Stefan - Boltzmann law:-

According to stefan's law, the total amount of heat radiated by a perfectly black body per second per unit area is directly propotional to the fourth power of its absolute temperature i.e.,

$$E \propto T^4$$

or $$E = aT^4$$

Where a is a constant, called as the stefan's constant.

The above law refers only to the emission and not the net loss. if a black body X at absolute temperature T is surrounded by another black body Y at absolute temperature T_0, then the amount of heat lost by black body X $= aT^4$

Amount of heat absorbed by the black body X from black body Y $= a T_0^4$

$\therefore$ Amount of heat lost by X per second per unit area $= a(T^4 - T_0^4)$

This is known as Stefan-Boltzman law

Derivation:

Let u be the energy density of radation in an enclosure at constant temperature T, V be the volm of enclosure & p the pressure of radation.

$\therefore$ The total energy of radation in the enclosure is $U = uV$

From the first law of thermodynamics

$$dQ = dU + p\,dV$$
$$= d(uV) + p\,dV$$
$$= d(uV) + \tfrac{1}{3}u\,dV \quad \left[\because p = \tfrac{1}{3}u\right]$$
$$= u\,dV + V\,du + \tfrac{1}{3}u\,dV$$
$$= V\,du + \tfrac{4}{3}u\,dV \quad \text{-------} (1)$$

From 2nd law of thermodynamics

$$dQ = T\,dS \quad \text{- - - -} (2)$$

Using ① & ②

$$ds = \frac{V\,du}{T} + \frac{1}{3}\,{u}/{T}\,dV \quad \cdots ③$$

Again $s = f(u, V)$

$$ds = \left(\frac{\partial s}{\partial u}\right)_V du + \left(\frac{\partial s}{\partial V}\right)_u dV \quad \cdots ④$$

From ③ & ④, we can write

$$\left(\frac{\partial s}{\partial u}\right)_V = \frac{V}{T} \text{ and } \left(\frac{\partial s}{\partial V}\right)_u = \frac{1}{3}\,{u}/{T} \quad \cdots ⑤$$

Since ds is a perfect differential, we have

$$\frac{\partial^2 s}{\partial V \partial u} = \frac{\partial^2 s}{\partial u \partial V}$$

From ⑤ $\quad \frac{\partial}{\partial V}\left(\frac{V}{T}\right) = \frac{1}{3}\frac{\partial}{\partial u}\left({u}/{T}\right) \quad \cdots ⑥$

Now temperature (T) is independent of V & is function of u only. So from ⑥

$$\frac{1}{T} = \frac{1}{3}\left(\frac{1}{T} - {u}/{T^2}\,{dT}/{du}\right)$$

$$\Rightarrow \quad 1 = \frac{1}{3} - \frac{1}{3}\,{u}/{T}\,.\,{dT}/{du}$$

$$\Rightarrow \quad 4\,{dT}/{T} = {du}/{u}$$

$$\therefore \quad \ln u = 4 \ln T + \ln a \;(\text{const.})$$

$$\therefore u = a T^4$$

We know that the energy E radiated per second per unit area from a perfectly black body at absolute temperature T and the energy of radiation u inside an enclosure at the same temperature are related by

$$E = \frac{1}{4} uc$$

Putting the value of u in the above equation

$$E = \frac{1}{4} a c T^4$$

$$= \sigma T^4$$

σ = constant and this is Stefan's law

Wien's Displacement Law :

If radation of a particular wavelength at a certain temperature is adiabatically altered to another wavelength then the temperature changes in the inverse ratio, i.e

$$\lambda_1 T_1 = \lambda_2 T_2 = \lambda_3 T_3 = \lambda_4 T_4 = \cdots$$

or, $\lambda T =$ Constant

Rayleigh-Jean's Law:

The deduction of the Rayleigh-Jean's law for the distribution of energy in the normal spectrum has been done by assuming that the energy is divided among all the possible modes of vibration. Accordingly, the energy density within the small range of wavelengths was determined by associating with every possible mode of vibration (frequency) an average energy kT.

The number of modes of vibration per unit vol^m in the frequency range ν and $\nu+d\nu$ is given by $= 8\pi\nu^2/c^3\, d\nu$

so energy density within frequency range ν and $\nu+d\nu$ is

$$u_\nu d\nu = \frac{8\pi\nu^2}{c^3}\, d\nu\, kT$$

In terms of wavelength,

$$\nu = c/\lambda \quad \& \quad d\nu = -c/\lambda^2\, d\lambda$$

$$\therefore u_\lambda d\lambda = \frac{8\pi}{c^3}\left(\frac{c}{\lambda}\right)^2\left(-\frac{c}{\lambda^2}d\lambda\right)\cdot kT = 8\pi kT/\lambda^4\; d\lambda$$

This law explains the energy distribution of long wavelength at higher temperature but fails for short wavelength.

8am
9am
10am
11am
12am
Lunch

Wien's Law of energy distribution:

Stefan-Boltzman law shows how the total energy of radiation is related to the temperature of the source of radiant energy. But the emitted radiation is not confined to a single wavelength but is spread over a continuous spectrum. How is this total energy distributed amongst the different wavelengths? The first step was taken by Wien in 1893. He showed that the spectral distribution of energy emitted by a black body at a temperature T can be expressed as

$$u_\lambda d\lambda = C\lambda^{-5} f(\lambda T)\, d\lambda$$

where $u_\lambda d\lambda$ is the energy density of radiation beth wavelength λ & $\lambda + d\lambda$. C = constant. $f(\lambda T)$ is a function of the product λT.