The Sphere

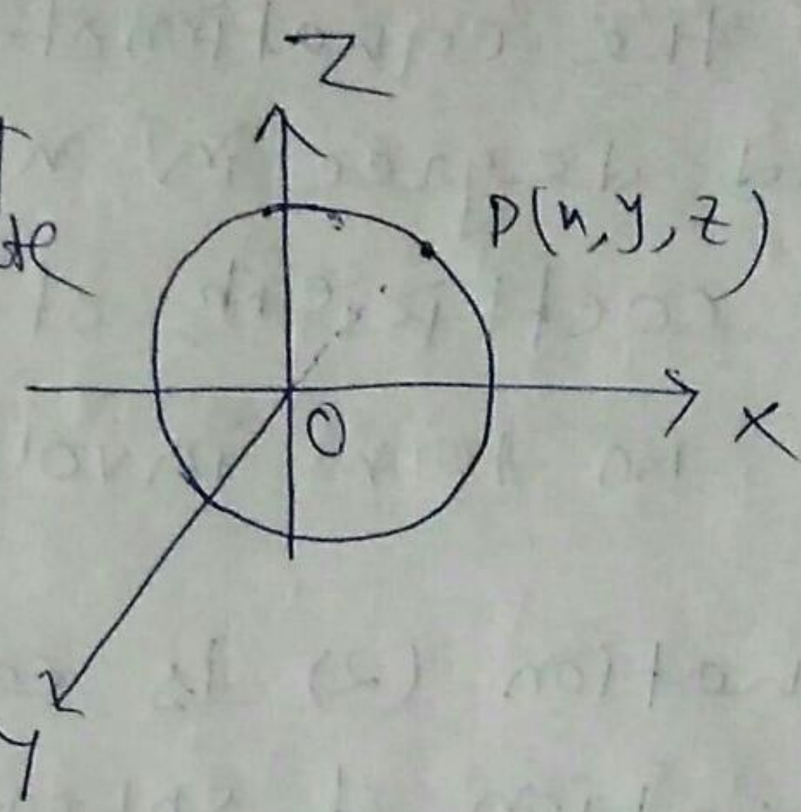
Def^n:- A sphere is the locus of a point which moves so that it's distance from a fixed point is always constant.

A sphere is a 3D-diagram.

Equation of a sphere in standard form:-

Let $P(x,y,z)$ be any point in 3D-coordinate system.

$O(0,0,0)$ be the fixed point.

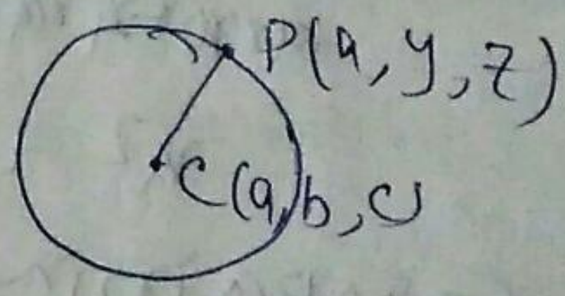


Let $OP = r$ (constant)

$$\Rightarrow \sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = r$$

$$\Rightarrow \boxed{x^2+y^2+z^2=r^2}$$ → Equation of a sphere in standard form.

Central form of the equation of a sphere :—



Let $P(x,y,z)$ be any point on the sphere and let $C(a,b,c)$ be the fixed, called centre of the sphere.

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Let CP = r

$$\Rightarrow \sqrt{(x-a)^2+(y-b)^2+(z-c)^2} = r$$

$$\Rightarrow (x-a)^2+(y-b)^2+(z-c)^2 = r^2 \quad -(1)$$

$$\Rightarrow x^2+y^2+z^2-2ax-2by-2cz+a^2+b^2+c^2-r^2=0 \quad -(2)$$

From equation (2) we note that

(i) the equation of a sphere is of 2nd degree in x, y, z.

(ii) coefficients of x^2, y^2, z^2 are equal.

(iii) No term involving xy, yz, zx.

Equation (2) is called general equation of sphere.

- Coordinate of centre and radius of a sphere from General equation of 2nd degree :-

Consider the general equation of 2nd degree =

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

Compairing with

$$x^2+y^2+z^2-2ax-2by-2cz+a^2+b^2+c^2-r^2=0$$

we get,

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$u = -a, v = -b, w = -c$

$d = a^2 + b^2 + c^2 - r^2$

$\Rightarrow r^2 = a^2 + b^2 + c^2 - d$.

So, $(-u, -v, -w)$ → coordinate of centre

$r = \sqrt{u^2 + v^2 + w^2 - d}$ → radius

Find the centre and radius of the following sphere —

(i) $x^2 + y^2 + z^2 + 2x - 4y + 6z - 2 = 0$

Compare it with $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

Then,

$u = 1, \quad v = -2, \quad w = 3, \quad d = -2$

So, Centre → $(-u, -v, -w) \equiv (-1, 2, -3)$

radius → $\sqrt{1 + 4 + 9 + 2} = \sqrt{16} = 4$.

(ii) $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z - 5 = 0$

Dividing throughout by 2 —

$x^2 + y^2 + z^2 - x + 2y + z - \frac{5}{2} = 0 \quad -1$

Compare & calculate → $(\frac{1}{2}, -1, -\frac{1}{2})$ (Centre)

→ $\sqrt{4} = 2$ (radius)

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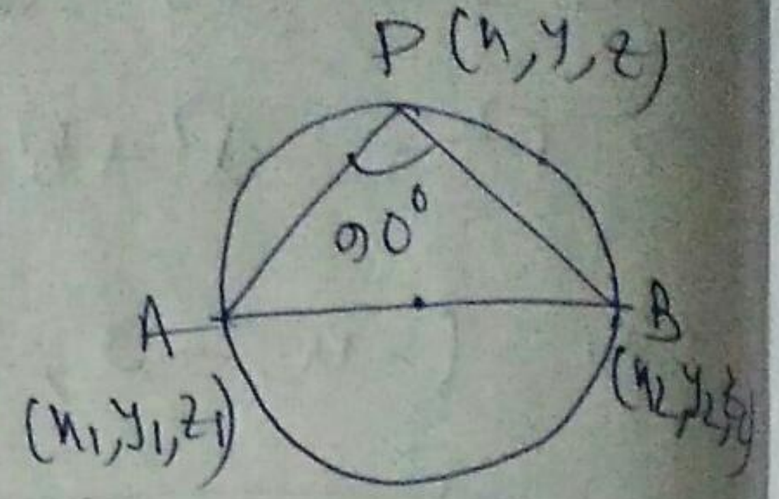
Equation of a sphere when coordinate of extreme points of diameter is given:-



Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be the extremities of the diameter of a given sphere.

Let $P(x, y, z)$ be any point on the sphere. Then PA and PB are at right angles.

Direction ratios of PA are

$(x-x_1, y-y_1, z-z_1)$.

DR's of PB are $-(x-x_2, y-y_2, z-z_2)$.

As PA and PB are at right angles,

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$.

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Find radius and centre of following spheres (if it represents a sphere):-

(i) $2x^2+3y^2+3z^2+2x+4y+3=0$

(ii) $2x^2+2y^2+2z^2+2x+3y+4z=0$

(iii) $x^2+y^2+z^2+x+2y+4=0$

(iv) $x^2+y^2+z^2+2xy+4x+7z+3=0$

(v) $4(x^2+y^2+z^2)+2x+4z+9=0$

(vi) $x^2+2y^2+z^2+4x+9y=0$

(vii) $2(x^2+y^2+z^2)+2x+2y+3z+4yz+7xz=0$

(viii) $x^3+2x^2+3y^2+4xy^3+2x+4y+9=0$

(ix) $x^2+y^2+z^2+7y-2z+2=0$

(x) $4(x^2+y^2+z^2)-2x+4y+6z=4$

(xi) $7x^2+9y+2x+4z=4$

(xii) $4x^2+4y^2+2x+4z+6y=9$

(xiii) $x^2+y^2+z^2=4$

(xiv) $x^2+2z^2+4x+27y=0$

(xv) $x^2+y^2+z^2+2x+4y+6z+8xyz=0$

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Find the equation of the sphere having —

(i) Centre at (0, 0, 0) and radius equal to 4 units.

(ii) Centre at (0, 2, -1) and radius 4 units

(iii) Centre at (1, 2, 3) and diameter equal to 8 units.

(iv) Centre at (0, 0, 0), and passing through the point (1, 2, 3)

(v) passes through (1, 0, 0), (0, 2, 3), (0, 0, 0), (2, 3, 0).

(vi) Passes through (1, -1, 1), (2, 1, 3), (-2, 0, 5), (-1, 4, 4).

(vii) ~~Coordinates of two extreme with a diameter~~

With a diameter, whose extreme points are (1, 2, 1), (-1, 0, 1).

(viii) a diameter with extreme points (4, 2, 3), (1, 2, 4).

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(1) Obtain the equation of the sphere passing through the points (0, −1, 2), (2, 3, 4) and having its centre on the straight line $2x-3y=0=5y+2z$.

(2) Find the equation of the sphere passing through the points (3, 1, −3), (−2, 4, 1), (−5, 0, 0) and whose centre lies on the plane $2x+y-z+3=0$

(3) The plane $x+2y+3z=6$ meets the co-ordinate axes in A, B, C. Find the equation of the sphere OABC, O being the origin; and determine the centre and radius of the sphere.

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