Title of the Unit: Gamma Decay, Nuclear reactions and Nuclear Models:

Unit Structure:

3.0 Nuclear Models

INTRODUCTION

As discussed in SLM 44, the size of the nucleus is very small and nuclear forces are far more complicated than other well-known forces. In fact, the picture of nuclear forces is still not clear. This picture is different from the case of atom, where the forces are known and atomic model is well established for deducing various properties in atomic domain. Due to the lack of detailed knowledge of nuclear forces, nuclear models, namely liquid drop model, shell model, Nilson model, Fermi gas model, collective model, Bohr Motelson model, interacting boson model, etc. have been developed, each of which is useful in a more or less limited fashion.

In order to understand and predict the properties of the nucleus, we have to know the forces completely. For knowing nuclear forces, we adopt a different approach. In nuclei, we choose an oversimplified theory, the treatment of which is mathematically possible, but the theory should be rich in physics. If this theory is fairly successful in accounting for at least a few properties of the nucleus, we can then improve the model by adding additional terms so that it is capable to account more nuclear properties. In this way, we construct a nuclear model, a simplified view of nuclear structure, which still contains the essentials of the nuclear properties. A good nuclear model must satisfy following two criteria:

- It must reasonably well account for previously measured nuclear properties.
- It must predict additional nuclear properties that can be measured in new experiments.

The development of nuclear models has taken place along the following lines. In the first type of nuclear models, nucleus has been treated like a drop of liquid, in which nucleons present in the nucleus interact very strongly among themselves. This is like molecules present in a drop of liquid, which interact among themselves very strongly. This treatment gave rise to models like *liquid drop model, collective model,* etc. The second type of models is constructed in analogy with the shell model of the atom. In these models, the nucleons are weakly interacting among themselves. This treatment gave rise to *Fermi gas model, shell model, Nilson model,* etc.

In this chapter, we discuss only two models, i.e. liquid drop model and shell model. In the end, a brief description of Collectiveis given.

3.1 LIQUID DROP MODEL

Bethe- Weizsacker in 1935 proposed on the basis of experimental facts that a nucleus resembles a drop of liquid. In 1939, Bohr and Wheeler further

developed this model to explain the phenomenon of nuclear fission.
Following are some of the similarities between a drop of liquid and nucleus, which prompted Weizsacker to develop the liquid drop model.

Similarities between Liquid Drop and Nucleus

1. Nuclear forces are analogous to the surface tension of a liquid.
2. The nucleons behave in a manner similar to that of molecules in a liquid drop.
3. The density of the nuclear matter is almost independent of *A*, showing resemblance to liquid drop where the density of a liquid is independent of the size of the drop.
4. The constant binding energy per nucleon is analogous to the latent heat of vaporization.
5. The disintegration of nuclei by the emission of particles is analogous to the evaporation of molecules from the surface of liquid.
6. The absorption of bombarding particles by a nucleus corresponds to the condensation of drops.
7. The energy of nuclei corresponds to internal thermal vibrations of drop molecules.

Based on these similarities, Weizsacker in 1935 and Bohr and Wheeler in 1939 developed liquid drop model. They ignored the finer features of nuclear forces but strong internucleon attraction is stressed.

Assumptions of the Liquid Drop Model

1. The nucleus consists of incompressible matter.
2. The nuclear force is identical for every nucleon.
3. The nuclear force saturates.
4. In an equilibrium state, the nuclei of atom remain spherically symmetric under the action of strong attractive nuclear forces.

3.1.1 Semiempirical Mass Formula

The analogy between nucleus and liquid drop has been used to set up a semiempirical formula for mass (or binding energy) of a nucleus in its ground state. The formula has been obtained by considering different factors of the nucleus binding.
The mass of the nucleus can be expressed in terms of the total binding energy *B* and the masses of *Z* protons and *N* neutrons as

$$\boxed{M = ZM_p + NM_n - B} \qquad (2.1)$$

The binding energy *B* of a nucleus is given by the sum of five terms as

$$B = B_1 + B_2 + B_3 + B_4 + B_5 \qquad (2.2)$$

which are explained in the following sections.

Volume Energy Term (B_1)

The volume term arises from the interaction of the nucleons through the strong force. When a liquid drop evaporates, the energy required for this process is the product of mass of the drop M_m and latent heat of vaporization L. This energy is used to break all the molecular bonds. This is same as the binding energy of the drop B. So

$$B = LM_m N \qquad (2.3)$$

where N is the number of molecules in the drop. Equation (2.3) can also be written as

$$\frac{B}{N} = LM_m = \text{constant} \qquad (2.4)$$

This means that B/N is independent of the number of molecules present in the liquid drop. As we know that in the liquid drop, a molecule interacts only with its nearest neighbours and number of neighbours is independent of the size of the drop. This characteristics of the system shows that range of interaction among the molecules is much smaller than the dimensions of the drop.

In SLM 44, we have seen that neutrons and protons are held together in nuclei by short- range attractive forces. These forces reduce the mass of the nucleus below that of its constituents by an amount proportional to the number of nucleons A. Since the volume of the nucleus is proportional to A, hence this term is regarded as a volume binding energy and in analogy to Eq. (2.4) is given by

$$\boxed{B_1 = a_v A} \qquad (2.5)$$

where a_v is a proportionality constant and subscript v is for volume.

Surface Energy Term (B_2)

The surface term is a correction to the volume term to take into account that the nucleons at the surface of the nucleus do not have the same level of interactions as nucleons in the interior of the nucleus. In the above discussion, we have assumed that all the molecules are surrounded by its neighbours, while in actual practice the molecules at the surface do not have any neighbours on all the sides. So these molecules are not as tightly bound as the molecules in the interior. Extending this argument to the nuclear case, some nucleons are nearer to the surface, and so they interact with fewer nucleons. Thus, the binding energy is reduced by an amount proportional to the surface area of the nucleus of radius r as the nucleons on the surface are less tightly bound than those in the interior. This term is proportional to the surface area of the nucleus of radius $r(= r_0A^{1/3})$. Therefore,

$$B_2 \propto -4\pi r^2$$

Or $$B_2 \propto -4\pi r_o^2 A^{2/3}$$

which is usually expressed as

$$\boxed{B_2 = -a_s A^{2/3}} \tag{2.6}$$

where negative sign is for decrease in energy and a_S is constant.

Coulomb Energy Term (B3)

The Coulomb term represents the energy incorporated in the nucleus as a result of the positive charge present in the nucleus. The only long-range force in the nucleus is the Coulomb repulsion between protons. The total work done in assembling a nucleus consisting of Z protons is given by

$$W = \frac{\frac{3}{5} Z^2 e^2}{4\pi \varepsilon_0 r}$$

where r is the radius of the nucleus.

For a single-proton nucleus

$$w = \frac{\frac{3}{5} e^2}{4\pi \varepsilon_0 r}$$

For a nucleus having Z protons

$$w' = \frac{\frac{3}{5} Z e^2}{4\pi \varepsilon_0 r}$$

For a single-proton nucleus no work is done against Coulomb repulsion in assembling the nucleus. Thus, the true Coulomb energy term for a nucleus containing Z protons is $W - w'$.

$$B_3 = -\left[\frac{\frac{3}{5} Z^2 e^2}{4\pi \varepsilon_0 r} - \frac{Z \frac{3}{5} e^2}{4\pi \varepsilon_0 r}\right]$$

i.e.

$$B_3 = -\frac{3}{5} \frac{Z(Z-1) e^2}{4\pi \varepsilon_0 r} \tag{2.7}$$

The negative sign indicates the repulsive term. As $r = r_0 A^{1/3}$, Eq. (2.7) can be written as

$$\boxed{B_3 = -a_c \frac{Z(Z-1)}{A^{1/3}}} \tag{2.8}$$

where a_C is constant.

Asymmetry Energy Term (B4)

The asymmetry term reflects the stability of nuclei with the proton and neutron numbers being approximately equal. This is a term, which depends on the neutron excess ($N - Z$) in the nucleus and it decreases with the increasing nuclear binding energy. For very few nuclei of low Z, $N - Z = 0$ and are more stable compared to their neighbours, i.e. their binding energies are maximum. The reduction in binding energy for higher A nuclei is directly proportional to $(N - Z)^2$ or square of excess of neutrons and is inversely proportional to mass number. So, we can write,

$$B_4 \propto \frac{(N-Z)^2}{A}$$

$$\boxed{B_4 = -a_a \frac{(A-2Z)^2}{A}} \tag{2.9}$$

As $A = N + Z$ and a_a is constant.

***Pairing Energy Term* (B_5)**

So far we have all the terms in the binding energy have smooth variation with respect to N or Z or A. However, in the actual binding energy versus A curve, there are several discontinuities, particularly when N or Z becomes equal to 2, 4, 8, 20, 28, 50, 82 or 126. These values correspond to shell closure for N or Z. The nuclei having N or Z equal to one of these numbers have large binding energy. This fact did not appear in the liquid drop model, which does not consider intrinsic spin of the nucleons and the shell effects.

It is interesting to classify all the stable nuclei into four groups, first having even Z–even N, second even Z–odd N, third odd Z–even N and last having odd Z–odd N. This classification is shown in Table 3.1.

TABLE 3.1 Number of stable isotopes

Z	*N*	*Number of stable nuclei*
Even	Even	165
Even	Odd	55
Odd	Even	50
Odd	Odd	5

(the five stable odd Z–odd N nuclei are: ${}^{2}_{1}H$, ${}^{6}_{3}Li$, ${}^{10}_{5}B$, ${}^{14}_{7}N$, ${}^{180}_{73}Ta$)

From Table 2.1, it is clear that even Z–even N nuclei, being most stable, are most abundant. Accordingly, odd Z–odd N nuclei are least abundant and hence least stable. The remaining nuclei have intermediate stability. Therefore, the binding energy also depends upon whether the number of protons and neutrons are odd or even. This pairing effect was incorporated by putting

$$\boxed{B_5 = a_p A^{-3/4}} \tag{2.10}$$

where

a_p = 33.5 MeV for even–even nuclei

= 0 for odd–even (or odd A) nuclei

= –33.5 MeV for odd–odd nuclei

Substituting the values of B_1, B_2, B_3, B_4 and B_5 from Eqs. (2.5), (2.6), (2.8),

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

(2.9) and (2.10) in Eq. (2.2), we get
Substituting the value of B from the above equation in Eq. (2.1), we get the semiempirical mass formula as

$$\boxed{M = ZM_p + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} - a_p A^{-3/4}} \quad (2.11)$$

The various constants found are,

$$a_v = 15.5 \text{ MeV}$$
$$a_s = 16.8 \text{ MeV}$$
$$a_c = 0.7 \text{ MeV}$$
$$a_a = 23.0 \text{ MeV}$$
$$a_p \begin{cases} = 34 \text{ MeV} & \text{for even–even nuclei} \\ = 0 \text{ MeV} & \text{for odd } A \text{ nuclei} \\ = -34 \text{ MeV} & \text{for odd–odd nuclei} \end{cases}$$

The contribution of various terms for few cases are given (in MeV) in Table 3.2.

TABLE 3.2 Contribution of various terms of semiempirical mass formula for some isotopes

Term	$^{40}_{20}$Ca	$^{120}_{50}$Sn	$^{238}_{92}$U
Volume	+620	+1860	+3689
Surface	−196	−409	−645
Coulomb	−80	−358	−973
Asymmetry	0	−77	−282
Pairing	+2	+1	+0.6
Resultant BE	346	1017	1789.6
BE/n	8.65	8.48	7.52

The semiempirical mass formula reproduces masses of various nuclei quite accurately, but does not account for all the features of the nuclear binding energy.

3.1.2 Mass of Most Stable Isobar

Isobars are nuclides that have same mass number A. The semiempirical mass formula can predict the atomic number Z_0 of most stable isobar for given mass number A.

Neglecting 1 in comparison to Z in the Coulomb term and rewriting Eq. (2.11) as

$$M(Z, A) = aA + bZ + cZ^2 + \delta \tag{2.12}$$

where

$$a = M_n - \left[a_v - a_a - \frac{a_s}{A^{1/3}}\right]$$

$$b = -4a_a - (M_n - M_p)$$

$$c = \left(\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right)$$

and

$$\delta = \mp a_p A^{-3/4}$$

Let us find the atomic number of most stable isotope for a given A. This can be calculated by taking the partial derivative of Eq. (2.12) with respect to Z keeping A as constant and equating the resultant equation to zero, i.e.

$$\left(\frac{\partial M}{\partial Z}\right)_A = b + 2cZ = 0$$

which gives

$$Z_0 = Z = -\frac{b}{2c}$$

where Z_0 is the atomic number of most stable isotope for given A. Substituting the values of b and c, we obtain

$$Z_0 = -\frac{-4a_a - (M_n - M_p)}{2\left[\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right]}$$

Since all the quantities in this expression are known, atomic number for most stable isobar can be calculated.

3.1.3 Achievements of Liquid Drop Model

- It predicts the atomic masses and binding energies of various nuclei accurately.
- It predicts emission of *a*- and *b*-particles in radioactivity.
- The theory of compound nucleus, which is based on this model, explains the basic features of the fission process.

3.1.4 Failures of Liquid Drop Model

- It fails to explain the extra stability of certain nuclei, where the numbers of protons or neutrons in the nucleus are 2, 8, 20, 28, 50, 82 or 126 (these numbers are called magic numbers).
- It fails to explain the measured magnetic moments of many nuclei.
- It also fails to explain the spin of nuclei.

- It is also not successful in explaining the excited states in most of the nuclei.
- The agreement of semiempirical mass formula with experimentally observed masses and binding energies is poor for lighter nuclei compared to the heavy ones.

3.2 Bohr- Wheeler Theory of Fission

Bohr and J. A. Wheeler put forward the theory of nuclear fission based on the liquid drop model the nucleus (1939). It is possible to calculate the activation energy E for fission of different nuclei on the basis of this theory.

If mechanical vibrations are set up within a liquid drop, it can lead to the break-up of the drop. To do this, energy must be supplied from outside. Since an atomic nucleus behaves like a charged liquid drop, similar vibrations may be generated in it if it gains some excitation energy which is possible if, for instance, the nucleus absorbs a neutron.

The vibrations set up in the nucleus deform it due to which its surface energy E_s, and electrostatic energy E_c are both changed.

In the fission process, the splitting of the nucleus is preceded by severe deformation of the original nucleus The surface forces tend to restore the original shape, while the electrical forces have the effect of increasing the deformation, because the surface energy is a minimum for the sphere while the electrical energy decreases with increased deformation. The various stages of deformation, leading to the final splitting of a liquid drop into two fragments are shown in Fig 3.1. We shall consider the cases of the lighter and heavier nuclei separately.

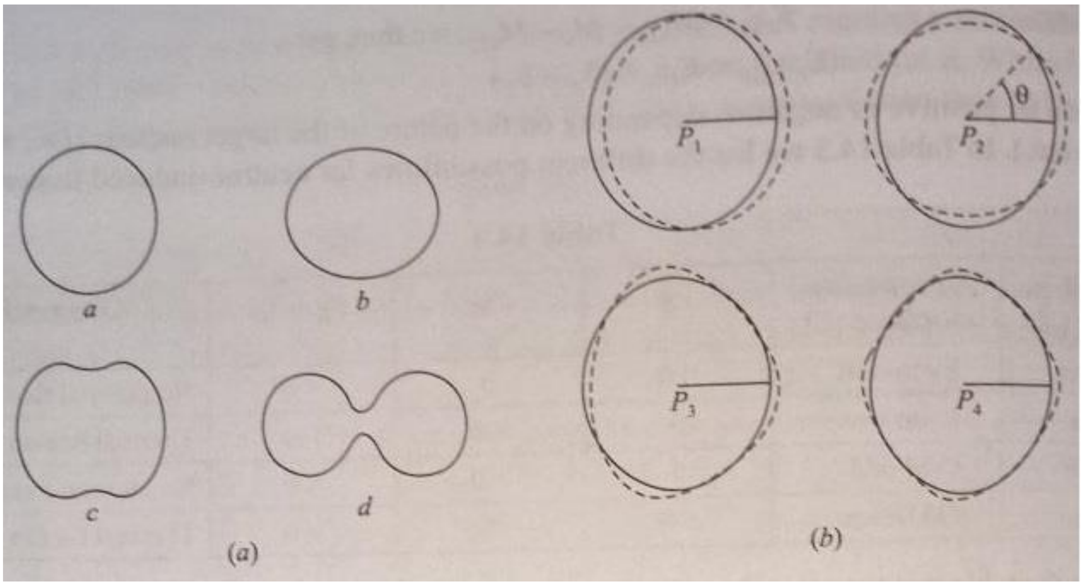

Fig. 3.1 (a) Various stages of deformation leading to the final splitting of a liquid drop. (b) Different modes of axially symmetric deformation of a liquid drop.

For a light nucleus, the electrical forces are small compared to the forces of surface tension. The final separation in this case can take place only if the stage is reached in which the two fragments are connected by a very narrow neck of the liquid. So the critical energy necessary to separate the two parts is given by the difference in energy between the original nucleus and the total energy of the fragments just separated.

In case of very heavy nucleus, the electrostatic forces within the nucleus play a predominant role. Hence even a slight initial deformation of the drop will tend to build up against the forces of surface tension. Thus, a complete separation of the fragments may be expected at an early stage of deformation. The surface and Coulomb's energies of an undeformed spherical nuclear droplet are given as below.

$$E_{so} = a_2 A^{\frac{2}{3}} = 4\pi r_o^2 S A^{\frac{2}{3}} \quad (2.13)$$

$$E_{Co} = \frac{a_3 Z^2}{A^{1/3}} = \frac{3ZA^2}{4\pi\varepsilon_o 5 r_o A^{1/3}} \quad (2.14)$$

Where, S is the surface energy per unit area and ro = 1.2 fm.

If Es and Ec represent the corresponding energies of the deformed nucleus, then the change in the combined surface and electrostatic energies due to deformation will be,

$$\Delta E = \Delta E_s - \Delta E_c = (E_s - E_{so}) + (E_c - E_{co}) \quad (2.15)$$

Bohr and Wheeler, by straight forward calculation shown that,

$$\Delta E = \alpha_2^2 \left(\frac{2}{5} a_2 A^{2/3} - \frac{1}{5} a_3 \frac{Z^2}{A^{1/3}}\right) \quad (2.16)$$

For Z sufficiently large, ΔE will become negative which means that spontaneous fission will occur instantaneously. The limiting condition for this to happen is,

$$\frac{1}{5} a_3 \frac{Z^2}{A^{1/3}} > \frac{2}{5} a_2 A^{2/3}$$

Substituting the values of the parameters a_2 and a_3 in equation (2.17), we get

$$\left(\frac{Z^2}{A}\right)_{lim} \cong 50$$

So, nuclei with $Z^2/A > 50$ will be unstable against spontaneous fission. For the heaviest natural element, uranium, $Z^2/A = 36$, which is well below the limiting value, so that all naturally occurring nuclei are stable w.r.t. small deformation.

Bohr and Wheeler on the basis of liquid drop model calculated the critical energy Ecrit that must be supplied with the neutron. According to their calculation,

$E_{crit} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/3}}$ MeV,

Where, A is the atomic mass of the compound nucleus and Z is the atomic number. For, ${}^{236}_{92}U$-fission one gets A=236 and Z=92; E_{crit} = 6.9 MeV.

3.3 SHELL MODEL

Atomic theory based on the shell model has provided remarkable clarification of the complicated details of atomic structure. Nuclear physicist, therefore, attempted to use a similar theory to study nuclear structure. In the atomic shell model, we fill the shells with electrons in order of increasing energy consistent with the requirement of the Pauli principle. When we do so, one obtains an inert core of filled shells, containing 2, 10, 18, 36, 54 and 86 electrons (atomic numbers of inert gases) and some valence electrons; the atomic properties are determined primarily by the valence electrons. When we compare some measured properties of atomic system with the predictions of the model, one finds remarkable agreement. The same kind of effect has been observed in nuclei. Experimentally it was found that nuclei that have 2, 8, 20, 28, 50, 82 and 126 nucleons (protons or neutrons), called magic numbers, are more abundant than other nuclei.

However, there exist several significant differences between atomic and nuclear cases. In the atomic case, the potential is provided by the Coulomb field of the nucleus; the orbits are generated by the external agent i.e. interaction between electrons and nucleus. We can solve the Schrödinger equation for this potential and calculate the energies of the sub-shells into which electrons can then be placed. In case of nucleus, there is no such external agent, the nucleons move in a potential which is not well defined that they themselves create.

Another appealing aspect of atomic shell theory is the existence of spatial orbits. It is often very useful to describe atomic properties in terms of spatial orbits of the electrons. The electrons can move in those orbits relatively free of collisions with other electrons. Nucleons which have a mass about 2000 times larger than that of electrons have a diameter comparable to the size of the nucleus, which is about 10^5 times smaller than that of an atom. How can we regard the nucleons as moving in well-defined orbits when a single nucleon can make many collisions during each orbit?

All these observations tempted nuclear physicists (Barlet, Guggenheimer et al.) to devise an independent particle model formally called the shell model. A shell structure means that nucleons move freely inside the nucleus similar to the electron motion in atom. This approach could explain the existence of first few magic numbers. However, physicist lost interest in this model till 1948 due to its failure to explain higher magic numbers.

In 1948, M.G. Mayer in USA brought together a considerable amount of convincing information showing the evidence for the closed shells, which led to the development of nuclear shell model which could explain all the magic numbers, namely 2, 8, 20, 28, 50, 82 and 126, which apparently represent closed shells in the nucleus. Some of the main aspects of this evidence based on the study of stable nuclei are as under:

1. Binding energy per nucleon vs. *A* curve. If we plot binding energy per nucleon versus *A* curve, it shows that binding energy suddenly increases when the number of nucleons is either 2, 8, 20, 28, 50, 82 or 126 indicating that these nuclei are exceptionally stable.
2. Number of stable isotopes. Relative stabilities of different elements are also indicated by the number of stable isotopes per element.

1.

19K = 3	20Ca = 6	21Sc = 1
49In = 2	50Sn = 10	51Sb = 2
81Tl = 2	82Pb = 4	83Bi = 1

It is clear that number of stable isotopes for *z* = 20, 50 and 82 are much larger compared to neighbouring isotopes.

3. Number of stable isotones. The numbers of stable isotones for *N* = 19, 20, 21; 49, 50, 51 and 81, 82, 83 are shown in Table 3.3.

TABLE 3.3 Number of stable isotones around different magic numbers

N	*Stable isotones*	N	*Stable isotones*	N	*Stable isotones*
19	0	49	1	81	1
20	5	50	6	82	7
21	1	51	1	83	1

It is clear from the above table that the numbers of stable isotones for N = 20, 50 and 82 are much larger as compared to neighbouring stable isotones.

4. A table of relative abundances of nuclei compiled from data on the composition of earth, sun, stars and meteorites shows pronounced peaks at

^{16}O	($N = Z = 8$)
^{40}Ca	($N = Z = 20$)
^{118}Sn	($Z = 50$)
^{88}Sr, ^{89}Y, ^{90}Zr	($N = 50$)
^{138}Ba, ^{139}La, ^{140}Ce	($N = 82$)
^{208}Pb	($Z = 82$, $N = 126$)

5. Binding energy of next neutron after a magic number is small. The separation energy of the last neutron for N = 7, 8, 9; 19, 20, 21 and 27, 28 29 is shown in Table 3.4.

TABLE 3.4 Binding energy of the last neutron around magic numbers

Nucleus	N	S_n (MeV)	*Nucleus*	N	S_n (MeV)	*Nucleus*	N	S_n (MeV)
^{15}O	7	13.2	^{39}Ca	19	13.3	^{47}Ca	27	7.3
^{16}O	8	15.7	^{40}Ca	20	15.7	^{48}Ca	28	9.9
^{17}O	9	4.14	^{41}Ca	21	8.4	^{49}Ca	29	5.1

From the above table it is clear that for neutron numbers = 9, 21 and 29, the separation energy of the last neutron suddenly decreases as compared to the case, when neutron numbers are 8, 20 and 28.

6. It is found that some isotopes are spontaneous neutron emitters. They are:

$^{17}_{8}O_{9}$, $^{81}_{36}Kr_{51}$, $^{137}_{54}Xe_{83}$, $^{89}_{36}Kr_{53}$

The end product of each series ends in N or Z equal to either 82 or 126.

7. Neutron absorption cross-section *s*, the probability of absorption of neutron by the nucleus is small for nuclides containing magic number of neutrons

N	19	20	21	49	50	51
s	2.0	0.41	12.0	19.0	0.65	6.4

8. α-decay energies are rather smooth functions of A for a given Z, but it shows striking discontinuities at N = 126 or Z = 82, the energy of α-particles increases. For example,

$$^{212}{}_{84}\text{Po}_{128} \rightarrow {}^{208}{}_{82}\text{Pb}_{126} + \alpha \text{ (9 MeV) Half-life } t_{1/2} =$$

$3 \text{ x } 10^{-7}$s where N = 126, Z = 82

$$^{210}{}_{84}\text{Po}_{126} \rightarrow {}^{206}{}_{82}\text{Pb}_{124} + \alpha\text{(6 MeV) } t_{1/2} = 3 \text{ x } 10^{-7}\text{s}$$

where N = 126, Z = 82

9. Similar behaviour is exhibited by *b*-emitters.

10. The electric quadrupole moment measures the departure of nuclear charge distribution from sphericity. This departure is a measure of nuclear deformation. A spherical nucleus has no or nearly zero quadrupole moments. It has been found that nuclei with proton or neutron number equal to one of the magic numbers are spherical in nature, i.e. all the three axes x, y and z are equal like that of a tennis ball. For such nuclei the quadrupole moment is either zero or nearly zero which is also observed experimentally.

However, in some nuclei, out of three, two axes are equal. In the case, where the unequal axis is shorter than the others, the nucleus has somewhat of a pumpkin shape, it is called *oblate*. Extreme case is that of a Hamburger as shown in Figure 1.7. In the other case where the third unequal axis is longer than the other two, the nucleus has somewhat of a football shape, it is called *prolate*. Extreme case is that of a cigar or Hot Dog as shown in Figure 1.6. In general, unequal axis differs in length by about 20%. However, in lighter nuclei, deformations are more. For example, in ^{24}Mg, all the three axes are unequal.

Thus all the facts given above show that magic numbers 2, 8, 20, 28, 50, 82 and 126 correspond to closed shells. The nuclei having any one of these magic number of protons or neutrons or both show more stability than the other nuclei.

Basic Assumptions of the Shell Model

Calculations similar to atoms were also performed for the nucleus also. Following assumptions were made for these calculations:

- Nucleons in a nucleus move independently in a common (mean) potential determined by the average motion of all the other nucleons.
- Protons and neutrons separately fill levels in the nucleus.
- Most of the nucleons are paired and a pair of nucleons contributes zero spin and zero magnetic moment. The paired nucleons thus form an inert core.
- The properties of odd A nuclei are characterized by the unpaired nucleon and odd–odd nuclei by the unpaired proton and neutron.

These assumptions indicate that the nucleus might have a shell structure. It means that nucleons moving in different shells inside the nucleus do not suffer any collisions similar to electrons in different shells in the atom. This assumption is apparently not acceptable as the nucleons have almost the same size as that of the nucleus. So obviously a question arises, why do not so many nucleons moving inside the nucleus suffer any collisions? How can we regard the nucleons as moving in well-defined orbits when a single nucleon can make many collisions during each orbit?

The answer to this question comes from Pauli's exclusion principle. Consider in a heavy nucleus, a collision between two nucleons in a state near the very bottom of the potential well. When the nucleons collide, they transfer energy to one another, but if all of the energy levels are filled up to the level of valence nucleon, there is no way for one of the nucleons to gain energy except to move up to the valence level. The other levels near the original level are filled and cannot accept an additional nucleon. Such a transfer from a low-lying level to the higher- lying level requires more energy than the nucleons are likely to transfer in a collision. Thus, the collisions cannot occur and the nucleons can indeed orbit as if they were transparent to one another!

The first step in developing the shell model is the choice of the potential. Different forms of potential $V(r)$ have been employed in order to obtain the required magic numbers. In the following, we consider two potentials to solve the Schrödinger equation.

3.3.1 The Square Well Potential

The problem can be mathematically simplified, if we assume a potential well with infinite walls as

$$V(r) = -V_0 \qquad r < r_0$$
$$= 0 \qquad r > r_0$$

The shape of the finite square well potential is shown in Figure 3.2.

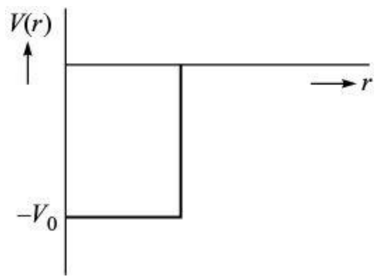

Figure 3.2 Square well potential. $-V_0$ is the depth of the well.

If we solve Schrödinger equation for square well potential, we get the following sequence of levels:

- 1s
- 1p
- 1d
- 2s
- 1f
- 2p
- 1g

- 2d
- 1h
- 3s
- 2f
- 1i
- 3p
- 2g

and so on, where *s, p, d, f, g, h, i*, ..., etc. stand for usual spectroscopic notation L = 0, 1, 2, 3,4, 5, 6, ..., respectively.

Because of the two different spin orientation of the nucleon, a level can contain (2L + 1) protons or neutrons. For example, number of nucleons in 1*s* (L = 0) shell will be 2(2 x 0 + 1) = 2 and number of nucleons in 1*f* (L = 3) shell will be 2(2 x 3 + 1) = 14. This model predicts the shell closures at nucleon number 2, 8, 18, 20, 34, 40, 58, etc. as shown in Table 3.5.

TABLE 3.5 Nuclear levels and magic numbers predicted by square well potential

Level	*Number of nucleons*	*Magic numbers*
1*s*	2	2
1*p*	6	8
1*d*	10	18
2*s*	2	20
1*f*	14	34
2*p*	6	40
1*g*	18	58
2*d*	10	68
1*h*	22	90
3*s*	2	92
2*f*	14	106
1*i*	26	132
3*p*	6	138

The numbers shown in column 3 of the above table are not the observed the magic numbers. The level sequence for square well potential is shown in figure 3.3.

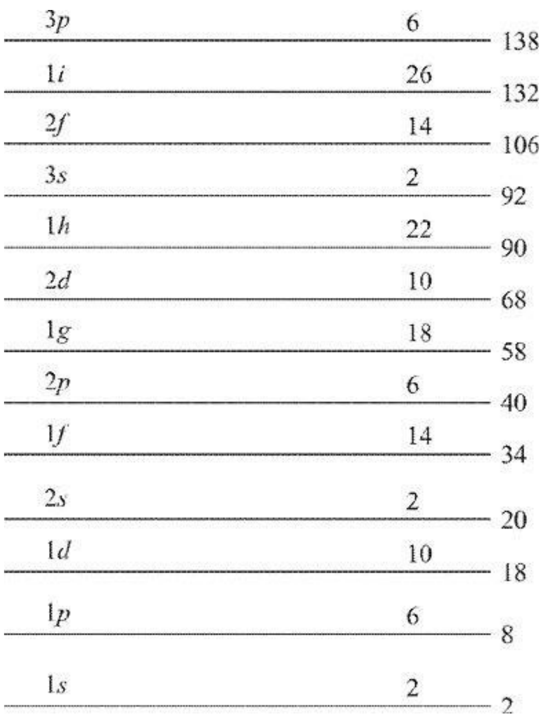

Figure 3.3 Sequence of levels of the square well potential.

The level sequence for squares well potential can be remembered in the following way. First in a vertical column write the level sequence 1*s*, 1*p*, 1*d*, 1*f*, 1*g*, 1*h*, 1*i*, etc. as shown in Figure 2.3. Then leave two vertical spaces as blank and again write the level sequence 2*s*, 2*p*, 2*d*, 2*f*, 2*g*, etc. Again, leave two vertical blank spaces and write the level sequence 3*s*, 3*p*, 3*d*, 3*f*, etc. as shown in Figure 3.4. In this sequence, 3*s* level will shift between 1*h* and 2*f*, similarly, 1*i level* will shift between 2*f* and 3*p* as indicated by arrow in the Figure 3.3. The resulting sequence will look as shown in Figure 3.5.

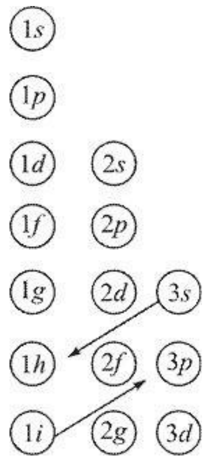

Figure 3.4 Way to remember the square well potential levels.

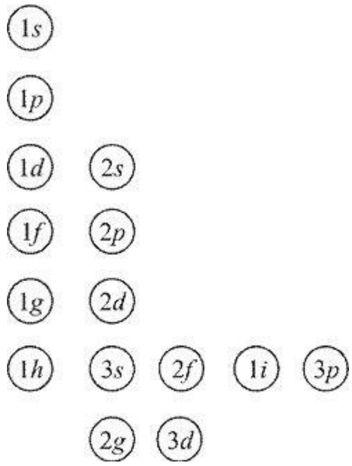

Figure 3.5 The resulting sequence is the sequence of square well potential levels.

Now, starting from the top and move horizontally from left to right, the first level is 1*s*, second 1*p*, third 1*d*, fourth 2*s*, fifth 1*f*, and so on.

3.3.2 The Harmonic Oscillator Potential

The shape of this potential is shown in Figure 3.6.

$$V(r) = -V_0 + \frac{1}{2}Kr^2 \tag{2.13}$$

This potential and the square well potential provide two contrasting viewpoints. The square well has infinite sharp edges. The harmonic oscillator potential diminishes steadily at the edges.

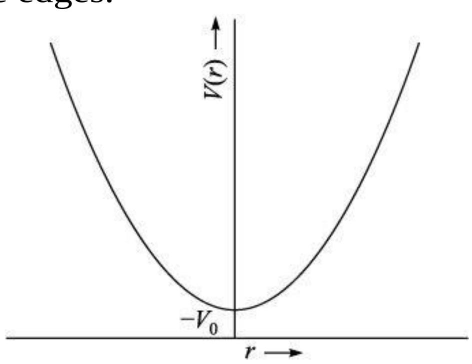

Figure 3.6 Harmonic oscillator potential.

Again solving Schrödinger equation for harmonic oscillator potential, we get the following sequence of levels as shown in first column of Table 3.6. The first level is 1*s*, second is 1*p*, third level contains two sub-shells 2*s*, 1*d*, having same energy, fourth level again contains two sub-shells, 2*p* and 1*f*, and so on.

TABLE 3.6 Nuclear levels and magic numbers predicted by harmonic oscillator potential

Level	*No. of nucleons in various levels*	*Magic number*
1*s*	2	2
1*p*	6	8
2*s*, 1*d*	2 + 10 = 12	20
2*p*, 1*f*	6 + 14 = 20	40
3*s*, 2*d*, 1*g*	2 + 10 + 18 = 30	70
3*p*, 2*f*, 1*h*	6 + 14 + 22 = 42	112
4*s*, 3*d*, 2*g*, 1*i*	2 + 10 + 18 + 26 = 56	168

In this case also each sub-shell contains 2(2L + 1) protons or neutrons. For example, in fourth shell, we have two sub-shells, 2*p* and 1*f*. For 2*p* (L = 1), it contains 2(2 L 1 + 1) = 6 nucleons and 2*f* (L = 3), it contains 2(2 L 3 + 1) = 14 nucleons. Total number of nucleons in fourth shell = 6 + 14 = 20. These numbers are shown in the third column of the Table 3.6.

This level sequence again does not reproduce experimentally observed magic numbers. The sequence of harmonic oscillator levels is shown in Figure 3.7. The levels are equally spaced. Harmonic oscillator level sequence can also be remembered in almost similar way as that of square well potential as shown below.

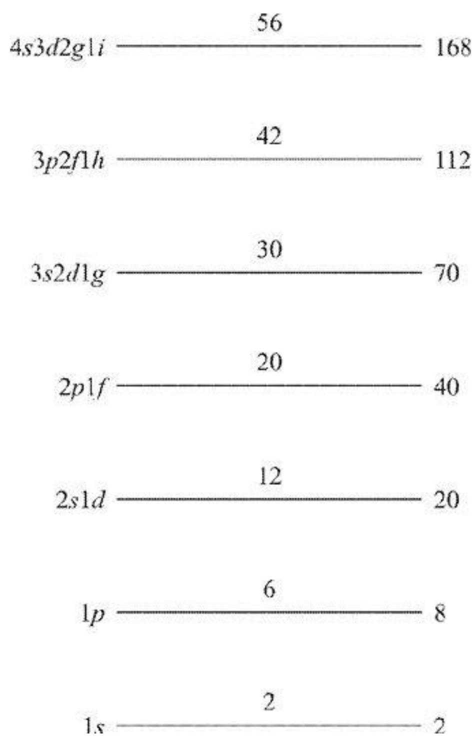

Figure 3.7 Level sequence as obtained for the harmonic oscillator levels.

As shown in Figure 3.8, write in the first vertical column the level sequence 1*s*, 1*p*, 1*d*, 1*f*, 1*g*, 1*h*, 1*i*, etc. Then leave two vertical spaces as blank and again write the level sequence 2*s*, 2*p*, 2*d*, 2*f*, 2*g*, etc. Again, leave two vertical blank spaces and write the level sequence 3*s*, 3*p*, 3*d*, 3*f*, etc. Now, these sequences read horizontally are the harmonic oscillator levels. For example, first level is 1*s*, second is 1*p*, third is 1*d*, 2*s* and so on.

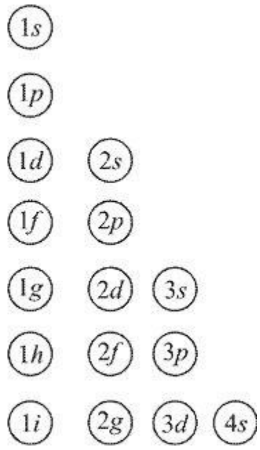

Figure 3.8 Way to remember the harmonic oscillator levels.

The other potential, which is a compromise between square well and harmonic

$$V(r) = \frac{-V_0}{1+e^{\frac{R-r}{d}}} \tag{2.14}$$

oscillator potential, is

This potential is known as Woods–Saxon potential. In this equation d = 0.524 fm, R is the mean nuclear radius and r = $r_0\ A^{1/3}$. Unlike square well potential, the Woods–Saxon potential does not have any sharp edges at all. The harmonic oscillator potential also does not have any edges. The shape of this potential is shown in Figure 3.9. This potential closely approximates the nuclear charge and matter distribution, falling smoothly to zero beyond the mean radius R. When the Schrödinger equation was solved for this potential, it predicted 2, 8, 20, 40, 58, 92, 112 as magic numbers. We again get the magic numbers 2, 8, and 20, but the higher magic numbers do not emerge from the calculations.

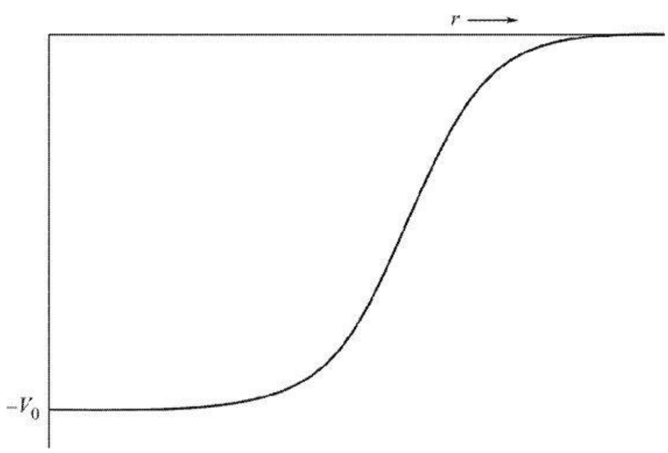

Figure 3.9 Wood-Saxon Potential

A way out of this difficulty, which proved to be remarkably successful was proposed independently in 1949 by M.G. Mayer in USA and O. Haxel, J.H.D. Jensen and H.E. Suess
in Germany. We have seen in Chapter 1 that each nucleon has a spin angular momentum $|\vec{s}| = \sqrt{s(s+1)}\hbar$ and orbital angular momentum $|\vec{\ell}| = \sqrt{\ell(\ell+1)}\hbar$. It was

$1s_{1/2}$	2
$1p_{3/2}$, $1p_{1/2}$	8
$1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$	20
$1f_{7/2}$	28
$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$	50
$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$	82
$1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $1i_{13/2}$	126
$2g_{9/2}$, $3d_{5/2}$, $1i_{11/2}$, $2g_{7/2}$, $4s_{1/2}$, $3d_{3/2}$, $1j_{15/2}$	184

proposed that there is a strong coupling between the orbital and spin angular momentum of each individual nucleon; referred as spin-orbit coupling. As a result of the spin-orbit coupling, the nucleon energy level for a given value ℓ of the orbital quantum number (except $\ell = 0$) splits into two sub-levels, characterized by total angular momentum quantum number $j = \ell + 1/2$ and $j = \ell - 1/2$ corresponding to spin components of +1/2 and –1/2 respectively. The sign of this term is chosen in such a way that $\ell + 1/2$ level goes down in energy whereas $\ell - 1/2$ goes up. Further, the total splitting is proportional to ℓ and becomes so large that for a given *n*, the level with largest ℓ value slides down to energy as low as those of the multiplet with quantum number $n - 1$.
The sequence of these levels is shown in Figure 3.10.

3.3.3 Predictions of the Shell Model

- Magic numbers.
- Even–even nuclei have ground state angular momentum or spin 0. There is no known exception to this rule.
- In odd *A* nuclei the spin will be determined by the last unpaired particle. For example, in $^{13}_{6}C_7$ and $^{13}_{7}N_6$ the levels fill as under

$$(1s_{1/2})^2 \mid (1p_{3/2})^4 \, (1p_{1/2})^1$$

Thus, in $^{13}_{6}C_7$ unpaired neutron is in $1p_{1/2}$ shell and, therefore, nucleus has a spin 1/2, whereas in $^{13}_{7}N_6$ last unpaired proton is also in $1p_{1/2}$ shell so its spin should also be 1/2. This is indeed observed experimentally.

Similarly, in $^{17}_{8}O_9$ and $^{17}_{9}F_8$ the filling of levels will be

$$(1s_{1/2})^2 \mid (1p_{3/2})^4 \, (1p_{1/2})^2 \mid (1d_{5/2})^1$$

the predicted spin is 5/2, which is also experimentally observed.

Similarly, for $^{33}_{16}S_{17}$ $(1s_{1/2})^2 \mid (1p_{3/2})^4 \, (1p_{1/2})^2 \mid (1d_{5/2})^6 \, (2s_{1/2})^2 \, (1d_{3/2})^1$

predicted and observed spin is 3/2. However, in $^{75}_{33}As_{42}$ and $^{61}_{28}Ni_{33}$

$(1s_{1/2})^2 \mid (1p_{3/2})^4 \ (1p_{1/2})^2 \mid (1d_{5/2})^6 \ (2s_{1/2})^2 \ (1d_{3/2})^4 \mid (1f_{7/2})^8 \mid (2p_{3/2})^4$
$(1f_{5/2})^1$

predicted spin is 5/2, whereas observed is 3/2, for both these nuclei.
Similar kind of exceptions has been observed for neutron numbers 57, 59 and 61 also.

One would also expect that for a high atomic mass number A, there will be many stable nuclei with spin 11/2 corresponding to an odd nucleon in the $1h_{11/2}$ state and similarly there should be many stable nuclei with spin 13/2 corresponding to an odd nucleon in the $1i_{13/2}$ state, but not even a single nucleus has ever been observed with ground state spin of 11/2 or 13/2. Many such exceptions have been eliminated by modifying the rules and stating that if the high spin shell (say $1f_{5/2}$) comes after low spin shell ($2p_{3/2}$), the high spin shell fills faster, pairing its particles before the low spin shell can be filled completely. According to this rule, we may write for ^{75}As and ^{61}Ni.

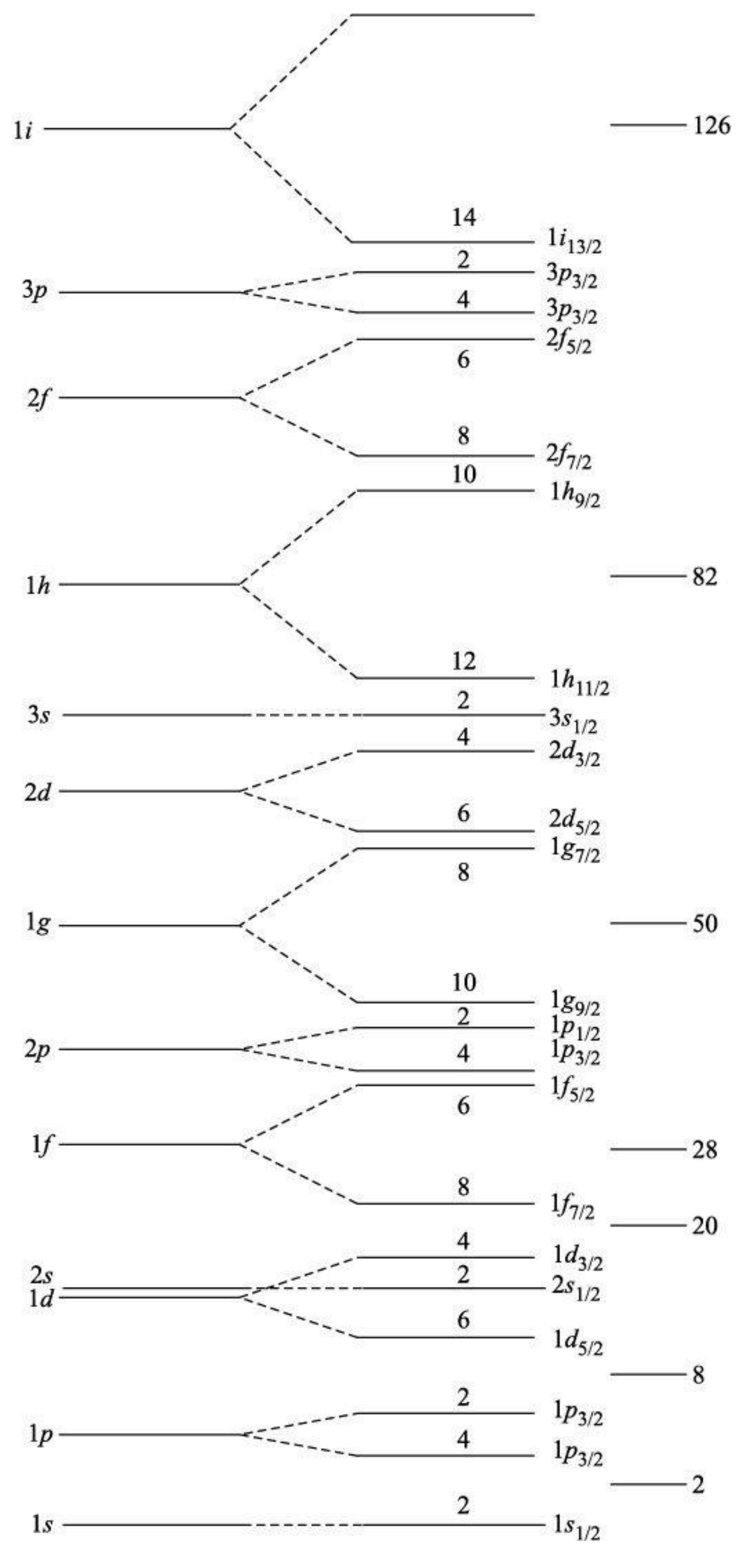

Figure 3.10 Level scheme due to spin-orbit coupling.

$(1s_{1/2})^2$ | $(1p_{3/2})^4$ $(1p_{1/2})^2$ | $(1d_{5/2})^6$ $(2s_{1/2})^2$ $(1d_{3/2})^4$ | $(1f_{7/2})^8$ | $(2p_{3/2})^3$ $(1f_{5/2})^2$

giving spin as 3/2 for both these nuclei.

We can say that there is a strong tendency for particles to form pairs in higher ℓ states even at some expense of energy. This can be put into the model in the form of pairing potential, which gives paired nucleons a lower energy than unpaired ones, and which increases with increasing ℓ.

The higher angular momentum states are usually formed in pairs. Thus, a level $1h_{11/2}$ may be filled in pairs while the odd nucleon goes to $3s_{1/2}$ or $2d_{3/2}$ shell. For example, the measured spin of $^{137}_{56}Ba_{81}$ is 3/2, while the one predicted by the shell model is 11/2.

$(1s_{1/2})^2$ | $(1p_{3/2})^4$ $(1p_{1/2})^2$ | $(1d_{5/2})^6$ $(2s_{1/2})^2$ $(1d_{3/2})^4$ | $(1f_{7/2})^8$ | $(2p_{3/2})^4$ $(1f_{5/2})^6$ $(2p_{1/2})^2$ $(1g_{9/2})^{10}$

| $(1g_{7/2})^8$ $(2d_{5/2})^6$ $(2d_{3/2})^4$ $(3s_{1/2})^2$ $(1h_{11/2})^{11}$ |

predicts spin as 11/2.

Due to pairing in higher *l* states, the alternative arrangement of nucleons is as under

$(1s_{1/2})^2$ | $(1p_{3/2})^4$ $(1p_{1/2})^2$ | $(1d_{5/2})^6$ $(2s_{1/2})^2$ $(1d_{3/2})^4$ | $(1f_{7/2})^8$ | $(2p_{3/2})^4$ $(1f_{5/2})^6$ $(2p_{1/2})^2$ $(1g_{9/2})^{10}$ |

$(1g_{7/2})^8$ $(2d_{5/2})^6$ $(2d_{3/2})^3$ $(3s_{1/2})^2$ $(1h_{11/2})^{12}$ |

This predicts spin as 3/2. A discrepancy occurs at $^{19}_{9}F_{10}$, which according to the shell model should have spin of 5/2.

$(1s_{1/2})^2$ | $(1p_{3/2})^4$ $(1p_{1/2})^2$ | $(1d_{5/2})^1$ $(2s_{1/2})^0$ $(1d_{3/2})^0$

Experimentally spin is 1/2. The odd proton goes to $2s_{1/2}$ state instead of $1d_{5/2}$ state. This discrepancy may be explained as the result of coupling between the nucleons outside the closed shell, i.e. the two neutrons each having spin 5/2 and a proton also having spin 5/2. Another discrepancy is for ^{23}Na, which has a spin of 3/2 while the one predicted by the shell model is 5/2.

$(1s_{1/2})^2$ | $(1p_{3/2})^4$ $(1p_{1/2})^2$ | $(1d_{5/2})^3$ $(2s_{1/2})^0$ $(1d_{3/2})^0$

Coupling between 3 protons in $1d_{5/2}$ shell gives spin of 3/2.

Finally, the parity of the system is given by $(-1)^{\ell}$, where ℓ is the orbital quantum number of the last odd nucleon. For a nucleon in a state *s*, *d*, *g*, ... corresponding to ℓ = 0, 2, 4, ... the parity is even (+), while for the states *p*, *f*, *h*, ... corresponding to ℓ = 1, 3, 5, ... the parity is odd (–).

3.3.4 Achievements of the Shell Model

- It explains the ground state spin and parities of all even–even nuclei without any exception.
- It explains the ground state spin and parities of most of odd *A* (even–odd or odd–even) nuclei.
- It also explains the spin and parities of odd–odd nuclei.
- It explains the extra stability of magic nuclei.
- It also explains the qualitative features of magnetic dipole and electric quadrupole moments of different nuclei.
- It is also able to explain many other properties, like nuclear isomerism of

different nuclei.

3.3.5 Failures of Shell Model

- Shell model fails to explain spin values for certain nuclei.
- Shell model is unable to explain the energy of first excited states in even–even nuclei.
- It is unable to explain magnetic moments of some nuclei.
- This model is also unable to explain quadrupole moments of many nuclei.
- Shell model is also unable to explain the ground states of odd A nuclei in the mass region $150 \leq A \leq 190$ and $A > 220$.

3.6 Collective Model

The success of both liquid-drop and nuclear shell models leads to a dilemma, as there is a basic contradiction between them. The liquid-drop model accounts for the behavior of nucleus as a whole, as in nuclear fission. Many nuclear phenomena however show that nucleons behave as individual and near-independent particles. The conclusion that follows is that the two models are incomplete parts of a more general one. The problem of electric quadrupole moments led to such a model. This new model is called the collective or unified model, proposed by Aage Bohr and Mottelson. It is a combination of the ideas of the liquid-drop and the shell model in which all the nucleons participate in a collective or unified manner.

In this model, the nucleons are assumed to exert a centrifugal pressure on the surface of the nucleus deforming it into a permanently non-spherical shape and may undergo oscillations (liquid-drop aspect). The nucleons then move in a non-spherical potential like that assumed to account for the quadrupole moments. The nuclear distortion reacts on the particles and somewhat modifies the independent particle aspect. The nucleus is considered as a shell structure that can oscillate in shape and size.

The simplest type of collective motion, identified experimentally, is connected with rotation of deformed nuclei. The rotational energy levels are obtained when the angular momentum is quantized and for even-even nuclei given by,

$$E_{rot} = \frac{\hbar}{2I} J(J+1) \qquad (3.15)$$

where I is the effective moment of inertia of the nucleus and J, the total angular momentum quantum number.

For a *spheroidal* nucleus, the deformation is symmetric relative to reflection in the nuclear center. So, J is restricted to even values: J = 0, 2,4, and parity should be even. According to this theory, the deformation would be maximum and rotational levels easily observable for nuclei with number of nucleons far from closed shell. The first excited state should be a 2+ state, the second a 4+ state with energy (5 x 4)/(3 x 2) = 3 times that of the first excited state. The excitation energy of successive states of this sequence should be 1 : 3: 7: 12- etc. Many nuclei are observed to follow this sequence. This is shown in Fig. 3.11. The effective moment of inertia is about one half that of a rigid body of the same shape.

Another interesting prediction of the model is that the y-ray transitions between these two states should be much faster, sometimes 100 times, than what the simple shell model would predict. This is also supported by experiments

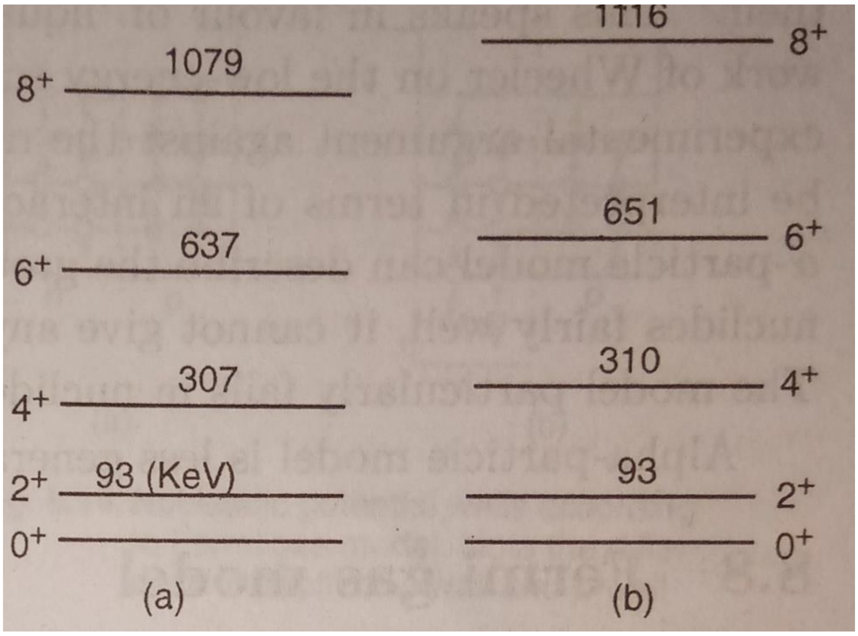

Fig. 3.11 Rotational states in ${}^{180}_{72}Hf$: (a) experimental levels
(b) levels calculated using equation (3.15).

Other forms of collective motion should also exist; for example, vibrational distortion. There is some evidence for this, but it is not so clear-cut as for the rotational states. A detailed study of the rotation and vibrational spectra of nuclei is beyond the scope of the book.

The collective model does not deny the validity of the shell model. The individual nucleons still pursue their quasi-independent' motions but in an ellipsoid potential, instead of a spherical one. One has therefore to understand in detail why the equilibrium distortion is favoured and how so great a degree of collective motion can arise if the individual nucleons are not tied together

4.0 Summary

In the first section of this unit, describes the origin of Gamma rays, its energy, spectrum and Internal conversion, nuclear isomerism and Gamma ray spectroscopy. In the second section, explains, Yukawa theory of nucleon-nucleon interaction, exchange forces between nucleons, binding energy of deuteron and the ground state of deuteron also classification of nuclear reaction, conservation laws and mass energy balance in nuclear reactions, compound nuclear model and Ghoshal's experiment for the confirmation of the theory of compound nucleus.

In the third section, the different nuclear models are discussed for the stability of nucleus, for the determination of binding energy, nuclear masses accurately, spin - parity of the ground state of nucleus, also explains the stability of magic nuclei and the qualitative features of magnetic dipole and electric quadrupole moments of different nuclei.

1.0 Glossary

Gamma rays spectrum, Internal Conversions, Nuclear Isomerism, Nucleaon-Nucleon Interaction, Nuclear reactions, compound nucleus, Nuclear models, Liquid drop model, Shell model, Nuclear Fission.

6.0 Self-assessment questions

1. (a) Describe qualitatively how γ - rays interact with matter while passing through it.
(b) $^{240}_{94}Pu$ decays by emission of two groups of α-particles with kinetic energies K_1 = 5.17 MeV and K_2 = 5.12 MeV. Calculate the energy of the accompanying γ- rays.

2.In the reaction $^{11}_{5}B+^{4}_{2}He\rightarrow^{14}_{7}N+^{1}_{0}n$, the masses of ^{11}B, ^{14}N and ^{4} He are 11.01280 u, 14.00752 u and 4.00387 u respectively. If the incident α-particle has a kinetic energy 5.250 MeV towards ^{11}B which is at rest and the kinetic energies of product nuclei ^{14}N and ^{1}n are 3.260 MeV and 2.139 MeV respectively, compute the mass of neutron in kg.

1. (a) Which experimental fact indicates saturation of nuclear force ?

b) Write down the asymmetry term in the Bethe-Weizsacker semiempirical formula for nuclear binding energy.
c) Using the extreme single particle shell model, determine the ground state spin-parities of Mg^{25}_{12} nucleus.
d) Indicate the processes by which Gamma rays absorbed in matter.
e) What do you mean by “Parity Violation” ?
f) What are the Bohr’s hypothesis about a compound nucleus.

4. Calculate the threshold energy required to initiate the reaction $P^{31}(n,p)Si^{31}$. Given, $m_n = 1.00898\,u$, $m_p = 1.008144\,u$, $M_P = 30.9836\,u$, $M_{Si} = 30.98515\,u$.

5. Find the Q-value of the nuclear reaction X(x,y)Y in terms of mass and kinetic energy of the incident, product particles and residual nucleus, if the product nucleus emitted of an angle 90° w.r.t. the direction of the incident particle.

6. a) Obtain the expression for the binding energy of a nucleus based on the liquid drop model. State the semi-empirical mass formula of Bethe- Weizsacker.

(b) What are the basic similarities between a liquid drop and a nucleus? Using semi empirical binding energy formula, show that nuclei with A > 160 should be α disintegrating.

7. A nucleus with A = 239 and Z = 92 is deformed from the spherical shape such that the deformation parameter α_2 = 0.1 while $\alpha_3 = \alpha_4$ = 0; Using Bohr-Wheeler theory, calculate the percentage in the surface and coulomb energy changes of the nucleus from those of the spherical shape.

8. What are magic numbers? Name a “doubly magic” nucleus. Give the experimental evidences in support of magic numbers and shell structure of nucleus in nuclei.

9. Predict the state of energy level of unpaired odd nucleon and spins and parities of the following nuclei from the single particle shell model.

(i) $^{27}_{13}Al$ (ii) $^{33}_{16}S$ (iii) $^{41}_{18}Ar$

10. a) Show that spin-orbit interaction force between nucleus overcomes the limitations of single particle shell model.
b) Discuss the success and limitations of the single particle shell model.
c) Find the total angular momentum and parity for the ground state of S^{33}_{16} nucleus using the shell model.

11.Describe Rutherford's experiment for observing the disintegration of nitrogen nuclei by bombardment with α particles.

12. Write down Bohr's independence hypothesis on compound nuclear reaction mechanism. Also explain with diagram the Ghosal's experimental results for the verification of compound nuclear theory.

13. In the reaction $^{11}_{5}B+^{4}_{2}He\rightarrow^{14}_{7}N+^{1}_{0}n$, the masses of ^{11}B, ^{14}N and ^{4} He are 11.01280 u, 14.00752 u and 4.00387 u respectively. If the incident α-particle has a kinetic energy 5.250 MeV towards ^{11}B which is at rest and the kinetic energies of product nuclei ^{14}N and ^{1}n are 3.260 MeV and 2.139 MeV respectively, compute the mass of neutron in kg.
14. a) What are the basic similarities between a liquid drop and an atomic nucleus. Develop the semi empirical mass formula discussing the physical basis of each term.
b) Using the semi empirical mass formula, find the atomic number of the most stable nucleus for a given mass number A. Hence explain which is the most stable among $^{6}_{2}He, ^{6}_{2}Be, ^{6}_{3}Li$. 2+5+3
15. a) Predict the state of energy level of unpaired odd nucleon and spins and parities of the following nuclei from the single particle shell model.
i) $^{13}_{6}C$ (ii) $^{27}_{13}Al$ (iii) $^{33}_{16}S$.

b) Discuss the success and limitations of the single particle shell model.
16. Explain briefly the important features of the collective model of nuclei. How does the collective model help in understanding the nuclear fission?

7.0 References

1. Nuclear Physics, Dr. S.N.Ghosal, S. Chand & Company PVT. Ltd.
2. Concept of Nuclear Physics, Dr. S.P.Kuila, New Central Book Agency (P) Ltd.
3. Introductory Nuclear Physics, Kenneth S. Krane, John Wiley & Sons
4. Nuclear Physics, Irving Kaplan, Addison-Wesley Publishing Compny.