ and touches the plane $2x-y-2z-4=0$ at $(1,2,-2)$.



$\Rightarrow$ Target is to find the coordinate of centre and radius.

Equation of the line OP at P(1, 2, -2) is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+2}{-2} = k \text{ (say)}$$

Point anywhere on that line is

$x = 2k+1$, $y = -k+2$, $z = -2k-2$

Now, let O is $(2k+1, -k+2, -2k-2)$

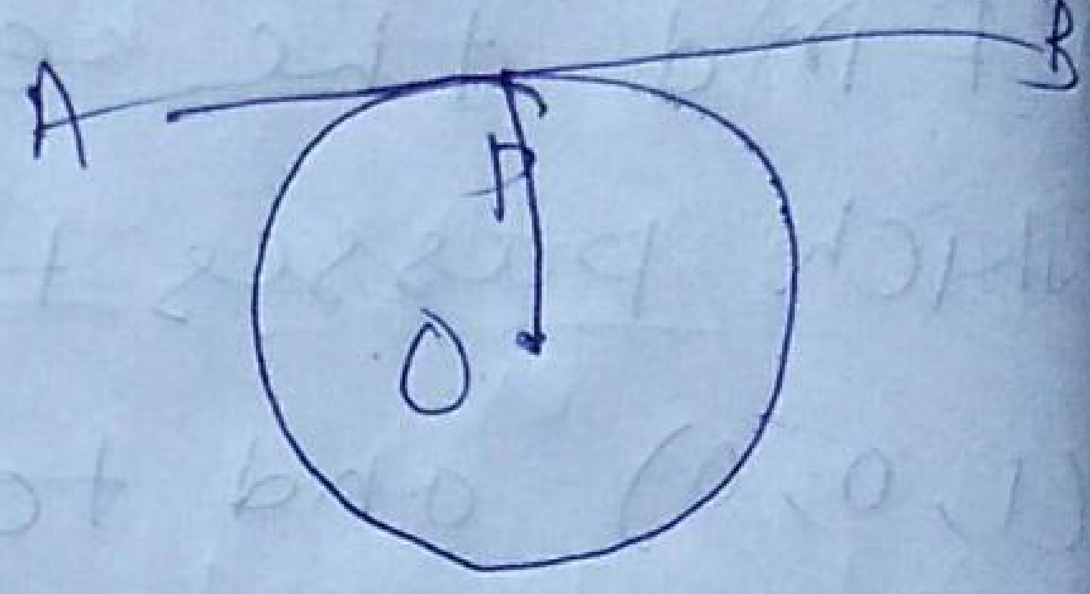
$OP^2 = OQ^2$

$\Rightarrow k = 2$

So, Radius $= OP$, $O(x, y, z)$
$= (-3, 4, 2)$

④ Find the equation of the sphere which ~~passes through~~ has centre $(2,3,-1)$ and touches the line

$$\frac{x+1}{-5} = \frac{y-8}{-3} = \frac{z-4}{4}.$$



$\Rightarrow$ Let $P(x,y,z)$ be any point on the line AB:

$$\frac{x+1}{-5} = \frac{y-8}{3} = \frac{z-4}{4} = k$$

$\Rightarrow x = -5k-1,\ y = 3k+8,\ z = 4k+4$

DR's of the line OP are

$(-5k-3,\ 3k+5,\ 4k+5)$

DR's of the line AB are

$-5, 3, 4.$

Since AP is tangent at P

$OP \perp AB$

So, $-5\cdot(-5k-3)+3(3k+5)+4\cdot(4k+5)=0$

$\Rightarrow k=-1$

Hence $P(x,y,z) = (4,5,0)$

So, radius $r = OP = \sqrt{(2-4)^2+(3-5)^2+(-1-0)^2}$

$= \sqrt{4+4+1} = 3$

Centre is given $(2,3,-1)$, radius is 3, so equn of sphere is $\rightarrow (x-2)^2+(y-3)^2+(z+1)^2=9$