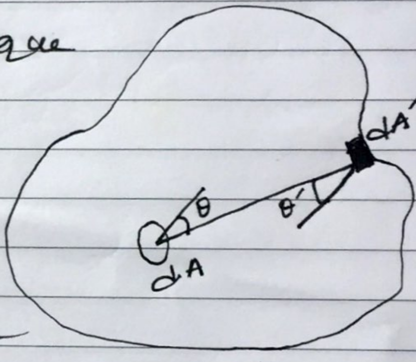
Kirchhoff's Law:

It states that any temperature, the ratio of the emissive power to the absorptive power for a given wavelength is the same for all bodies and is equal to the emissive power of a perfectly black body.

Derivation:

Consider an enclosure having walls opaque to radiations of all wavelengths and insulated thermally from the surroundings. The enclosure is at uniform temperature T. The enclosure is filled with temperature radiation emitted by the walls. A body A having emissive power (e_λ) & absorptive power (a_λ) being placed inside the enclosure.



i) Whatever be the initial temperature of the body, it will ultimately attain the temperature T of the enclosure.

ii) Since the total energy absorbed by the body is independent of its position or orientation w.r.t the walls of the enclosure. As the different surfaces of the body have different absorptive coefficients, this is possible when the radiation inside is isotropic

∴ The amount of energy emitted by an elemental area dA of the body in the direction betn θ, $\theta + d\theta$ and ϕ, $\phi + d\phi$ is

$$e_\lambda d\lambda \, dA \cos\theta \sin\theta \, d\theta \, d\phi$$

$\therefore$ The amount of energy emitted by dA for all wavelength and all possible values of θ & φ is

$$dA \int_0^\infty e_\lambda \, d\lambda \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\varphi = \pi \, dA \int_0^\infty e_\lambda \, d\lambda$$

For the whole body, emission is

$$\pi(\Sigma dA) \int_0^\infty e_\lambda \, d\lambda \quad \text{------} \ldots (1)$$

But the amount of energy emitted by an elemental area dA′ of the enclosure in the direction dA of the body is

$$dQ_\lambda = E_\lambda \, d\lambda \, dA' \cos\theta' \, \frac{dA \cos\theta}{r^2}$$

r = distance betn dA′ & dA

E_λ = Emissive power of the surface of the enclosure

θ' = Angle made by the normal to dA′ with the direction of emission

$\therefore$ The amount of energy absorbed by dA of the body is

$$a_\lambda dQ_\lambda = a_\lambda E_\lambda \, d\lambda \, dA \cos\theta \, \frac{dA' \cos\theta'}{r^2}$$

$$= a_\lambda E_\lambda \, d\lambda \, dA \cos\theta \, d\omega = a_\lambda E_\lambda \, d\lambda \, dA \cos\theta (\sin\theta \, d\theta \, d\varphi)$$

Where $d\omega = dA' \cos\theta' / r^2$ = solid angle substended by dA′ at dA

$= \sin\theta \, d\theta \, d\varphi$

$\therefore$ Total energy absorbed by dA from the total surface of the enclosure for all wavelength is

$$dA \int_0^\infty a_\lambda E_\lambda \, d\lambda \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \int_0^{2\pi} d\varphi = \pi \, dA \int_0^\infty a_\lambda E_\lambda \, d\lambda$$

$\therefore$ For the whole body, the absorption is

$$\pi(\Sigma dA) \int_0^\infty a_\lambda E_\lambda \, d\lambda \quad \ldots (2)$$

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In equilibrium state,

$$\int_0^{\infty} e_\lambda \, d\lambda = \int_0^{\infty} a_\lambda E_\lambda \, d\lambda$$

$\therefore$ For any value of λ, $e_\lambda = a_\lambda E_\lambda$

i.e, at any temperature the ratio of the emissive power to the absorptive power of a substance is constant, being equal to the emissive power of a perfectly black body. This is Kirchhoff's law

Application:

The most striking application of Kirchhoff's law was made in the explanation of Fraunhofer lines.

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