CHAPTER 6

Displacement Measurement

It is sometimes said that the measurement of displacement—linear or angular—is fundamental to all measurements. Many measurements, such as force, strain, pressure, temperature, level etc. boil down to the measurement of displacement in the ultimate analysis.

The displacement transducers can be broadly classified into the following categories:

1. Pneumatic
2. Electrical
3. Optical
4. Ultrasonic
5. Magnetostrictive
6. Digital

Of course, the common method of measuring displacements is using mechanical devices like scales—simple or Vernier, measuring tapes, micrometers, spherometers, etc. But these self-sufficient devices are so common that it is pointless to discuss them here. We will discuss only those transducers which can be used as components in an instrumentation system.

6.1 Pneumatic Transducers

The mostly used pneumatic transducer, called the *nozzle-flapper transducer* is an important pneumatic transducer that finds many applications in small displacement measurement. Though it can be used to measure small displacements, it is generally used to determine the null position in a servosystem.

Nozzle-flapper Transducer

Consider the arrangement shown in Fig. 6.1. A gas at a fixed pressure p_s flows through a nozzle past a restriction in the tube. An obstruction, called flapper[1] is placed close to the nozzle. Owing to the presence of the flapper, there will be a back pressure that will alter the pressure of the gas in the volume between the restriction and the nozzle, to p_o. A pressure transducer, such as a piezoelectric device[2] is attached to the volume to monitor the pressure there.

[1]Meaning, 'a piece of something attached on one side only, that covers an opening', *Pocket Oxford Dictionary*, Oxford University Press (2004).

[2]See Section 5.2 at page 130.

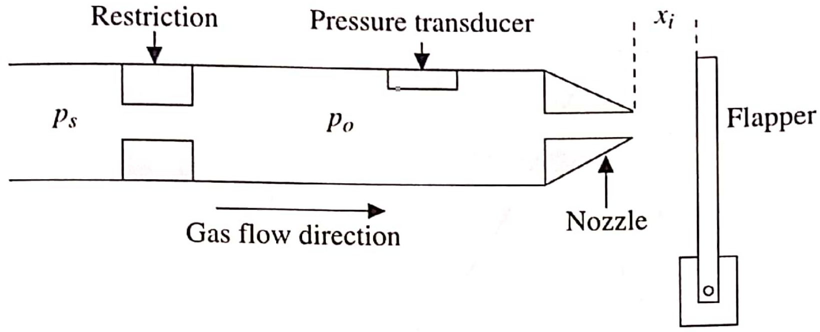

Fig. 6.1 Schematic diagram of nozzle-flapper transducer.

Clearly, the distance between the nozzle and the flapper will determine the pressure p_o which means, p_o can be calibrated in terms of the displacement x_i of the flapper. An approximate relation between p_o and x_i can be derived as follows.

If C_d is the discharge coefficient (see Section 11.2 at page 447 for definition)
d_s is the diameter of the supply orifice
ρ is the density of the fluid

then, assuming that the fluid is incompressible, the mass flow rate G_s through the supply orifice is given by

$$G_s = C_d \left(\frac{\pi d_s^2}{4}\right) \sqrt{2\rho(p_s - p_o)} \tag{6.1}$$

The flow from the nozzle spreads over a cylindrical volume of height x_i and diameter d_n which is the nozzle diameter. Therefore, the mass supply-rate through the nozzle G_n is given by

$$G_n = C_d(\pi d_n x_i)\sqrt{2\rho p_o} \tag{6.2}$$

neglecting the ambient pressure which is small in comparison to p_o and assuming the same discharge coefficient for the flow through the nozzle. Under equilibrium condition $G_s = G_n$. Then from Eqs. (6.1) and (6.2), we have

$$\frac{d_s^2}{4}\sqrt{p_s - p_o} = d_n x_i \sqrt{p_o}$$

$$\Rightarrow \quad \frac{p_o}{p_s} = \frac{1}{1 + (16 d_n^2 x_i^2 / d_s^4)} \tag{6.3}$$

If we put $p_N = \dfrac{p_o}{p_s}$ and $x_N = \dfrac{d_n x_i}{d_s^2}$, Eq. (6.3) turns out to be

$$p_N = \frac{1}{1 + 16 x_N^2} \tag{6.4}$$

A plot of p_N vs. x_N is shown in Fig. 6.2.

The operating point is chosen to keep the output pressure the same for equal displacement of the flapper on either side of its main position. Normally, the variation of p_N is nearly linear between 0.15 and 0.75. In industries, the supply pressure p_s is usually 20 psig (1.33 kg/cm^2). Which means, p_o varies between 3 and 15 psig.

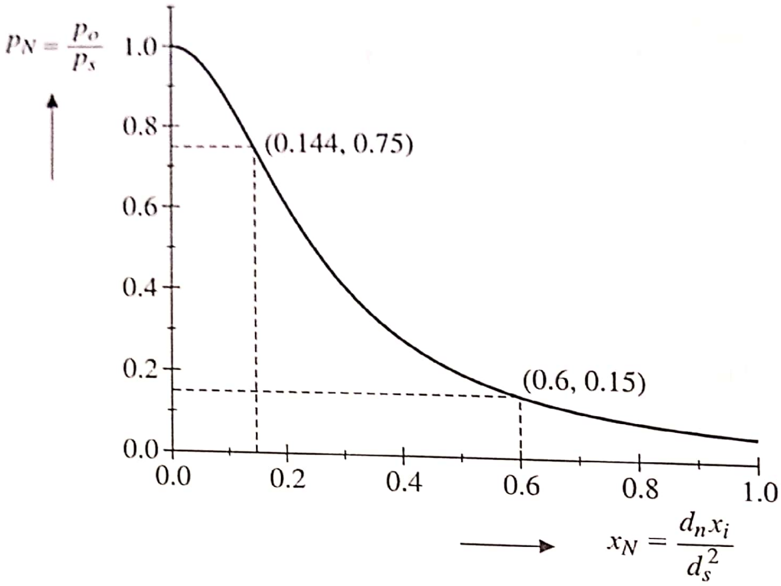

Fig. 6.2 Plot of p_N vs. x_N as given by Eq. (6.4). The operating area is between points (0.144, 0.75) and (0.6, 0.15) as shown in the diagram.

As low a displacement as 0.1 mm of the flapper produces an appreciable change in the output pressure p_o as may be seen from Example 6.1. This high sensitivity has made it rather popular in mechanical instrumentation. However, because of the approximately linear range of the transducer, it finds more application as a sensitive null detector in a servosystem rather than a final readout device.

Example 6.1

Calculate the variation of the output pressure for a nozzle-flapper transducer when the supply pressure is 20 psig, restriction and nozzle diameters are both 0.5 mm and the flapper movement is ±0.05 mm.

Solution

Given: $d_n = d_s = 0.05$ cm, $p_s = 20$ psig and $x_i = \pm 0.005$ cm. We know, $p_N|_{\max} = 0.75$ and $p_N|_{\min} = 0.15$. Also

$$x_N = \pm \frac{(0.05)(0.005)}{(0.05)^2} = \pm 0.1$$

Therefore, from Eq. (6.4) the output pressure variation is given by

$$\Delta p_o = (p_N|_{\max} - p_N|_{\min}) \cdot \frac{p_s}{1 + 16x_N^2}$$

$$= (0.75 - 0.15) \cdot \frac{20}{1 + 16(0.1)^2} = 10.34 \text{ psig}$$

Example 6.2

A pneumatic displacement gauge shown in Fig. 6.3 operates on the principle that the flow through the orifices of diameters D_1 and D_2 is governed by the separation distance x between the outlet and the workpiece.

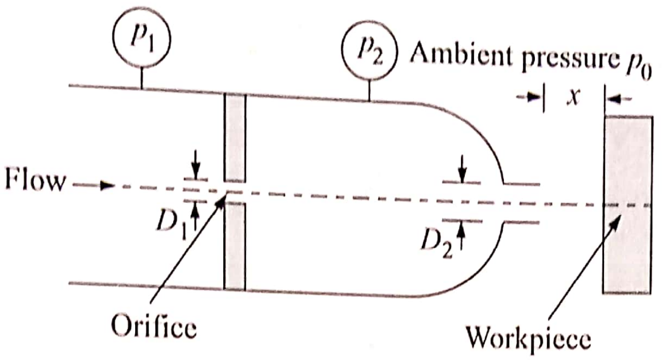

Fig. 6.3 Example 6.2

(a) Obtain an expression for the pressure ratio $R = p_2/p_1$ as a function of the diameter ratio of the two orifices. Assume, ambient pressure $p_0 = 0$ and the discharge coefficients for the orifices to be equal.

(b) Find the displacements for the pressure ratios of $R = 0.4$ and $R = 0.9$, if the orifice diameters are $D_1 = 0.5$ mm and $D_2 = 1.0$ mm.

Solution

(a) Modifying Eq. (6.3) suitably, the relation is given by

$$R = \frac{p_2}{p_1} = \frac{D_1^4}{D_1^4 + 16D_2^2x^2}$$

(b) From the above equation, we get the expression for the required displacement as

$$x = \sqrt{\frac{(1-R)D_1^4}{16RD_2^2x^2}} = \sqrt{\frac{1-R}{R}} \cdot \frac{D_1^2}{4D_2}$$

For $R = 0.4$
$$x = \sqrt{\frac{0.6}{0.4}} \cdot \frac{(0.5)^2}{4} = 0.076 \text{ mm}$$

For $R = 0.9$
$$x = \sqrt{\frac{0.1}{0.9}} \cdot \frac{(0.5)^2}{4} = 0.021 \text{ mm}$$

6.2 Electrical Transducers

Here our aim is to convert displacement to an electrical format. An electrical circuit consists basically of three variable passive components, namely resistance, inductance and capacitance. All three of them can be utilised to construct devices for transducting displacement.

Resistive Transducer: Potentiometer

The familiar potentiometer, or *pot* in common parlance, is widely used as a transducer. Basically, it consists of a resistance element provided with a movable contact. The motion of the contact can be translational, rotational, or helical which is a combination of the two former motions (Fig. 6.4).

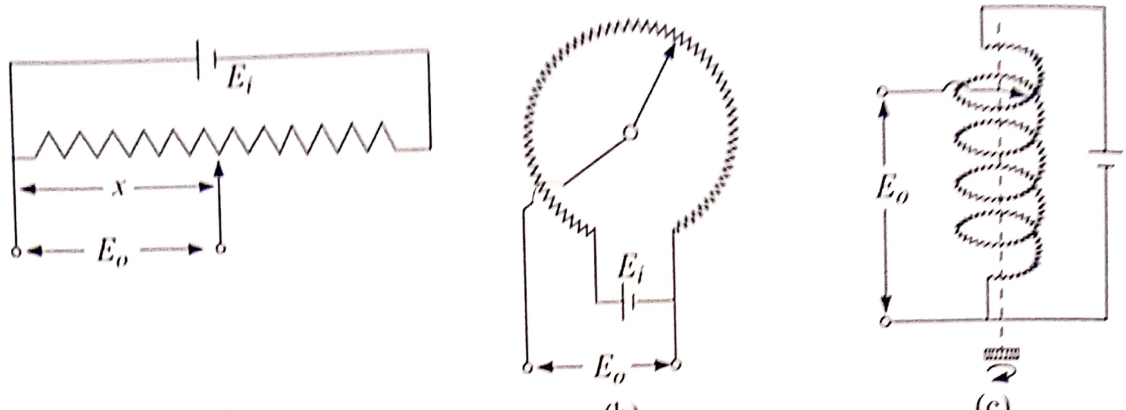

Fig. 6.4 Schematic representations of potentiometers: (a) translational, (b) rotary, and (c) helical.

Construction

Potentiometers are generally constructed in three forms—single slide wire, wire-wound, and cermet.

Single slide wire. The only advantage that the single slide wire offers is the stepless variation of resistance is as the wiper travels over it. But since the length of the wire is limited by the desired stroke in a translational device and by the diameter in a rotational one, this type of potentiometer is limited to rather small values of resistance. Although the resistance per unit length can be increased by decreasing the area of cross-section of the wire, it is done only at the expense of its strength and resistance to wear.

Wire-wound. In this case the resistance wire is wound on a straight or circular card or a mandrel (Fig. 6.5), depending on the type of the device—translational or rotational—used.

Fig. 6.5 Wire-wound resistance elements on: (a) straight insulating card and (b) circular insulating card.

The wire-wound construction produces a stepwise increase in resistance (Fig. 6.6) as the wiper moves from one turn of the wire to another, thus imposing a restriction on the resolution of the transducer. For example, if there are 400 turns on a 20 mm long card, the resolution is $20 \div 400 = 50$ µm. In fact, the practical limit of winding is 20 to 40 turns/mm which restricts the resolution to 25 to 50 µm. For rotational devices the resolution R can be figured out from the relation

$$R = \frac{360 \times 10^{-3}}{\pi n D}$$

where D is the diameter of the potentiometer in metres and n is the number of turns/mm.

It may be noted in this context that a higher resolution demands thinner wires which, in turn, means a higher total resistance. Thus resolution and resistance are interdependent.

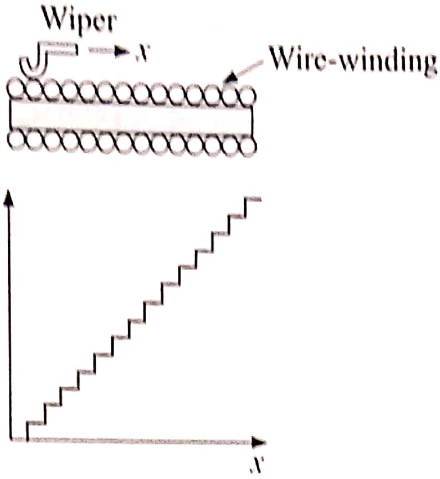

Fig. 6.6 Stepwise increase of resistance of wire-wound potentiometers.

Wires of nickel-chromium (nichrome), nickel-copper (constantan), silver-palladium or some other precious metals are used as resistive elements, their diameters varying between 25 and 50 μm. To avoid surface oxidation, they are annealed in a reducing atmosphere.

On the other hand, hard alloys like phosphor-bronze, beryllium-copper or other precious metal alloys are used to construct wipers and are shaped in such a way that they slide with minimum friction and at the same time maintain a firm contact with the winding.

Cermet. Precious metal particles fused into a ceramic base constitute cermet. This has many advantages such as:

1. Stepless variation of resistance offering a very high resolution
2. Large power ratings because it is not easily fusible
3. Low cost
4. Moderate temperature coefficients
5. Utility in ac applications

Apart from these, hot moulded carbon, carbon films, thin metal films are also used to construct potentiometers.

Characteristics

While choosing a potentiometer for an application, it is necessary to consider the following characteristics:

Loading effects. The resistance element is excited with dc or ac voltage and the input-output relation is ideally linear.

But in practice the voltage measuring arrangement loads the output and as a result the relation is far from linear as will be evident from the following analysis.

From Fig. 6.7(a) it is clear that resistance of the length $x_i = (x_i/x_t)R_p \equiv KR_p$, say. Here, x_t is the total length of the potentiometer wire and R_p is the total resistance of the potentiometer. The circuit in Fig. 6.7(a) can be redrawn to the form shown in Fig. 6.7(b) so that its Thevenin equivalent looks like Fig. 6.7(c). It is clear from Fig. 6.7(c) that the Thevenin equivalent resistance R_o of the potentiometer circuit is $K(1-K)R_p$ and the Thevenin equivalent input voltage E_i' is KE_i, where E_i is the actual input voltage.

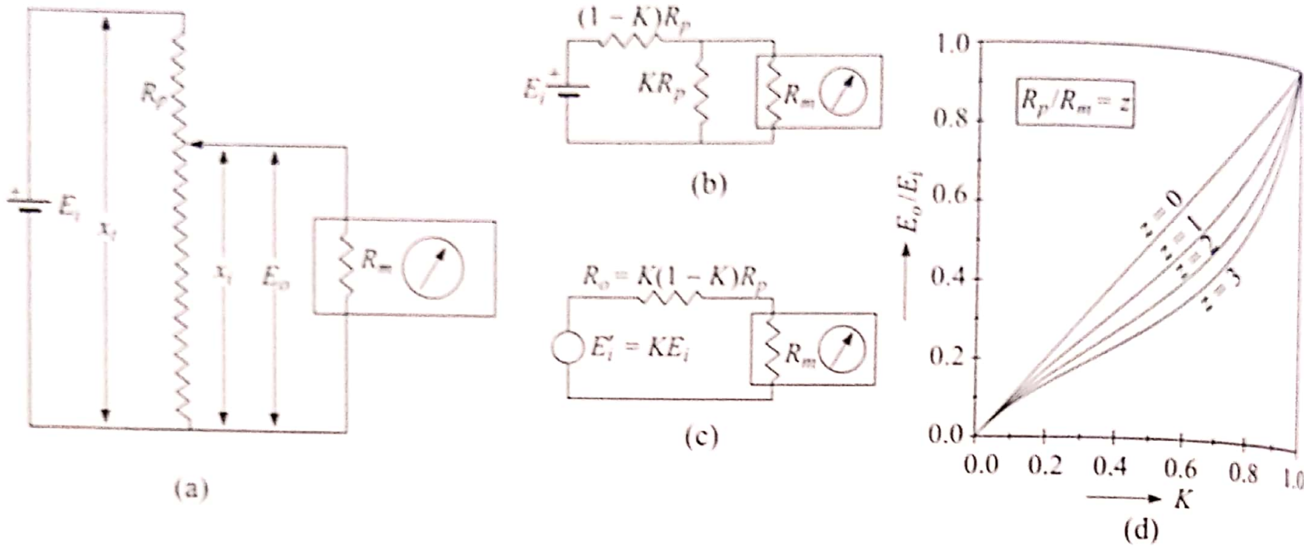

Fig. 6.7 Potentiometer loading effect: (a) circuit arrangement, (b) re-drawn circuit, (c) Thevenin equivalent circuit, and (d) characteristic curves.

Thus, if E_o is the output voltage, we have

$$\frac{E_o}{E_i} = \frac{1}{1 + \dfrac{R_o}{R_m}} = \frac{K}{1 + K(1-K)\dfrac{R_p}{R_m}} \tag{6.5}$$

In actual practice, $R_m \neq \infty$ and, therefore, the characteristics curve is nonlinear. Table 6.1 as well as Fig. 6.7(c) will give an idea about the error caused by the loading effect.

Table 6.1 Error caused by loading of the potentiometer

R_p/R_m	1.0	0.1	< 0.1
Maximum error (%)	12	1.5	$15(R_p/R_m)$

Power rating. The typical available power rating is 5 W at room temperature. The maximum excitation voltage can be calculated from the relation

$$(E_i)_{\max} = \sqrt{PR_p} \text{ volt}$$

where P is the rated power in watts.

Linearity and sensitivity. For high sensitivity, the output voltage E_o and for that matter input voltage E_i should be high. But the maximum value of E_i is determined by the resistance of the potentiometer R_p and its power rating. The value of R_p has to be kept low in comparison to the resistance of the measuring instrument R_m to achieve linearity. This requirement, thus, is in conflict with the desire for high sensitivity. Typical values of sensitivity are 200 mV/mm for translational or 200 mV/deg for rotational devices.

The advantages and disadvantages of potentiometric displacement transducers are given in Table 6.2.

Table 6.2 Advantages and disadvantages of potentiometers

Advantages	*Disadvantages*
1. Inexpensive and simple to set up.	1. Mechanical loading owing to wiper friction.
2. Rather large displacements can be measured.	2. Electrical noise from the sliding contact.
3. Sufficient output to drive control circuits.	3. Wear and misalignment owing to friction.
4. Frequency response and resolution limited for the wire-wound, but unlimited for others.	4. Quick manipulation generates heat and associated problems.

Example 6.3
The output of a potentiometer is to be read by a 10 kΩ voltmeter, holding non-linearity to 1%. A family of potentiometers having a thermal rating of 5W and resistances ranging from 100 Ω to 10 kΩ in 100 Ω steps are available. Choose from this family the pot that has the greatest possible sensitivity and meets other requirements. What is the sensitivity if pots are single-turn (360 °) units?

Solution
To hold linearity to 1%, $R_p = R_m/15 = 666.7\,\Omega$. Pots available in this range are 600 Ω and 700 Ω. To ensure a high sensitivity we should choose 700 Ω, but then the nonlinearity goes above 1%. So, we have no alternative but to choose the 600 Ω pot. With this pot, the maximum excitation voltage is $\sqrt{5 \times 600} \cong 54.8$ V, and, therefore, the required sensitivity is $54.8/360 \cong 152$ mV/degree.

Example 6.4
In a potentiometer transducer, the potentiometer has a total resistance of 24 kΩ for a total wiper travel of 120 mm. During a measurement the wiper moves between 20 mm and 60 mm over the potentiometer.

(a) If the voltmeter of 15 kΩ is used to read the output voltage of the transducer, find out the error due to the loading effect at the two measuring points.

(b) If the error due to the loading effect in the above instrumentation is to be kept within ± 3%, what should be the resistance of the voltmeter?

Solution
(a) The wiper travels between 20 mm and 60 mm. The resistance R between these points is

$$R = \frac{40}{120} \times 24 = 8\text{ k}\Omega$$

Let the excitation voltage be E. Therefore, the voltage V across R is

$$V = \frac{8}{24}E = \frac{E}{3} \cong 0.3333E\text{ V}$$

Now, the 15 kΩ resistance of the voltmeter lies parallel to R. Their combined resistance R_c is

$$R_c = \frac{15 \times 8}{15 + 8} = 5.217\text{ k}\Omega$$

Therefore, the voltage V_e developed across R_e is

$$V_e = \frac{5.217E}{(24-8)+5.217} = 0.2459E \text{ V}$$

Note: This result can be obtained by putting $K = x_i/x_t = 40/120 = 1/3$, $R_p = 24\,\text{k}\Omega$ and $R_m = 15\,\text{k}\Omega$ in Eq. (6.5).

Therefore, the error ε in the measurement is

$$\varepsilon = \frac{(0.3333-0.2459)E}{0.3333E} \times 100 = 26.22\%$$

(b) To keep the error within 3%, if V_{cx} is the voltage to be developed across the combined resistance of R and the unknown resistance R_{mx} of the measuring voltmeter, we have

$$3 = \frac{(0.3333 - V_{cx})E}{0.3333E} \times 100$$

This gives $$V_{cx} = 0.3333E - \frac{3 \times 0.3333}{100}E = 0.3233E \text{ V}$$

So $$0.3233E = \frac{R_{cx}}{16 + R_{cx}}E$$

or $$R_{cx} = \frac{16 \times 0.3233}{1 - 0.3233} = 7.6442\ \text{k}\Omega$$

Now

$$R_{cx} = \frac{8R_{mx}}{8 + R_{mx}}$$

This gives $$R_{mx} = \frac{8R_{cx}}{8 - r_{cx}} = \frac{8 \times 7.6442}{8 - 7.6442} \cong 172\ \text{k}\Omega$$

Example 6.5

A potentiometer is used to measure the displacement of a hydraulic ram. The potentiometer is 25 cm long, has a total resistance of 2500 ohms and is operating at 4 W with a voltage source. It has linear resistance-displacement characteristics. Determine

(a) Sensitivity of the potentiometer in volts/cm (without loading effect)

(b) Loading error in the measurement of displacement at actual input displacement of 15 cm, when the potentiometer is connected to a recorder having a resistance of 5000 ohms.

Solution

Given, $L = 25$ cm, $R_p = 2500\,\Omega$, $P = 4$ W. Therefore, current in the circuit is

$$I = \sqrt{\frac{P}{R_p}} = \sqrt{\frac{4}{2500}} = 0.04 \text{ A}$$

and excitation voltage is

$$V = 2500 \times 0.04 = 100 \text{ V}$$

(a) Sensitivity $= \frac{100}{25} = 4$ V/cm.

(b) Actual input displacement $x = 15$ cm. Therefore, resistance across x is

$$R_x = \frac{15}{25} \times 2500 = 1500\,\Omega$$

and actual voltage across R_x is

$$V_x = 15 \times 4 = 60 \text{ V}$$

The recorder has been connected parallel to R_x. Their combined resistance is

$$\frac{1500 \times 5000}{1500 + 5000} = 1153.85\,\Omega$$

Hence the total resistance of the circuit is now

$$(1153.85 + 1000) = 2153.85\,\Omega$$

Therefore,

$$\text{Voltage across } R_x = \frac{60}{2153.85} \times 1153.85 = 32.14 \text{ V}$$

$$\text{Loading error} = \frac{60 - 16.07}{60} \times 100 = 46.4\%$$

Inductive Transducers

Inductive transducers can be of various types. We will consider only three, namely

1. Linear variable differential transformer
2. Rotary variable differential transformer
3. Synchros

Linear variable differential transformer (LVDT)

The linear variable differential transformer (LVDT) is the most commonly used variable inductance transducer in industry. It is an electromechanical device designed to produce an ac voltage output proportional to the relative displacement of a transformer and an iron core, as illustrated in Fig. 6.8.

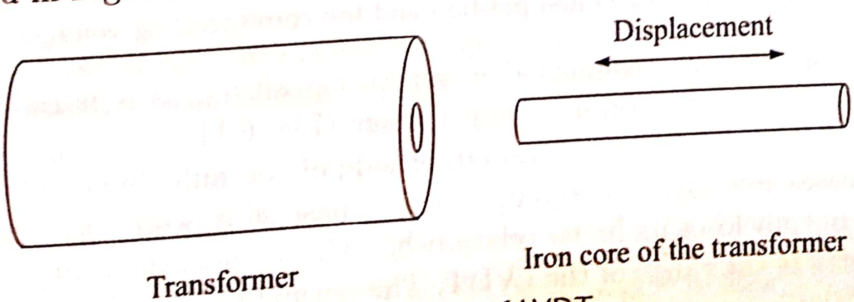

Fig. 6.8 Basics of LVDT.

The basic construction of the LVDT is shown in Fig. 6.9. It consists of one primary winding and two secondary windings. Secondary windings are identical in respect of their number of turns and their placement on both sides of the primary winding. A sinusoidal voltage e_i of amplitude 1 to 15 V and frequency 50 Hz to 20 kHz can be used to excite the primary though 1 V at 2 kHz to 10 kHz is common. A movable core of high μ produces signals proportional to its displacement by changing the mutual inductance between the coils. Nickel-iron alloy, slotted longitudinally to minimise eddy current loss, is normally used to construct the core.

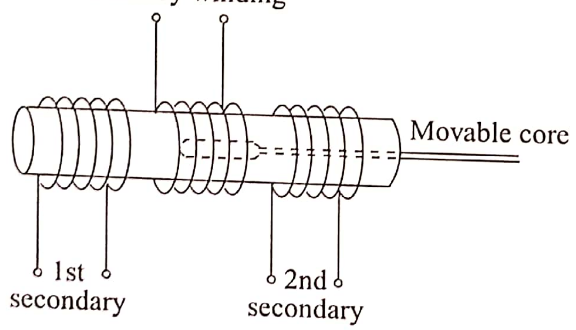

Fig. 6.9 Construction of LVDT.

When the core is in the middle position, a sinusoidal voltage of equal amplitude appears across the two secondaries (Fig. 6.10).

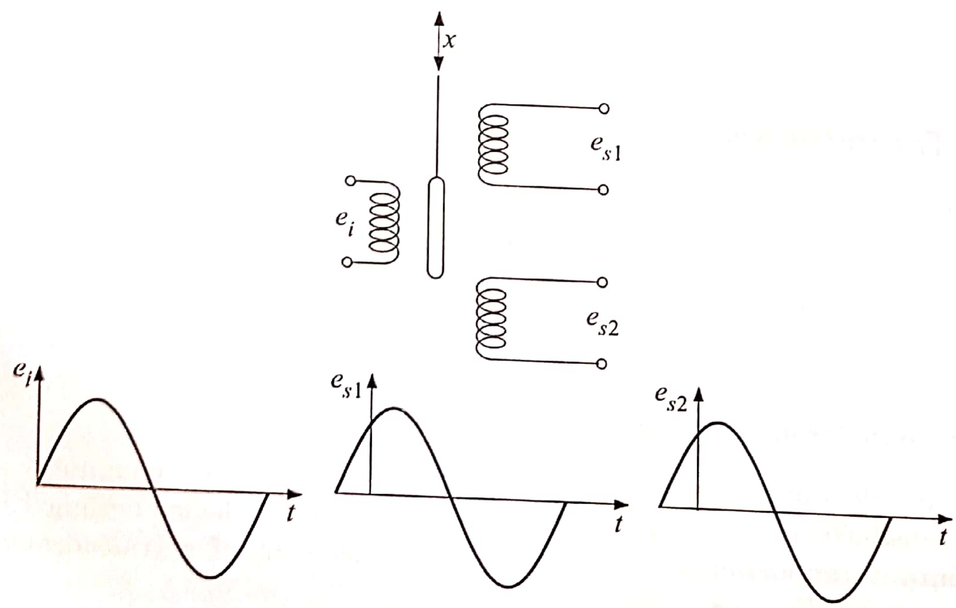

Fig. 6.10 Core in null position and the corresponding voltages.

And if the secondaries are connected in series opposition, as is normally the case, these voltages cancel each other to produce a null voltage (Fig. 6.11).

With the displacement of the core on either side of the null, the combined voltage of the secondaries increases linearly, undergoing a 180° phase-shift while passing through the null (Fig. 6.12). The output loses its linear relationship with displacement beyond some limits and this property restricts the range of the LVDT. The normal range is from ±10 μm to ±10 mm.

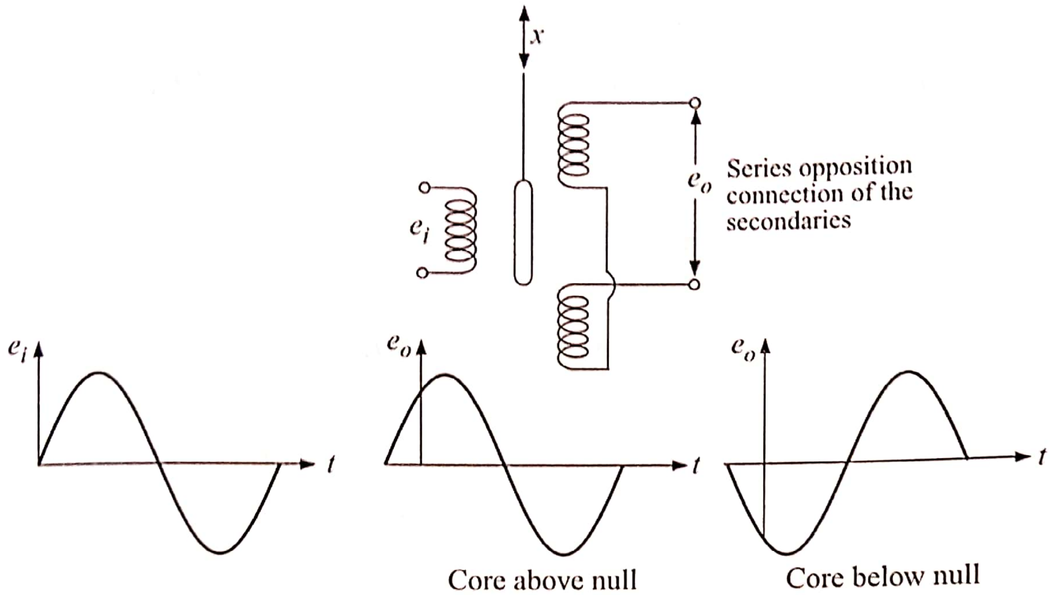

Fig. 6.11 Series opposing connection of secondaries and voltages for different core positions.

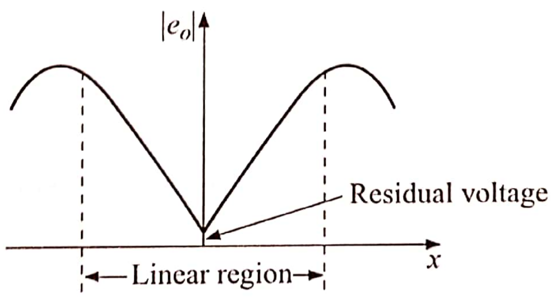

Fig. 6.12 Magnitude of output voltage for core displacement of an LVDT.

Circuit analysis. A simplified analysis of the circuit also reveals that the amplitude linearly varies with the difference in mutual inductances. When the secondary is open circuited, the equation for the primary can be written as

$$i_p R_p + L_p \frac{di_p}{dt} = e_i \tag{6.6}$$

where symbols have their usual meaning. On Laplace transformation Eq. (6.6) yields

$$(sL_p + R_p)I_p = E_i$$

or

$$I_p = \frac{E_i}{sL_p + R_p} \equiv \frac{E_i/R_p}{\tau_p s + 1}$$

where $\tau_p = L_p/R_p$. Now, if e_{s1} and e_{s2} are voltages generated in the secondary coils owing to their mutual inductances of coefficients M_1 and M_2, equations for the secondaries and their Laplace transforms are

$$e_{s1} = M_1 \frac{di_p}{dt} \qquad e_{s2} = M_2 \frac{di_p}{dt}$$

$$E_{s1} = sM_1 I_p \qquad E_{s2} = sM_2 I_p$$

Therefore

$$E_o = E_{s1} - E_{s2} = (M_1 - M_2)sI_p = \frac{(M_1 - M_2)s/R_p}{\tau_p s + 1} E_i$$

Thus

$$\frac{E_o(s)}{E_i(s)} = \frac{s(M_1 - M_2)/R_p}{\tau_p s + 1}$$

whence

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{j\omega(M_1 - M_2)/R_p}{j\omega\tau_p + 1} = \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}} \angle\phi \qquad (6.7)$$

where

$$\phi = \frac{\pi}{2} - \tan^{-1}\omega\tau_p$$

In Eq. (6.7) the expression preceding $\angle\phi$ represents the amplitude ratio of the output and the input. Since ω, R_p, τ_p and the input amplitude are constants for a given set-up, the amplitude of the output A_o can be written as

$$A_o = K(M_1 - M_2) = K'x$$

where K and K' are constants and x is the displacement.

The value of $(M_1 - M_2)$ keeps on increasing with the displacement of the core up to a certain point and then it starts falling as the core moves past one of the secondaries.

Excitation frequency. For a good dynamic response of an LVDT, the excitation frequency must be much higher than the core-movement frequencies. This is necessary for distinguishing them in the amplitude modulated output signal. The rule of the thumb is to set

$$\frac{\text{Maximum core-movement frequency}}{\text{Excitation frequency}} = \frac{1}{10}$$

Residual voltage. Whereas the output voltage at null position is ideally zero, harmonics in the excitation voltage and stray capacitance coupling between primary and secondary usually result in a small but non-zero voltage which is called the *residual voltage*. Under usual conditions this is $< 1\%$ of the full-scale output and may be quite acceptable. Methods of reducing this null when it is objectionable are available.

Wiring variation. Most LVDTs are wired as shown in Fig. 6.11. This wiring arrangement is known as *open wiring*. Since the number of coil windings is uniformly distributed along the transformer, the voltage output is proportional to the iron core displacement when the core slides through the transformer. The corresponding equation is:

$$x = Se_o$$

where x is the displacement of the iron core with respect to the transformer, and S is the sensitivity of the transformer.

Capacitive Transducers

Capacitive transducers can directly sense a variety of things—motion, chemical composition, electric field—and, indirectly, pressure, acceleration, fluid level, and fluid composition. The technology is low cost and stable and uses simple conditioning circuits. Capacitive displacement detectors can detect 10^{-8} m displacements with good stability and high speed under wide environmental variations.

Generally, parallel-plate capacitors are used as transducers. According to the theory, the capacitance C of such a capacitor is given by

$$C = \frac{\varepsilon A}{x} \text{ farad} \tag{6.8}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of the intervening medium (farad/metre)
x is the distance between the plates (metre)
A is the overlapping area of the plates (metre^2)

Therefore, a variable capacitance device can be constructed by effecting variation in either of

1. Distance x between the plates [Fig. 6.18(a)]
2. Effective overlapping area A between the plates [Fig. 6.18(b)]
3. Relative permittivity ε_r of the intervening medium between the plates [Fig. 6.18(c)]

We will consider them in that order.

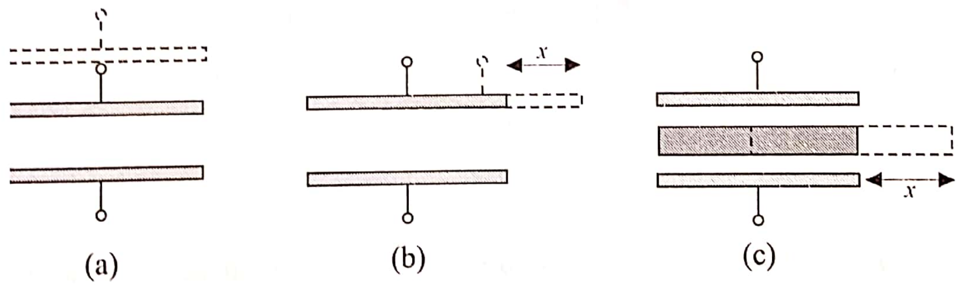

Fig. 6.18 Three kinds of variation in capacitative transducers: (a) change in the gap, (b) change in the area and (c) change in the permittivity.

Change in the gap x between the plates

Since capacitance varies inversely as x, the plot of C vs. x is a rectangular hyperbola. That means

$$\text{Sensitivity} = S = \frac{dC}{dx} = -\frac{\text{constant}}{x^2}$$

is not constant. This is rather inconvenient for measurements. In fact S decreases as x increases.

Linearisation. The linearisation of the input-output relationship is usually achieved by resorting to three different techniques:

1. By measuring the per cent change in capacitance
2. Using a charge amplifier
3. Measuring impedance
4. Differential arrangement

By measuring per cent change in capacitance the input output relation can be linearised. We observe that

$$\frac{dC}{dx} = -\frac{\varepsilon A}{x^2} = -\frac{C}{x}$$

$$\Rightarrow \quad \frac{dC}{C} = -\frac{dx}{x} \tag{6.9}$$

Equation (6.9) indicates that the per cent changes of C and x are linearly related provided, of course, the changes are small.

By using a charge amplifier the input-output relation can be linearised as shown in Fig. 6.19. With currents i and i_x, capacitors C and C_x, and voltages e_i and e_o as indicated in Fig. 6.19, we have

$$e_i = \frac{\int i\,dt}{C} \tag{6.10}$$

$$e_o = \frac{\int i_x dt}{C_x}$$

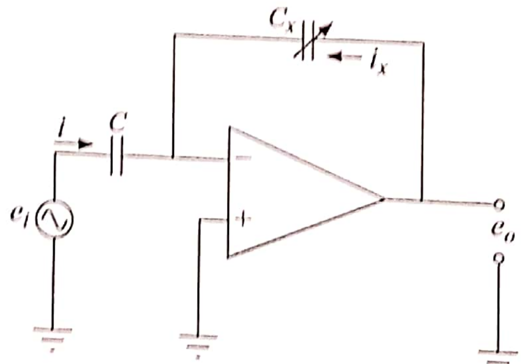

Fig. 6.19 Use of the op-amp to linearise the input-output relation.

Now, because the op-amp has a virtual ground at the input, $i = -i_x$. Hence,

$$e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i\,dt}{C_x} \qquad [\text{because } i_x = -i] \tag{6.11}$$

$$= -\frac{C}{C_x} e_i \qquad [\text{using Eq. (6.10)}] \tag{6.12}$$

$$= -\frac{Ce_i}{\varepsilon A} x \qquad [\text{using Eq. (6.8)}] \tag{6.13}$$

In Eq. (6.13), since all other factors are constant, the output voltage varies linearly with the displacement.

Measuring impedance rather than the capacitance is another way of linearisation. Because the impedance is given by

$$X_C = \frac{1}{2\pi f C} = \frac{x}{2\pi f \varepsilon_r \varepsilon_0 A} \tag{6.14}$$

where f is the frequency of the exciting voltage. It is obvious from Eq. (6.14) that the X_C vs. x curve is linear.

In a typical commercially available transducer, e_i is a 50 kHz sine wave. The output is rectified and fed to a dc voltmeter calibrated directly in distance units.

Having a differential arrangement of capacitors, as shown in Fig. 6.20 is another technique of producing a linear transfer characteristic.

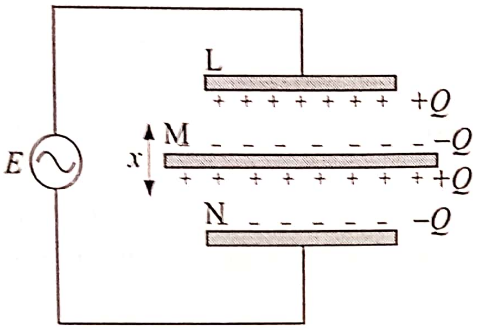

Fig. 6.20 Differential arrangement of capacitors to linearise the output.

Here, a three-plate capacitor is used, keeping the end plates (L and N) fixed and allowing the middle plate M to move. It can be shown that the voltage differential ΔE has a linear relation with the displacement $|x|$ of the movable plate M. At any instant,

$$E_{\mathrm{LM}} = \frac{Q}{C_{\mathrm{LM}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{LM}}} \tag{6.15}$$

$$E_{\mathrm{MN}} = \frac{Q}{C_{\mathrm{MN}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{MN}}} \tag{6.16}$$

$$C_{\mathrm{LN}} = \frac{\varepsilon A}{2d}$$

where Q is the amount of charge on any plate and d is the distance between two adjacent plates.

When M is right at the midway, $C_{\mathrm{LM}} = C_{\mathrm{MN}}$ and therefore, the voltage differential is zero. If M is displaced upwards by a distance x, then

$$C_{\mathrm{LM}} = \frac{\varepsilon A}{d - x} \tag{6.17}$$

$$C_{\mathrm{MN}} = \frac{\varepsilon A}{d + x} \tag{6.18}$$

where A is the area of the plates and d is the distance between L and M (or M and N) when M is at the midway. Plugging values from Eqs. (6.17) and (6.18) in Eqs. (6.15) and (6.16), we get

$$E_{\mathrm{LM}} = E \cdot \frac{d - x}{2d}$$

$$E_{\mathrm{MN}} = E \cdot \frac{d + x}{2d}$$

which give

$$\Delta E = E_{\mathrm{LM}} - E_{\mathrm{MN}} = \frac{E}{d}x$$

This arrangement, with appropriate instrumentation, can measure displacements between 10^{-8} mm and 10 mm with an accuracy of about 0.1%.

Effect of fringing flux. The variation in spacing x of parallel plates is often used for displacement detection if $x < l$ or w where l and w are the length and width of the electrodes respectively. As long as the l and w of the plates are close compared to the plate spacing, the equations given above will produce more or less accurate results. But as the plate spacing increases relative to the l and w of the plates, more flux lines connect from the edges and backs of the plates' This is called *fringing*. With fringing, the measured capacitance can be much larger than calculated.

Fringing incorporates nonlinearity in the C vs. x^{-1} relation. This effect may very largely be eliminated by introducing what is called a *guard ring* as shown in Fig. 6.21.

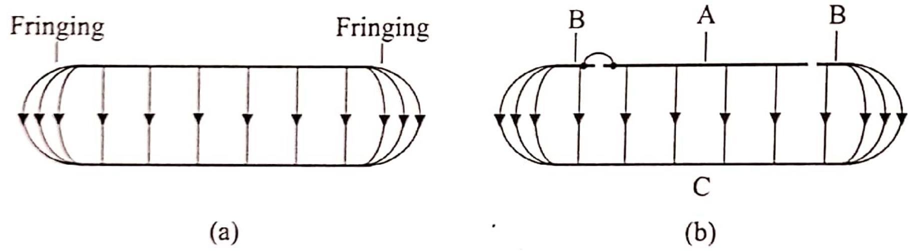

Fig. 6.21 (a) Fringing of flux at the ends of plates of a parallel-plate capacitor. (b) Guard ring (B) to eliminate the effect of fringing in a capacitor. B is at the same electrical potential with A.

One of the circular plates A of the capacitor is surrounded by a concentric annular plate B in the same plane. The inner radius of B is slightly larger than the radius of A and a metallic connection between A and B is made so that they will have the same potential. The other plate C of the capacitor is placed parallel to A and B. It is obvious that the flux between the plates will have edge effects on B, but that between A and C will be practically parallel. However, some correction will be necessary to calculate the capacity of the arrangement because the area of the plates should be A′ and not equal to that of A because of its connection to B.

Example 6.7

Figure 6.22 shows a circuit with a variable air gap parallel plate capacitor as the sensing element. Show that the circuit acts as a velocity sensor for very small displacements, and find the proportionality constant between the voltage e_o and the input velocity v. Nominal (zero displacement) capacitance C is 50 pF and the nominal (zero displacement) distance between the capacitor plates x_0 is 5 mm.

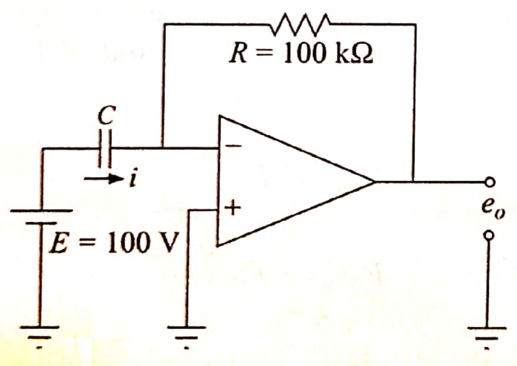

Fig. 6.22 Variable air gap parallel plate capacitor (Example 6.7).

Solution

We know, $C = \frac{\varepsilon A}{x}$, where terms have their usual meaning. Therefore,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{\varepsilon A}{C}\right) = -\frac{\varepsilon A}{C^2}\frac{dC}{dt} = -\frac{Cx}{C^2}\frac{dC}{dt}$$

$$= -\frac{x}{C}\frac{dC}{dt} \qquad \text{(i)}$$

We also know, $C = \frac{Q}{E}$. Therefore,

$$\frac{dC}{dt} = \frac{1}{E}\frac{dQ}{dt} = \frac{i}{E}$$

where i denotes current. Substituting the value of $\frac{dC}{dt}$ in Eq. (i), we get on rearranging

$$i = -\frac{EC}{x}\frac{dx}{dt}$$

Therefore,

$$e_o = -iR = \frac{ECR}{x}\frac{dx}{dt} = \frac{(100)(50 \times 10^{-12})(100 \times 10^3)}{0.5}\frac{dx}{dt} = 1.0 \times 10^{-3}v$$

Example 6.8

Figure 6.23(a) shows the variable displacement type capacitor sensor with push-pull arrangement.

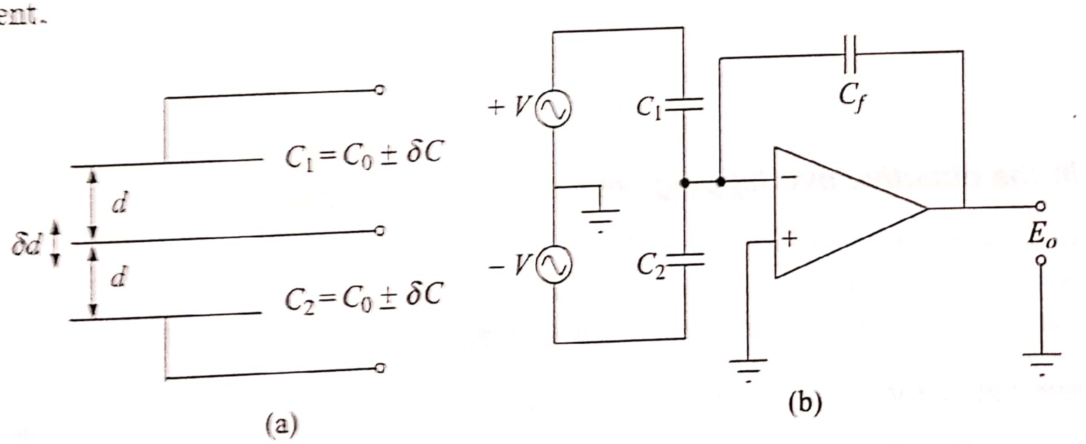

Fig. 6.23 Variable displacement type capacitor sensor (Example 6.8).

If A is the plate area, d the normal plate displacement, ε the permittivity and δd the input displacement,

(a) Calculate $\delta C/C_0$ where C_0 is the nominal capacitance and δC the change in capacitance corresponding to δd.

(b) Show that the output voltage is

$$E_o = V\frac{C_0}{C_f}\left[\frac{2(\delta d/d)}{1-(\delta d/d)^2}\right]$$

when the push-pull configuration is connected as shown in Fig. 6.23(b).

Solution

We know, the capacitance C is given by

$$C_0 = \frac{\varepsilon A}{d}$$

where terms have their usual significance.

(a) Owing to the displacement δd of the middle plate, C_1 becomes

$$C_1 = \frac{\varepsilon A}{d - \delta d} = \frac{\varepsilon A}{d} \cdot \frac{1}{1 - (\delta d/d)} = \frac{C_0}{1 - (\delta d/d)}$$

Thus,

$$\frac{\delta C_1}{C_0} = \frac{1}{1 - (\delta d/d)} \qquad \frac{\delta C_2}{C_0} = \frac{1}{1 + (\delta d/d)}$$

Note: Since C_1 and C_2 are connected in series, it is easy to see that the total capacitance between the end plates remains the same, irrespective of the movement of the central plate.

(b) The given circuit to which the push-pull capacitors are connected in Fig. 6.23(b) is a charge amplifier. Because the voltage V is connected in the opposite way to the two capacitors, the charge at the input to the op-amp is

$$VC = -VC_1 + VC_2 = -VC_0\left[\frac{1}{1 - (\delta d/d)} - \frac{1}{1 + (\delta d/d)}\right] = -VC_0\left[\frac{2(\delta d/d)}{1 - (\delta d/d)^2}\right]$$

Now from Eq. (6.12), we get

$$e_o = -\frac{VC}{C_f} = \frac{VC_0}{C_f} \cdot \frac{2(\delta d/d)}{1 - (\delta d/d)^2}$$

Change in the effective overlapping area A between the plates

In the x-variation motion detectors as discussed above, when the displacement is as large as the dimension of the electrodes, the accuracy of measurement suffers from the vanishing signal level. The area variation measurement is then preferred.

Parallel-plate capacitor. For a parallel-plate capacitor, having two exactly equal rectangular parallel plates placed one on top of another, if the width of the plate is given by w and the overlap length by l then the area of overlap is lw. Hence,

$$C = \frac{\varepsilon l w}{x}$$

Now, if one of the plates is displaced along l, the area of overlap changes. But here, sensitivity $= dC/dl = \varepsilon w/x =$ constant. That means, the C vs. l plot is a straight line and, therefore, no extra circuitry is needed to make the calibration linear.

But strictly speaking, some nonlinearity is introduced owing to the fringing effect and this feature restricts the precision of the measurement. This type of transducer is suitable for measurement of linear displacements between 1 cm and 10 cm, the maximum attainable precision being about 0.005% which is quite satisfactory.