Ultraviolet Catastrophe :

We know that in thermal equilibrium, the energy corresponding to each degree of freedom is $\frac{1}{2}kT$. So, in thermal equilibrium, the energy density of radiation per unit frequency interval per unit volume is

$$du = u(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3}\bar{E}\, d\nu$$

$$u(\nu) = \frac{8\pi\nu^2}{c^3}\bar{E}$$

~~Again~~ Again the average energy of a harmonic oscillator in thermal equilibrium is $\bar{E} = kT$ and so the spectrum of black body radiation is expected to be

$$u(\nu) = \frac{8\pi\nu^2}{c^3}\bar{E} = \frac{8\pi\nu^2 kT}{c^3}$$

This result is very bad news – the energy density of radiation diverges at high frequencies. Einstein expressed this result forcibly –

$$\int_0^{\infty} u(\nu)\, d\nu = \int_0^{\infty} \frac{8\pi\nu^2 kT}{c^3}\, d\nu \longrightarrow \infty$$

This is the famous result known as the Ultraviolet Catastrophe – the total energy in black-body radiation diverges.

Saha Ionization formula :

Let the energies of two states, A & B, be E_A & E_B, and their statistical weights g_A & g_B, respectively. In local thermodynamic equilibrium the number of particles in two states, N_A & N_B, statisfy Boltzman equation:

$$\frac{N_A}{N_B} = \frac{g_A}{g_B} \exp\left[-(E_A - E_B)/kT\right]$$

Now we shall consider two ions, 'i' & 'i+1' of the same element. The ionization potential, i.e. the energy needed to ionize "i" from the ground state is χ, and the statistical weights of the ground states of the two ions are g_i & g_{i+1}, respectively. The number densities, $[cm^{-3}]$, of the two types of ions and free electrons are n_i, n_{i+1} & n_e, respectively. We shall use the Boltzman equation to estimate the number ratio n_{i+1}/n_i. The statistical weight of an ion in the ~~upper~~ lower ionization state to be used in the upper equation is just g_i. The statistical weight of an ion in the upper ionization state is g_{i+1} multiplied by the number of possible states in which a free electron may put. As in every cell of phase space with vol^m h^3 there are two possible states for an electron, as there are two possible orientations of its spin. The energy of a free electron with a momentum p w.r.t the ground state of an ion in a lower ionization state is $E = \chi + p^2/2m$. The number of cells ~~avail~~ available for free electrons with a momentum betn p & $p+dp$ is $V_e 4\pi p^2 dp/h^3$, where $V_e = 1/n_e$ is the vol^n in ordinary space available per electron, and n_e is the free electron density. Now we integrate over all ~~avail~~ available cells, taking the Boltzman factor into account

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}}{g_i}\frac{2V_e}{h^3}\int_0^\infty e^{-(\chi+p^2/2m)/kT}\, 4\pi p^2 dp$$

$$= \frac{g_{i+1}}{g_i}\frac{1}{n_e}\frac{2}{h^3}(2mkT)^{3/2} e^{-\chi/kT}\, 2\pi \int_0^\infty e^{-x} x^{1/2} dx$$

8am $= \frac{g_{i+1}}{g_i} \frac{1}{n_e} \frac{2}{h^3} (2\pi mkT)^{3/2} e^{-\chi/kT}$ $\left[\chi = p^2/2mkT \right]$

9am $\therefore \frac{n_{i+1} n_e}{n_i} = \frac{(2\pi mkT)^{1.5}}{h^3} \, 2 \, \frac{g_{i+1}}{g_i} e^{-\chi/kT}$

10am This equation is known as Saha equation.

1. In an ideal gas obeying MB statistics there are N number of particles at a temperature T. Find the internal energy of the gas and the heat capacity at constant vol^m.

2. Six distinguishable particles are distributed over three nondegenerate levels of energies 0, E and 2E. Calculate the total number of micro-states of the system. Find the total energy of the distribution for which the probability is a maximum.

3. Five identifiable particles are distributed in three nondegenerate levels with energies 0, E & 2E. Determine the most probable distribution for a total energy 3E.

4. A linear harmonic oscillator moves with a constant energy along the x-axis. What will be the phase trajectory?

5. Derive an expression for the average thermal speed of a particle of mass m in an ideal Boltzmann gas at a temperature T. What is the value of this speed for an electron system obeying MB statistics at 300 K?

6. Starting from the MB velocity distribution of a system of gas particles, find the number of particles having kinetic energy bet^n E & E+dE. How does the energy distribution curve differ from the velocity distribution curve?

7. A system of distinguishable particles has two nondegenerate single-particle energy state 0 & E. At what temperature would the probability of a particle occupying the excited state be half that of occupying the ground state?

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ay	4	11	18	25	Notes
day	5	12	19	26	Appointment
esday	6	13	20	27	