VU CBCS Semester-IV 2019: Planck's quantum, Planck's constant and light as a collection of photons; Blackbody Radiation: Quantum theory of Light; Photo-electric effect and Compton scattering. De Broglie wavelength and matter waves; Davisson-Germer experiment. Wave description of particles by wave packets. Group and Phase velocities and relation between them. Two-Slit experiment with electrons. Probability. Wave amplitude and wave functions.

Particle Nature of Wave

Photoelectric effect:

In 1888 Hertz and afterwards other scientists observed that when the surface of metals like zinc is irradiated with ultraviolet light the metal gets positively charged i.e. the metal loses negative charge. 1n 1899 P. Lenard (Philipp Lenard, German Physicist, 1862-1947, Supporter of Hitler) showed that the loss of negative charge is due to emission of negatively charged electrons from the metal surface. The following laws were discovered experimentally prior to 1905:

- If the frequency of the incident radiation is smaller than the metal's threshold frequency - a frequency that depends on the properties of the metal—no electron can be emitted regardless of the radiation's intensity (Philipp Lenard, 1902).
- No matter how low the intensity of the incident radiation, electrons will be ejected *instantly* the moment the frequency of the radiation exceeds the threshold frequency ν_0.

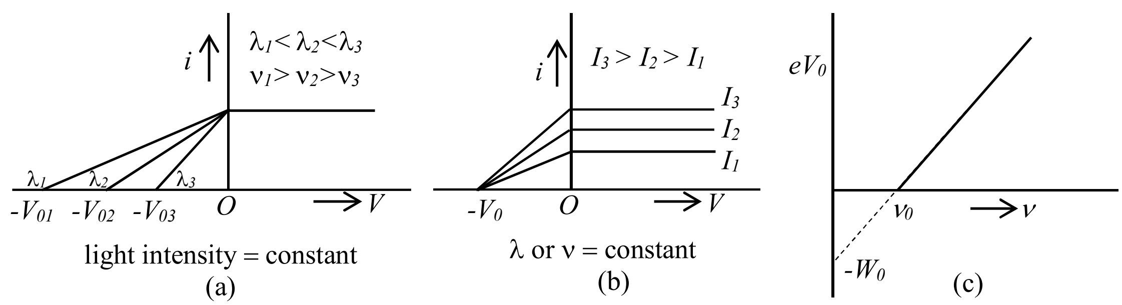


Fig.X

- At any frequency above ν_0, the photo current and hence the number of electrons ejected per second increases with the intensity of the light but does not depend on the light's frequency.
- The stopping potential (V_0) and so the kinetic energy of the ejected electrons depends on the frequency but not on the intensity of the beam; V_0 and so the maximum kinetic energy of the ejected electrons $\left(\frac{1}{2}mv_m{}^2 = eV_0\right)$ increases *linearly* with the incident frequency.

Einstein showed that the plot of maximum kinetic energy of the electrons or of eV_0 with frequency (ν) of the incident light is a straight line (Fig.-X(c)) which can be given by:

$$eV_0 = h\nu - W_0$$

The slope of the straight line graph does not depend on the metal and as determined from experimental results, it is equal to Planck's constant h. If the electron emitting metal surface is clean and oxide free then the intercept W_0 is characteristic of the metal and it is equal to $h\nu_0$.

Einstein, extending Planck's quantum condition to radiation, proposed that light is made up of discrete energy packets or quanta – photons – each of which have energy $h\nu$, where ν is the frequency of light. In photo electric effect a photon, incident on an electron, is completely absorbed by it. At normal temperatures the maximum energy of an electron inside the metal is less than the minimum energy required by an electron to come out of the metal surface by an amount equal to W_0 which is called work function. So if $h\nu < W_0$ the photon absorbing electron cannot come out of the metal surface. But if $h\nu > W_0$ the electron can emit from the metal surface. And the emitted electron possesses a kinetic energy $h\nu - W_0$ with which it can reach to the anode even if the anode is given no positive potential with respect to the cathode. Moreover to stop such an emitted electron from leaving the metal surface a

negative potential (say $-V_0$) should be applied to the anode with respect to the cathode. V_0 is called stopping potential. Clearly $eV_0 = h\nu - W_0$ and it is equal to the maximum kinetic energy of the electrons.
The equation: $eV_0 = h\nu - W_0$ is Einstein's photoelectric equation.

Problems:

JAM 2014

Q.8 In a photoelectric effect experiment, ultraviolet light of wavelength 320 nm falls on the photocathode with work function of 2.1 eV. The stopping potential should be close to

(A) 1.8 V (B) 1.6 V (C) 2.2 V (D) 2.4 V

Ans. $eV_0 = h\nu - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240}{320} - 2.1$ electron Volt $= 1.775$ electron Volt

$\Rightarrow V_0 = \frac{1.775}{e}$ electron Volt $= 1.775$ Volt $\Rightarrow (A)$.

JAM 2011

Q.6 Light described by the equation E= (90 V/m)[sin(6.28 × 10^{15} s^{-1}) t + sin(12.56 × 10^{15} s^{-1}) t] is incident on a metal surface. The work function of the metal is 2.0 eV. Maximum kinetic energy of the photoelectrons will be

(A)) 2.14 eV (B) 4.28 eV (C) 6.28 eV (D) 12.56 eV

Ans.: $E = E_0 \sin \omega t$ represents the electric field vector of light having angular frequency ω i.e. frequency $\nu = \omega/2\pi$ where E_0 is the amplitude of the electric field vector.

So $E = (90\ V/m)[\sin(6.28 \times 10^{15} s^{-1})t + \sin(12.56 \times 10^{15} s^{-1})t]$

$= (90\ V/m)\sin(6.28 \times 10^{15} s^{-1})t + (90\ V/m)\sin(12.56 \times 10^{15} s^{-1})t$

represents two light waves of frequencies $\nu_1 = \frac{6.28 \times 10^{15}}{2\pi} s^{-1} = 10^{15} s^{-1} = 10^{15} Hz$ and

$\nu_2 = \frac{12.56 \times 10^{15}}{2\pi} s^{-1} = 2 \times 10^{15} s^{-1} = 2 \times 10^{15} Hz$

Clearly the maximum kinetic energy will be determined by the larger frequency. So, in this problem the maximum kinetic energy will be:

$h\nu_2 - W_0\ \ Joule = \frac{h\nu_2}{e} - \frac{W_0}{e}\ \ eV$

$= \frac{6.626 \times 10^{-34} \times 2 \times 10^{15}}{1.6 \times 10^{-19}} - 2.0\ eV = 8.28\ eV - 2.0\ eV = 6.28\ eV \Rightarrow (C)$

JAM 2007:

20. A beam of light of wavelength 400 nm and power 1.55 mW is directed at the cathode of a photoelectric cell. (given: hc = 1240 eV nm, e = 1.6×10^{-19} C). If only 10% of the incident photons effectively produce photoelectrons, find the current due to these electrons. If the wavelength of light is now reduced to 200 nm, keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. What are the values of the stopping potentials for the two wavelengths? [21]

Ans. $P_{eff} = 1.55 \times 10^{-3} \times \frac{10}{100}\ W = 1.55 \times 10^{-4}\ W$

Number electrons emitted from the cathode per second is equal to the number of photons participating in photoemission per second:

$$= \frac{\text{effective power}}{\text{photon energy}} = \frac{1.55\times10^{-4}}{h\nu} = \frac{1.55\times10^{-4}}{hc/\lambda}$$

$$= \frac{1.55\times10^{-4}}{[1240/400]\times1.6\times10^{-19}} = \frac{0.5\times10^{-4}}{1.6\times10^{-19}}$$

Current $= \frac{0.5\times10^{-4}}{1.6\times10^{-19}} \times 1.6 \times 10^{-19} = 0.05\ mA.$

$$(K.E)_{max} = eV_0 = h\nu - W_0 = \frac{hc}{\lambda} - W_0 \qquad \Rightarrow eV_{01} = \frac{hc}{\lambda_1} - W_0; \qquad eV_{02} = \frac{hc}{\lambda_2} - W_0$$

$$eV_{02} - eV_{01} = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = \left(\frac{1240}{200} - \frac{1240}{400}\right) \text{ electron Volt} = 3.1 \text{ electron Volt}$$

$$\Rightarrow 5eV_{01} - eV_{01} = 3.1 \text{ electron Volt} \quad \Rightarrow 4eV_{01} = 3.1 \text{ electron Volt} \quad \Rightarrow V_{01} = 3.1/4 \text{ V}$$

$$\Rightarrow V_{01} = 0.7525 \text{ V}; \quad V_{02} = 5 \times 0.775 \text{ eV} = 3.875 \text{ V}$$

Problem of estimation of Plank Constant (Example 1.2, Zettili):

When two ultraviolet beams of wavelengths $\lambda_1 = 80$ nm and $\lambda_2 = 110$ nm fall on a lead surface, they produce photoelectrons with maximum energies 11.390 eV and 7.154 eV, respectively.

(a) Estimate the numerical value of the Planck constant.

(b) Calculate the work function, the cutoff frequency, and the cutoff wavelength of lead.

Solution

(a) From (1.22) we can write the kinetic energies of the emitted electrons as $K_1 = hc/\lambda_1 - W$ and $K_2 = hc/\lambda_2 - W$; the difference between these two expressions is given by $K_1 - K_2 = hc(\lambda_2 - \lambda_1)/(\lambda_1\lambda_2)$ and hence

$$h = \frac{K_1 - K_2}{c}\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}. \tag{1.24}$$

Since 1 eV $= 1.6 \times 10^{-19}$ J, the numerical value of h follows at once:

$$h = \frac{(11.390 - 7.154) \times 1.6 \times 10^{-19}\text{ J}}{3 \times 10^8\text{ m s}^{-1}} \times \frac{(80 \times 10^{-9}\text{ m})(110 \times 10^{-9}\text{ m})}{110 \times 10^{-9}\text{ m} - 80 \times 10^{-9}\text{ m}} \simeq 6.627\times10^{-34}\text{ J s.} \tag{1.25}$$

This is a very accurate result indeed.

(b) The work function of the metal can be obtained from either one of the two data

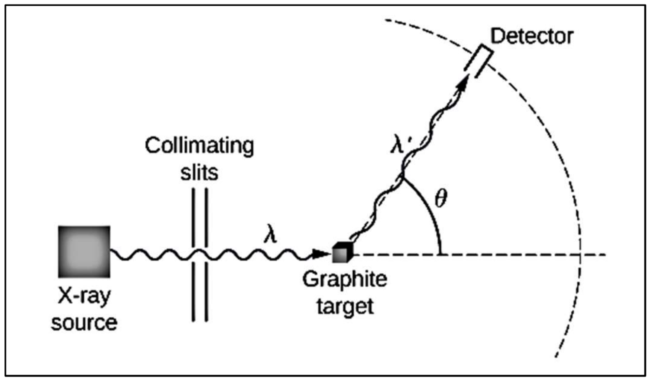
$$W = \frac{hc}{\lambda_1} - K_1 = \frac{6.627 \times 10^{-34}\text{ J s} \times 3 \times 10^8\text{ m s}^{-1}}{80 \times 10^{-9}\text{ m}} - 11.390 \times 1.6 \times 10^{-19}\text{ J}$$

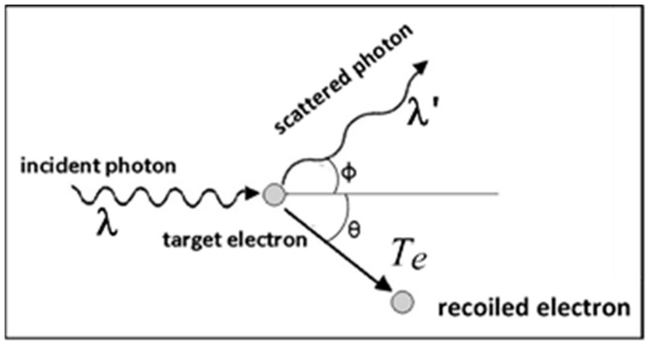
$$= 6.627 \times 10^{-19}\text{ J} = 4.14\text{ eV.} \tag{1.26}$$

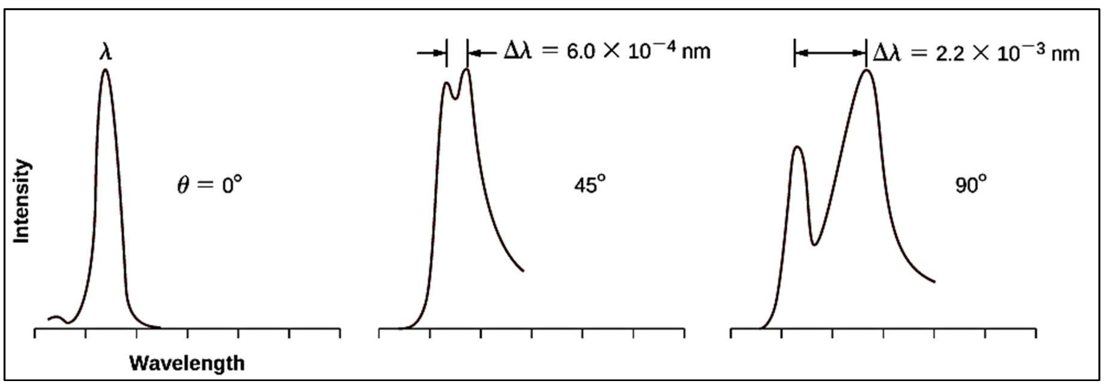
The cutoff frequency and wavelength of lead are

$$\nu_0 = \frac{W}{h} = \frac{6.627 \times 10^{-19}\text{ J}}{6.627 \times 10^{-34}\text{ J s}} = 10^{15}\text{ Hz}, \qquad \lambda_0 = \frac{c}{\nu_0} = \frac{3 \times 10^8\text{ m/s}}{10^{15}\text{ Hz}} = 300\text{ nm.} \tag{1.27}$$

Compton Effect:







High energy photons: X-rays and γ-rays.
Electron is assumed to be free.
Collision is considered to be elastic.

Conservation of energy:

$$h\nu = h\nu' + T_e = h\nu' + mc^2 - m_0c^2$$

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_0c^2 = mc^2 \qquad \Rightarrow \frac{h}{\lambda} - \frac{h}{\lambda'} + m_0c = mc.$$

Squaring

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'} + 2m_0ch\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + m_0{}^2c^2 = m^2c^2$$

$$\Rightarrow \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'} + \frac{2m_0ch}{\lambda\lambda'}(\lambda' - \lambda) = m^2c^2 - m_0{}^2c^2$$

$$\Rightarrow \frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-2\frac{h^2}{\lambda\lambda'}+\frac{2m_0ch}{\lambda\lambda'}(\lambda'-\lambda)=\frac{m_0{}^2c^2}{1-\beta^2}-m_0{}^2c^2=\frac{m_0{}^2c^2\beta^2}{1-\beta^2}=\frac{m_0{}^2v^2}{1-\beta^2}$$

[Where $\beta = v/c$]

$$\Rightarrow \frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-2\frac{h^2}{\lambda\lambda'}+\frac{2m_0ch}{\lambda\lambda'}(\lambda'-\lambda)=m^2v^2 \quad \text{...........(A)}$$

Conservation of momentum:

Relativistic energy relation $\Rightarrow E^2 = p^2c^2 + m_0{}^2c^4$.

For photon, $E = h\nu$ and rest mass $m_0 = 0$. Therefore $h\nu = pc$.

$$\Rightarrow photon\ momentum\ p = \frac{h\nu}{c} = \frac{h}{\lambda}.$$

Therefore

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\varphi + mv\cos\theta \qquad \Rightarrow \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi = mv\cos\theta \qquad \text{...........(B) and}$$

$$0 = \frac{h}{\lambda'}\sin\varphi - mv\sin\theta \qquad \Rightarrow \frac{h}{\lambda'}\sin\varphi = mv\sin\theta\ . \qquad \text{...........(C)}$$

Squaring and adding (B) and (C):

$$\frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-2\frac{h^2\cos\varphi}{\lambda\lambda'}=m^2v^2 \quad \text{...........(D)}$$

(A) – (D)

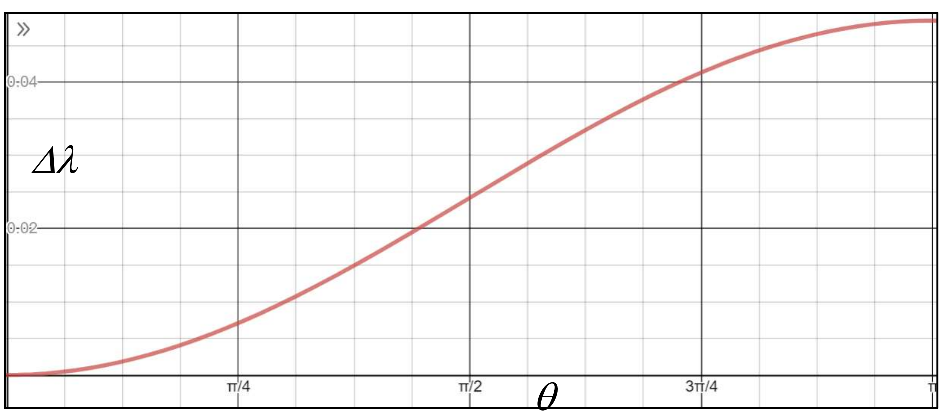
$$-2\frac{h^2}{\lambda\lambda'}(1-\cos\varphi)+\frac{2m_0ch}{\lambda\lambda'}(\lambda'-\lambda)=0$$

Wavelength shift:

$$\Rightarrow \lambda'-\lambda=\frac{h}{m_0c}(1-\cos\varphi)=\lambda_c(1-\cos\varphi) \quad \text{.......... (E)}$$

$$\Rightarrow \lambda' = \lambda + \lambda_c(1-\cos\varphi). \quad \text{.......... (F)}$$

$$\lambda_c = \frac{h}{m_0c} = \text{Compton wavelength} = \frac{6.626\times10^{-34}}{9.11\times10^{-31}\times3\times10^8} = 0.02424\ \text{Å}.$$



Recoil angle of electron:

We have $\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\varphi = mv\cos\theta$ ....(B) and $\quad \frac{h}{\lambda'}\sin\varphi = mv\sin\theta\,.$ ....(C)

$$\tan\theta = \frac{\frac{h}{\lambda'}\sin\varphi}{\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\varphi} = \frac{\sin\varphi}{\frac{\lambda'}{\lambda}-\cos\varphi}$$

From (F), $\frac{\lambda'}{\lambda} = 1 + \frac{\lambda_c}{\lambda}(1-\cos\varphi) = 1 + \alpha(1-\cos\varphi)$.

Then $\tan\theta = \frac{\sin\varphi}{1+\alpha(1-\cos\varphi)-\cos\varphi} = \frac{\sin\varphi}{(1+\alpha)(1-\cos\ \ )} = \frac{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}{2(1+\alpha)\sin^2\frac{\varphi}{2}} = \frac{\cos\frac{\varphi}{2}}{(1+\alpha)\sin\frac{\varphi}{2}}$

$\cot\theta = (1+\alpha)\tan\frac{\varphi}{2}$ .... (G)

Recoil energy of electron:

$$T_e = h\nu - h\nu' = h\nu\left(1-\frac{\nu'}{\nu}\right) = h\nu\left(1-\frac{\lambda}{\lambda'}\right) = h\nu\left(1-\frac{1}{1+\alpha(1-\cos\varphi)}\right)$$

$$= h\nu\frac{\alpha(1-\cos\ \ )}{1+\alpha(1-\cos\varphi)}.$$

Useful way to calculate T_e if λ (or ν) and φ are given:

First find $\lambda' = \lambda + \lambda_c(1-\cos\varphi)$

Now $T_e = h\nu - h\nu' = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$

Use $hc = 1240\ nm.eV$

Then: $T_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) = 1240\left(\frac{1}{\lambda \text{ in nm}} - \frac{1}{\lambda' \text{ in nm}}\right)\ eV$

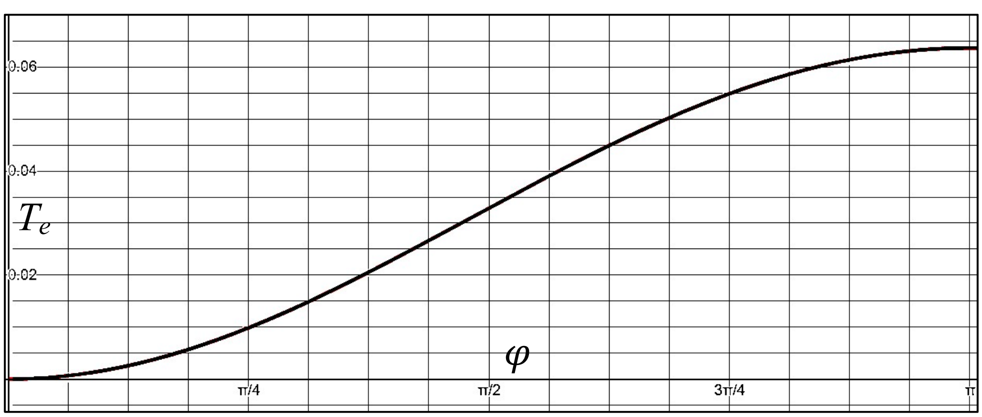
Plot $\mathbf{T_e} = \frac{hc}{\lambda}\frac{\alpha(1-\cos\varphi)}{1+\alpha(1-\cos\varphi)}$ $\boldsymbol{vs.\ \varphi}$. Given $\boldsymbol{\lambda = 0.709}$ Å.

Ans.: $\alpha = \frac{h}{m_0 c\lambda} = \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}\times0.709\times10^{-10}} = 34.2328\times10^{-3}$

$$\frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{0.709\times10^{-10}} = 28.036671\times10^{-16}\ J$$

$$\mathrm{T}_e = \frac{hc}{\lambda}\frac{\alpha(1-\cos\varphi)}{1+\alpha(1-\cos\ \ )} = \frac{hc}{\lambda}\cdot\frac{34.2328\times10^{-3}(1-\cos\ \ )}{1+34.2328\times10^{-3}(1-\cos\ \ )}$$

We plot $\frac{0.034(1-\cos\ \ )}{1+0.034(1-\cos\ \ )}\ vs.\ \varphi$



$$(\mathrm{T}_e)_\pi = \frac{hc}{\lambda}\frac{\alpha(1-\cos\ \ )}{1+\alpha(1-\cos\pi)} = \frac{hc}{\lambda}\cdot\frac{2\alpha}{1+2\alpha};\ (\mathrm{T}_e)_{\pi/2} = \frac{hc}{\lambda}\cdot\frac{\alpha}{1+\alpha}.$$

Comparison between Photoelectric Effect and Compton Effect:

	Photoelectric Effect	Compton Effect
Participating Radiation (photon)	Visible or UV light $\lambda \sim 10^2\ nm$ $E \sim eV$	X-Rays and γ-Rays $\lambda \sim 10^{-1} - 10^{-3}\ nm$ $E \sim keV - MeV$
What happens to the photon	Completely absorbed by the electron (or other scattering particle) Energy completely transferred to the electron	Energy partly transferred to electron, Direction changes up to 180°, Wavelength increases, For electrons $\Delta\lambda$ *may be up to* $2\lambda_c = 2 \times 0.002426\ nm$
Participating electron	Conduction electrons or electrons loosely bound the atoms of a metal	Loosely bound atomic electrons of non-metals e.g. graphite
Electrons free or bound	Electrons have negative energy of several eV In magnitude equal or slightly greater than the work function. Work function of Ag and Na are $4.54\ eV$ & $2.28\ eV$ Magnitude of electron energy is comparable to the energy of photon (eV). So electrons are considered as bound.	Several eV. Magnitude may be greater than that of electrons of Photoelectric effect. Ionisation potential of Carbon is $11.26\ eV$ Magnitude of electron energy is small compared to the photon energy (keV). So electrons are considered as free.
What happens to the electron	Emits from the metal	Recoils. If recoil energy is high then emits from the material.
Energy of emitted / recoiled electron	Several eV. Non-relativistic treatment is allowable	Several eV to keV. Relativistic treatment is required

Problems:

JAM 2017

Q.31 A photon of frequency ν strikes an electron of mass m initially at rest. After scattering at an angle ϕ, the photon loses half of its energy. If the electron recoils at an angle θ, which of the following is (are) true?

(A) $\cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

(B) $\sin\theta = \left(1 - \frac{mc^2}{h\nu}\right)$

(C) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$.

(D) Change in photon wavelength is $\frac{h}{mc}(1 - 2\cos\phi)$.

Ans.: (A) $\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\varphi) \quad \Rightarrow c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_0 c}(1 - \cos\varphi)$

$$\Rightarrow c\left(\frac{2}{\nu} - \frac{1}{\nu}\right) = \frac{h}{m_0 c}(1 - \cos\varphi)$$

$[Since \quad loss\ of\ energy = h\nu - h\nu' = h\nu/2 \ \Rightarrow \nu' = \nu/2]$

$\Rightarrow \frac{c}{\nu} = \frac{h}{m_0 c}(1 - \cos\varphi) \quad \Rightarrow (1 - \cos\varphi) = \frac{m_0 c}{h}\frac{c}{\nu} = \frac{m_0 c^2}{h\nu}$

$\Rightarrow \cos\varphi = 1 - \frac{m_0 c^2}{h\nu} \Rightarrow\Rightarrow (A)\ \ is\ correct.$

$Loss\ of\ energy\ of\ Compton\ scattered\ photon$
$= kinetic\ energy\ gained\ by\ the\ recoiled\ electron$

$$= T_e = h\nu\frac{\alpha(1-\cos\varphi)}{1+\alpha(1-\cos\ \ )},\ \text{ where } \alpha = \frac{\lambda_C}{\lambda} = \frac{h/m_0c}{\lambda} = \frac{h/m_0c}{c/\nu} = \frac{h\nu}{m_0c^2}.$$

$$\text{Here } T_e = \frac{h\nu}{2} = h\nu\frac{\alpha(1-\cos\ \ )}{1+\alpha(1-\cos\varphi)} \qquad \Rightarrow 2\alpha(1-\cos\varphi) = 1+\alpha(1-\cos\varphi);$$

$$\Rightarrow 1-\cos\varphi = \frac{1}{\alpha} \qquad \Rightarrow \cos\varphi = 1-\frac{1}{\alpha} = 1-\frac{m_0c^2}{h\nu} \qquad \Rightarrow\Rightarrow (\boldsymbol{A})\ \boldsymbol{is\ correct}.$$

(B) $$\tan\theta = \frac{\sin\varphi}{(1+\alpha)(1-\cos\ \ )} = \frac{\sqrt{1-\cos^2\varphi}}{(1+\alpha)/\alpha} = \frac{\sqrt{1-\left(1-\frac{1}{\alpha}\right)^2}}{(1+\alpha)/\alpha} = \frac{\sqrt{\frac{2}{\alpha}-\frac{1}{\alpha^2}}}{\frac{\alpha+1}{\alpha}} = \frac{\sqrt{\frac{2\alpha-1}{\alpha^2}}}{\frac{\alpha+1}{\alpha}} = \frac{\sqrt{2\alpha-1}}{\alpha+1}$$

$$\sin\theta = \sqrt{\frac{1}{1+\cot^2\theta}} = \sqrt{\frac{1}{1+\frac{(1+\alpha)^2}{2\alpha-1}}} = \sqrt{\frac{1}{\frac{\alpha^2+4\alpha}{2\alpha-1}}} = \sqrt{\frac{2\alpha-1}{\alpha^2+4\alpha}}$$

Given relation in (B) : $\sin\theta = 1-\frac{m_0c^2}{h\nu} = 1-\frac{1}{\alpha} \qquad \Rightarrow\Rightarrow (\boldsymbol{B})\ \boldsymbol{is\ wrong}.$

(C) We have $\frac{h}{\lambda'}\sin\varphi = mv\sin\theta.$

$$\Rightarrow \frac{mv}{h/\lambda'} = \frac{\sin\ \ }{\sin\theta} \qquad \Rightarrow\Rightarrow (\boldsymbol{C})\ \boldsymbol{is\ correct}.$$

(D) Change in photon wavelength $\lambda'-\lambda = \frac{h}{m_0c}(1-\cos\varphi) \qquad \Rightarrow\Rightarrow (\boldsymbol{D})\ \boldsymbol{is\ wrong}.$

JAM 2016

Q.57 X-rays of 20 keV energy is scattered inelastically from a carbon target. The kinetic energy transferred to the recoiling electron by photons scattered at 90° with respect to the incident beam is ________keV.
(Planck constant = 6.6×10^{-34} Js, Speed of light = 3×10^{8} m/s, electron mass = 9.1×10^{-31} kg, Electronic charge = 1.6×10^{-19}C)

Ans.: Given: $h\nu = 20keV$; Also $\alpha = \frac{h\nu}{m_0c^2} = \frac{20\times1000\times1.6\times10^{-19}}{9.1\times10^{-31}\times(3\times10^8)^2} = \frac{2\times1.6}{9.1\times9}$ =0.039072

$$T_e = h\nu\frac{\alpha(1-\cos\varphi)}{1+\alpha(1-\cos\ \ )} \ \Rightarrow T_e = 20keV\times\frac{0.039072}{1+0.039072} \approx 0.75keV$$

Alternately: $h\nu = 20keV \Rightarrow \lambda = \frac{1240}{20\times1000} = 0.062\ nm$

$$\lambda' = \lambda + \frac{h}{m_0c}(1-\cos\varphi) = 0.062\ nm + \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times3\times10^8} = 0.062\ nm + 0.002427nm$$

$$= 0.064427nm$$

$$h\nu' = \frac{1240}{0.064427}eV = 19.25\ keV$$

$$T_e = h\nu - h\nu' = 0.75\ keV$$

JAM 2015: Section C

Q.8 X-rays of wavelength 0.24 nm are Compton scattered and the scattered beam is observed at an angle of 60° relative to the incident beam. The Compton wavelength of the electron is 0.00243 nm. The kinetic energy of scattered electrons in eV is ___________.

Ans.: $\lambda'-\lambda = \frac{h}{m_0c}(1-\cos\varphi) = \lambda_c(1-\cos\varphi) = 0.00243\times(1-\cos60°)$

$$= 0.00243/2 = 0.001215\ nm$$

$\lambda' = 0.241215\ nm$

Loss of energy of the X-ray is the gain in kinetic energy of the electron. So:

$T_e = \left(\frac{1240}{0.24} - \frac{1240}{0.241215}\right)\ eV = (5166.67 - 5140.64)\ eV = 26.03\ eV$

JAM 2013

Q.18 A beam of X-rays of wavelength 0.2 nm is incident on a free electron and gets scattered in a direction with respect to the direction of the incident radiation resulting in maximum wavelength shift. The percentage energy loss of the incident radiation is

Ans.: $\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\varphi) = \lambda_c(1 - \cos\varphi) = 0.002426 \times (1 - \cos 180°)$

$= 0.004856\ nm$

$\lambda' = 0.204856\ nm$

% loss of energy of the X-rays:

$\left(\frac{1240}{0.2} - \frac{1240}{0.204856}\right)/\frac{1240}{0.2}\ \% = (6200 - 6053.03)/6200\ \% = 2.37\ \%.$

JAM 2010

Q.19 (a) A photon of initial momentum p_0 collides with an electron of rest mass m_0 moving with relativistic momentum P and energy E. The change in wavelength of the photon after scattering by an angle θ is given by, $\Delta\lambda = 2c\lambda_0 \frac{p_0 + P}{E - cP}\sin^2\frac{\theta}{2}$, where c is the speed of light and λ_0 is the wavelength of the incident photon before scattering. What will be the value of $\Delta\lambda$ when the electron is moving in a direction opposite to that of the incident photon with momentum P and energy E? Show that the value of $\Delta\lambda$ becomes independent of the wavelength of the incident photon when the electron is at rest before collision. (12)

(b) In a Compton experiment, the ultraviolet light of wavelength 2000 Å is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the Compton shift, if the observation is made at 90° with respect to the direction of the incident light? (9)

Ans.: (a) In case of head on collision the photon will bounce back by 180°.

Then $\Delta\lambda = 2c\lambda_c \frac{p_0+P}{E-cP}\sin^2\frac{\theta}{2} = 2c\lambda_c \frac{p_0+P}{E-cP}\sin^2\frac{\pi}{2} = 2c\lambda_c \frac{p_0+P}{E-cP}$

If the electron is initially at rest then: $P = 0$.

Then $\Delta\lambda = 2c\lambda_c \frac{p_0+P}{E-c}\sin^2\frac{\theta}{2} = 2c\lambda_c \frac{p_0}{E}\sin^2\frac{\theta}{2} = 2c\lambda_c \frac{h/\lambda}{E}\sin^2\frac{\theta}{2}$

(b) Resolving power $= \frac{\lambda}{\Delta\lambda} = \frac{\lambda}{\lambda_c(1-\cos\varphi)} = \frac{2000}{0.02426\times(1-\cos 90°)} = \frac{2000}{0.02426} = 82440.23$

JAM 2008

Q.11 A photon of wavelength λ is incident on a free electron at rest and is scattered in the backward direction. The fractional shift in its wavelength in terms of the Compton wavelength λ_c of the electron is

(A) $\frac{\lambda_c}{2\lambda}$ **(B)** $\frac{2\lambda_c}{3\lambda}$ **(C)** $\frac{3\lambda_c}{2\lambda}$ **(D)** $\frac{2\lambda_c}{\lambda}$

Ans.: $\frac{\lambda'-\lambda}{\lambda} = \frac{\lambda_c}{\lambda}(1-\cos\varphi) = \frac{\lambda_c}{\lambda}(1-\cos\pi) = \frac{2\lambda_c}{\lambda}.$

JAM 2006:

23. A photon of energy E_{ph} collides with an electron at rest and gets scattered at an angle 60° with respect to the direction of the incident photon. The ratio of the relativistic kinetic energy T of the recoiled electron and the incident photon energy E_{ph} is 0.05.

(a) Determine the wavelength of the incident photon in terms of the Compton wavelength $\lambda_c\left(=\frac{h}{m_e c}\right)$, where h, m_e, c are Planck's constant, electron rest mass and velocity of light respectively. [12]

(b) What is the total energy E_e of the recoiled electron in units of its rest mass ? [9]

Ans.: (a) $0.05 = \frac{T_e}{h\nu} = \frac{h\nu - h\nu'}{h\nu} = 1 - \frac{\nu'}{\nu} = 1 - \frac{\lambda}{\lambda'}$

$\lambda = 0.95\lambda' = 0.95\Delta\lambda + 0.95\lambda \quad \Rightarrow 0.05\lambda = 0.95\Delta\lambda \quad \Rightarrow \lambda = 19\Delta\lambda$

$\lambda = 19\Delta\lambda = 19\lambda_c(1-\cos\varphi) = 19\lambda_c(1-\cos 60^\circ) = \frac{19}{2}\lambda_c = 9.5\lambda_c.$

(b) $E_e = T_e + m_0c^2 = 0.05h\nu + m_0c^2 = \frac{0.05hc}{\lambda} + m_0c^2 = \frac{0.05hc}{9.5\lambda_c} + m_0c^2$

$= \frac{0.05hcm_0c}{9.5h} + m_0c^2 = \frac{0.05\quad _0c^2}{9.5} + m_0c^2 = \frac{m_0c^2}{190} + m_0c^2.$

$= \frac{191m_0c^2}{190}\, J = \frac{191\quad ^2}{190}\quad \textit{in the units of rest mass}$

Zettili: Page-21

Calculate the de Broglie wavelength for

(a) a proton of kinetic energy 70 MeV kinetic energy and

(b) a 100 g bullet moving at 900 m s^{-1}.