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App. Math
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# Probability

① If $P(A) = 3/8$, $P(B) = 1/2$, $P(A \cap B) = 1/4$ then find $P(\bar{A}/\bar{B})$ and $P(\bar{B}/\bar{A})$

② Two coins are thrown simultaneously. find the probability of getting atmost one head.

③ If A and B are independent, then show that $A^c$ and $B^c$ are independent.

④ State and prove Bayes' theorem of probability.

⑤ Is the function defined as follows a density function?

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

If so, determine the probability that the variate having this density will full in the interval $(1, 2)$?

⑥ there are 6 persons are seated at a row. Find the probability that 2 persons always seat together.

⑦ Two distinct natural numbers are choosen at random from the first 100 natural numbers. find the probability that

i) The first one is divisible by 2 and 2nd one divisible by 3

ii) both are divisible by either 2 or 3

iii) both are ~~divisible~~ neither divisible by 2 nor 3

iv) the two numbers $x$ and $y$ satisfy the equation $3x-4y=0$

(8) If two dice are thrown simultaneously, find the probability of the sum of numbers coming up is greater than 9.

(9) A die is rolled. If the outcome is an odd number, find the probability that it is a prime.

(10) Two dice are thrown. Find the probability that the numbers appeared has the sum 8, if it is known that the second die is always exhibits 4.

(11) A bag contains 5 white, 7 red and 8 black balls. If three balls are drawn one by one without replacement, find the prob. of getting all white balls.

(12) Test ~~wheathere~~ whethere the events A and B are independent if $P(A)=0.6$ $P(B)=0.2$ and $P(A \cup B)=0.68$

(13) For any two events A and B, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(14) A couple has 2 children. find the prob. that both are boys, if it is known that

i) one of the children is a boy

ii) the older child is a boy.

(15) A problem in mathematics is given to 3 students whose chance of solving it are 1/2, 1/3, 1/4 respectively. What is the probability that the problem is solved.

① Probability (Solun) ①

As $P(A) = 3/8$, $P(B) = 1/2$, $P(A \cap B) = 1/4$

Now $P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$, As we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/2 + 3/8 - \frac{1}{4}$

$= \frac{4+3-2}{8}$

$= 5/8$

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$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 5/8$

$= 3/8$

$P(\bar{B}) = 1 - P(B) = 1 - 1/2 = 1/2$

$\therefore P(\bar{A} | \bar{B}) = \frac{3/8}{1/2} = 3/4$

Similarly $P(\bar{B}/\bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})}$

$= P(\bar{A} \cap \bar{B}) / 1 - P(A)$

$= \frac{3/8}{1 - 3/8} = \frac{3/8}{5/8} = 3/5$

② $S = \{HH, TT, HT, TH\}$

P(getting atmost one head) $= n(A)/n(S)$

$= 3/4$

③ As A and B independent then we have $P(A \cap B) = P(A) \cdot P(B)$

we have to prove $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

Now $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

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$= 1 - \{ P(A) + P(B) - P(A \cap B) \}$

$= 1 - P(A) - P(B) + P(A) \cdot P(B)$

$= (1 - P(A)) - P(B)(1 - P(A))$

$= (1 - P(A))(1 - P(B))$

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

Hence $\bar{A}$ and $\bar{B}$ is independent (proved)

(4) ?

(5) A function $f(x)$ is said to be density function

if $\int_{-\infty}^{\infty} e^{-x} dx = 1$

$\Rightarrow \int_{0}^{\infty} e^{-x} dx = -e^{-x} \Big|_{0}^{\infty}$

$= -(e^{-\infty} - e^{0})$

$= -(0 - 1)$

$= 1$

Hence given $f(x)$ is a density function.

Now $P(1 < x < 2) = \int_{1}^{2} e^{-x} dx$

$= -e^{-x} \Big|_{1}^{2}$

$= -(e^{-2} - e^{-1})$

$= -\left(\frac{1}{e^2} - \frac{1}{e}\right)$

$= \left(\frac{1}{e} - \frac{1}{e^2}\right) = \left(\frac{e-1}{e^2}\right)$ Ans

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(6) there 6 person seat together, then the total no. of ways

$n(S) = 6! = 720$

No of ways of 2 person seat together is

$n(A) = 5! \times 2!$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{5! \times 2!}{6!} = \frac{\cancel{5!} \times 2}{6 \times \cancel{5!}}$

$= 2/6 = 1/3$

(7) If two distinct natural no are choosen at random from first 100 natural no. then total no of outcomes

$n(S) =$

$= {}^{100}C_2 = 50 \times 99$

total no of natural no. which is divisible by $2 = [100/2]$
$= 50$

" " " by $3 = [100/3] = 33$

$n(A) = {}^{50}C_1 \times {}^{33}C_1 = 50 \times 33$

i) $P(A) = \frac{50 \times 33}{50 \times 99} = 1/3$

ii) $P($both are divisible by 2 or 3$)$
$= P(A \cup B) = n(A \cup B) / n(S)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 50 + 33 - 16 = 83 - 16 = 67$

Hence $P(A\cup B) = 67/_{50\times 99} = \frac{67}{50\times 99}$

iii) $P$( both are neither divisible by 2 nor divisible by 3)

$=$

$= 1 - P(A\cap B)$

$= 1 - \frac{n(A\cap B)}{n(S)} = 1 - \frac{16}{100}$

iv) as $x \in \{1, 2, \ldots 100\}$ and $y \in \{1, 2, \ldots 100\}$

$3x - 4y = 0 \Rightarrow \boxed{y = \frac{3x}{4}}$ Similarly $x = \frac{4y}{3}$
if $y = 3, x = 4$
$y = 6 \quad x = 8$

if $x = 4, \quad y = 3$
$x = 8, \quad y = 6$
$x = 12 \quad y = 9$
$x = 16, \quad y = 12$
$\vdots$
$x = 100, \quad y = 75$ i.e we taken the values of $x$ as multiple of 4 and $y$ is multiple of 3

$P(A) = n(A)/n(S) = \frac{25\times 33}{50\times 99} = \frac{1\times 33}{198} = \frac{1}{6}$

(8) Two dice are thrown,
$n(S) = 6^2 = 36$
A: coming up no is greater than 9
$n(A) = \{(4,6), (6,4), (5,5), (5,6), (6,5), (6,6)\}$
$= 6$
$\therefore P(A) = n(A)/n(S) = 6/_{36} = \frac{1}{6}$

(9) A die is rolled,
# of outcomes = 6 = n(S)
A : number is odd
B : " " prime

$P(B/A) = P(A \cap B) / P(A)$

Now $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{3}{6} + 3/6 - \frac{4}{6}$

$= \frac{3+3-4}{6} = 2/6 = 1/3$

$P(B/A) = \frac{1}{3} / \frac{1}{2} = 2/3$

(10) As two dice is thrown, then total no. of outcomes $n(S) = 6^2 = 36$

A : numbers appeared has the sum 8

B : second die is always exhibits 4

Now we have to find $P(A/B)$

Now $P(A/B) = \frac{P(A \cap B)}{P(B)}$

=

$P(A \cap B) = P(\text{sum 8 and 2nd die exhibits 4})$

$= \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$

$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$

$\therefore P(A/B) = \frac{1/36}{6/36} = 1/6$

(11). A bag contains 5W, 7R, 8B balls,

$W_i$ : getting white balls in 1st, 2nd, 3rd

$R_i$ : " Red " " " "

$B_i$ : " Black " " " "

where $i = 1, 2, 3$

Now we have to find $P(W_1 \cap W_2 \cap W_3)$ (as three balls are drawn one by one without replacement)

Now we can write

$$P(W_1 \cap W_2 \cap W_3) = P(W_1)\, P(W_2/W_1)\, P(W_3/W_1 \cap W_2)$$

$$= \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}$$

$$= \frac{1}{114}$$

(12) As $P(A) = 0.6$, $P(B) = 0.2$, $P(A \cup B) = 0.68$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.68 = 0.6 + 0.2 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.8 - 0.68$

$= 0.12$

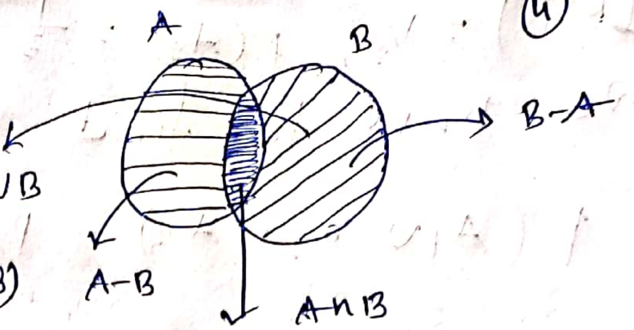
Now $P(A \cap B) = 0.12 = 0.6 \times 0.2$

$= P(A) \cdot P(B)$

Hence A and B is independent (proved)

(13) For any two events A and B we have to prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Using the venn diagram we can write




$A \cup B = (B-A) \cup (A \cap B) \cup (A-B)$

Taking prob. on both sides we have.

$P(A \cup B) = P(B-A) + P(A \cap B) + P(A-B)$ [as given three events mutually exclusive].

$= [P(B) - P(A \cap B)] + P(A \cap B) + [P(A) - P(A \cap B)]$

$= P(A) + P(B) + P(A \cap B) - 2P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (proved)

(14) A couple has 2 children.

$S = \{BB, GG, BG, GB\}$

A: both are boys

B: one of the children is boy

C: the older child is a boy.

Now we have to find $P(A/B)$ and $P(A/C)$

i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

ii) $P(A/C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{1/4}{2/4} = 1/2$

(15) Ai: 3 students solved this problem

that is we have

$P(A_1) = 1/2$, $P(A_2) = \frac{1}{3}$, $P(A_3) = \frac{1}{4}$

We have to find $P(A_1 \cup A_2 \cup A_3)$

$\therefore$ $P(A_1 \cup A_2 \cup A_3) = P$ ( problem is solved )

$= P$ ( atleast one student solved it )

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_3 \cap A_1) + P(A_1 \cap A_2 \cap A_3)$$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_1) \cdot P(A_2) - P(A_2) \cdot P(A_3) - P(A_3) \cdot P(A_1) + P(A_1) \cdot P(A_2) \cdot P(A_3)$$

[ as $A_i$, $i = 1, 2, 3$ mutually independent ]

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{2} \times \frac{1}{3} - \frac{1}{3} \times \frac{1}{4} - \frac{1}{4} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$$

$$= \frac{6+3+3}{12} - \frac{4+2+3}{24} + \frac{1}{24}$$

$$= 1 - \frac{9}{24} + \frac{1}{24}$$

$$= \frac{15}{24} + \frac{1}{24} = \frac{16}{24} = \frac{2}{3}$$

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