(1) Superposition of two simple harmonic motions of the same frequency at right angles to each other.

Let the two simple harmonic vibrations acting along the x and y axes be represented respectively, by,

$$x = a_1 \cos(\omega t + \alpha_1)$$
$$y = a_2 \cos(\omega t + \alpha_2)$$

$$y = a_2 \cos[(\omega t + \alpha_1) - (\alpha_1 - \alpha_2)]$$

$$= a_2 \cos(\omega t + \alpha_1) \cdot \cos(\alpha_1 - \alpha_2) + a_2 \sin(\omega t + \alpha_1) \cdot \sin(\alpha_1 - \alpha_2)$$

$$= a_2 \cdot \frac{x}{a_1} \cos(\alpha_1 - \alpha_2) + a_2 \sqrt{1 - \frac{x^2}{a_1^2}} \sin(\alpha_1 - \alpha_2)$$

$$= \frac{a_2}{a_1} x \cos(\alpha_1 - \alpha_2) + \frac{a_2}{a_1} \sqrt{a_1^2 - x^2} \sin(\alpha_1 - \alpha_2)$$

$$\Rightarrow \frac{a_1}{a_2} y - x \cos(\alpha_1 - \alpha_2) = \sqrt{a_1^2 - x^2} \sin(\alpha_1 - \alpha_2)$$

Squaring both sides.

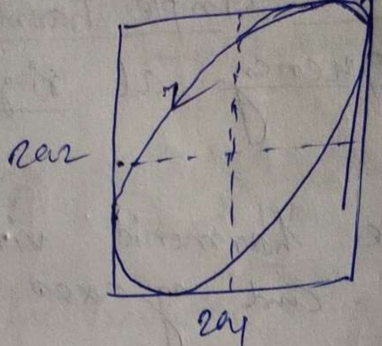
$$\frac{a_1^2}{a_2^2} y^2 + x^2 \cos^2(\alpha_1 - \alpha_2) - 2\frac{a_1}{a_2} xy \cos(\alpha_1 - \alpha_2) = (a_1^2 - x^2) \cdot \sin^2(\alpha_1 - \alpha_2)$$

$$\Rightarrow x^2 + \frac{a_1^2}{a_2^2} y^2 - \frac{2a_1}{a_2} xy \cos(\alpha_1 - \alpha_2) = a_1^2 \sin^2(\alpha_1 - \alpha_2)$$

$$\Rightarrow \boxed{\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2} \cdot \cos(\alpha_1 - \alpha_2) = \sin^2(\alpha_1 - \alpha_2)} \quad (1)$$

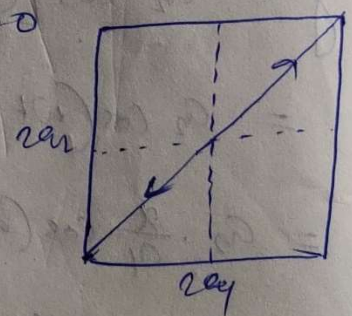
This equation represents an ellipse confined inside a rectangle of sides $2a_1$ & $2a_2$. The major axis of the ellipse makes an angle δ with the x axis

Case I :



Case I: If $\alpha_1 - \alpha_2 = 0$, i.e two simple harmonic motion are in phase, then equation ① becomes.

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2} = 0$$



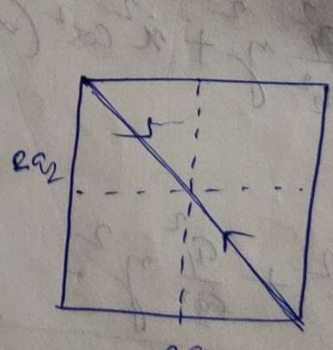
$$\frac{x}{a_1} - \frac{y}{a_2} = 0$$

$$y = \frac{a_2}{a_1} x$$

Which is a straight line passing through origin with a slope $\frac{a_2}{a_1}$.

Case II: If $\alpha_1 - \alpha_2 = \pi$ i.e. two simple harmonic motions are in opposite phase, - The equ^n ① reduces to.

$$\frac{x}{a_1} + \frac{y}{a_2} = 0$$

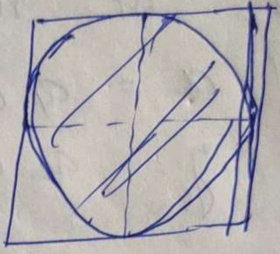


$$y = - \frac{a_2}{a_1} x.$$

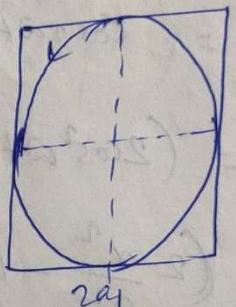
~~case III~~ ~~If~~ Which is also a straight line passing through origin with a slope $-a_2/a_1$.

Case III If $\alpha_1 - \alpha_2 = \pi/2$ i.e. two motion are quadrature, we have from equ^n (1)

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} = 1$$



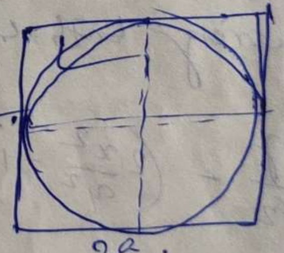
Which is an equation of ellipse of semi axes a_1 & a_2, the axes of ellipse coinciding with the x & y axes.



Case IV If $a_1 = a_2 = a$ and $\alpha_1 - \alpha_2 = \pi/2$

then equ^n (1) reduces to

$$x^2 + y^2 = a^2.$$



Which is a circle of radius a and centre at origin. Here the resultant motion is a uniform circular motion of angular frequency ω.

Phase measurement using lissajous figure / Pattern.

Y input signal → $V_y(t) = V_y \sin(2\pi f_y t + \phi)$ → unknown sine wave

x input signal → $V_x(t) = V_x \sin(2\pi f_x t)$ → Known sine wave

Case I: $\phi = 0$

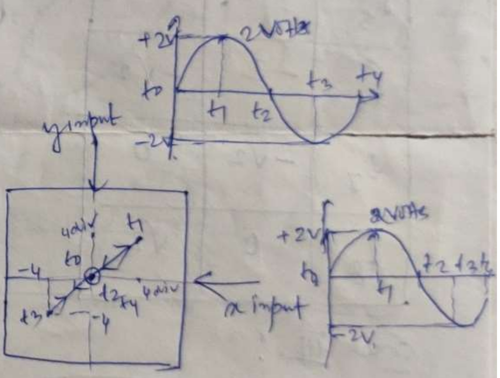
Let $V_y = V_x = 2$ Volts

$f_y = f_x = 50$ Hz

$\omega_y = \omega_x = 2\pi \times 50$ rad/sec $=$ 314 rad/sec.

$V_y(t) = 2 \sin 314t$

$V_x(t) = 2 \sin 314t$



t	x input	y input
t_0	0	0
t_1	2	2
t_2	0	0
t_3	-2	-2
t_4	0	0

If $S = 2$ div/volt

→ 0.

→ $\frac{2\ \text{div}}{\text{Volt}} \times 2\ \text{Volt} = 4$ div.

Pattern: straight diagonal line 1st & 3rd quadrant

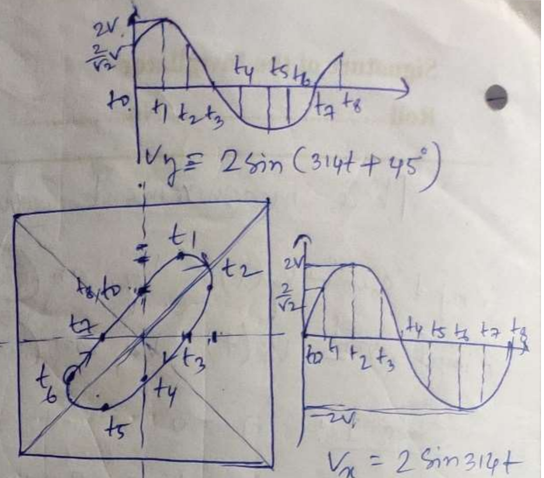
$V_x = V_y$ → slope $45°$

$V_y > V_x$ → slope $> 45°$

$V_y < V_x$ → slope $< 45°$.

Case II : $\phi = 45^\circ$

t	V_x	V_y
t_0	0	$2/\sqrt{2}$
t_1	$2/\sqrt{2}$	2
t_2	2	$\sqrt{2}$
t_3	$\sqrt{2}$	0
t_4	0	$-\sqrt{2}$
t_5	$-\sqrt{2}$	-2
t_6	-2	$-\sqrt{2}$
t_7	$-\sqrt{2}$	0
t_8	0	$\sqrt{2}$



Major axis → I & III quadrant & ~~max~~

Minor " → II & IV "

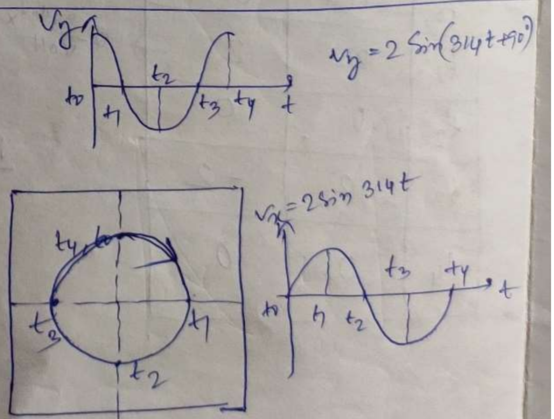
Rotation → Clockwise

Phase shift → +ve (45°)

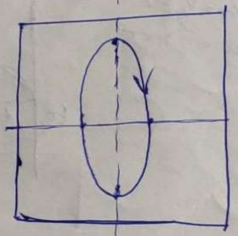
Major axis make 45° angle with X axis.

Case III :

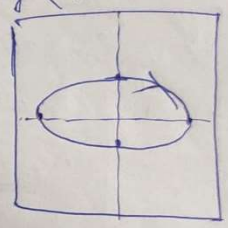
t	V_x	V_y
t_0	0	2
t_1	2	0
t_2	0	-2
t_3	-2	0
t_4	0	2



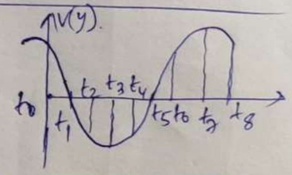
if $V_y > V_x$

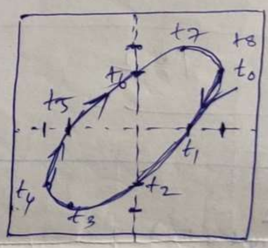


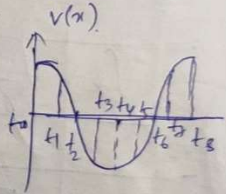
& if $V_y < V_x$



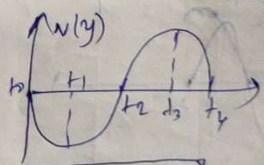
$V(x) = 2\cos 314t$
$V(y) = 2\cos(314t + 45^\circ)$

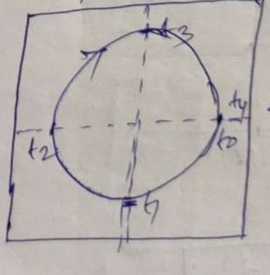


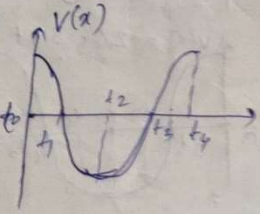




$V(x) = 2\cos 314t$
$V(y) = 2\cos(314t + 90^\circ)$







$V(y) = V_y \sin(2 \times 314t + \phi)$

$V(y) = V_y \sin(2\omega t + \phi)$

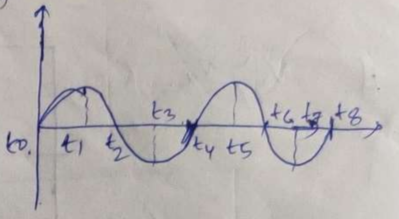
$V(x) = V_x \sin \omega t$

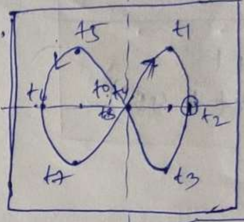
For $\phi = 0$

$V_x = V_y = 2$

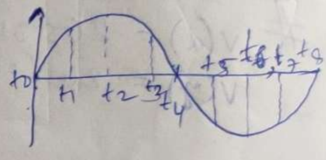
$\omega = 2\pi \times 50$

$V_y = 2\sin(2 \times 314t)$





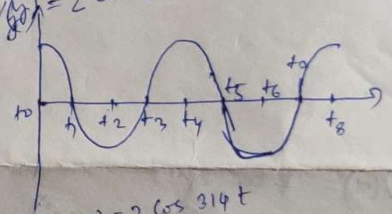
$V(x) = 2\sin 314t$



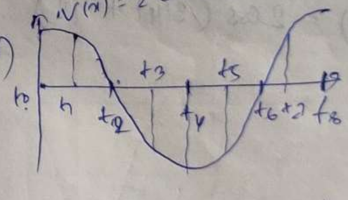
$f_x : f_y = 1 : 2$

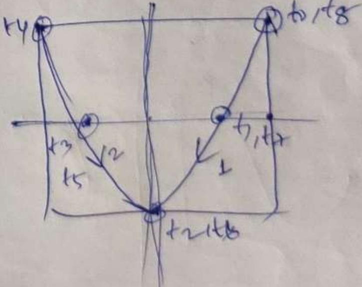
For $\phi = 0$

$V(y) = 2\cos(2 \times 314t)$



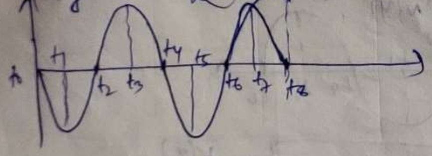
$V(x) = 2\cos 314t$





For $\phi = \pi/2$

$V(y) = 2\cos(2 \times 314t + \pi/2)$



$V(x) = 2\cos 314t$