The Compton Effect :

Upon incident on a block of material, the x-ray of wavelength λ_0 scatters and the intensity of scattered radiation is found to peak at two wavelengths - One is the same as the incident wavelength λ_0 while the other is λ_1, where $\lambda_1 > \lambda_0$. The shift $\Delta\lambda = \lambda_1 - \lambda_0$ is called the Compton shift and depends only on the scattering angle and not on the initial wavelength λ_0 and material of the target.

Classically, the oscillating electric field of the incident radiation, of specific frequency $\nu_0 = c/\lambda_0$ interacts with the electrons contained in the atoms of the target and forces them to vibrate with same frequency, thus scattering at the same wavelength λ_0 as the incident x-ray. Hence, classical picture can not explain the presence of larger wavelength λ_1.

Compton & Debye regarded the incident x-ray beam as a collection of photons and not as waves, each of energy $E_0 = h\nu_0 = hc/\lambda_0$. They suggested that λ_1 could be attributed to scattering of x-ray photons from loosely bound electrons in the atom of the target, where they loose some of its energy in the inelastic collision, $E_1 < E$. Therefore, their frequency is reduced implying larger wavelength $\lambda_1 = c/\nu_1 = hc/E_1$. Since the electrons participating in the scattering process are treated almost free and initially stationary and does not involve entire atoms, this kind of explains why $\Delta\lambda$ is independent of the material of scatterer.

To calculate the compton shift, let a photon of energy E_0 and momentum p_0 is incident on a stationary electron of rest mass energy m_0c^2.

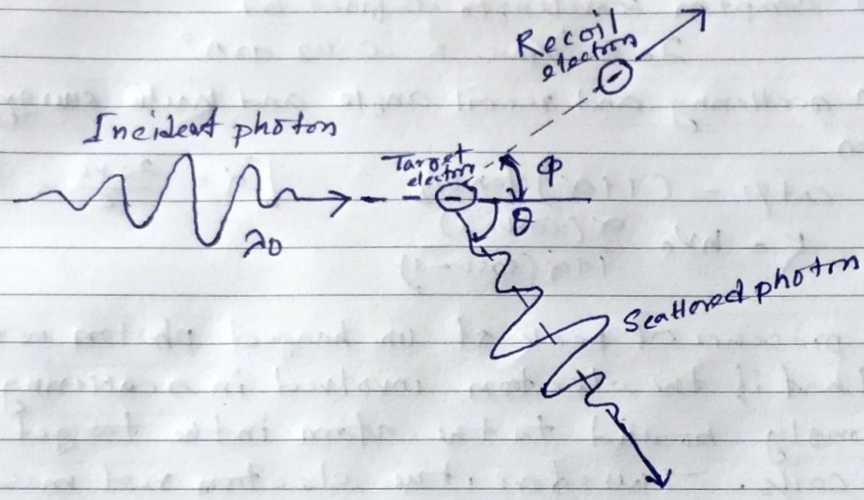
$$E_0 = h\nu_0 = hc/\lambda_0 \quad \text{and} \quad p_0 = E_0/c = h/\lambda_0 \quad \text{---- (1)}$$

After the collision, the photon is scattered at angle θ and moves off with total energy E_1 and momentum p_1,

$$E_1 = h\nu_1 = hc/\lambda_1 \quad \& \quad p_1 = E_1/c = h/\lambda_1 \quad \text{---- (2)}$$

electron recoils at an angle ϕ with K.E K, total energy E and momentum p,

$$E^2 = p^2c^2 + m_0^2c^4 \quad \& \quad K = E - m_0c^2$$



Momentum conservation leads to

$$p_0 = p_1 \cos\theta + p\cos\phi \quad \text{---- (3)}$$

$$0 = p_1 \sin\theta - p\sin\phi \quad \text{-- (4)}$$

Squaring & adding equ^n (3) & (4)

$$p^2 = p_0^2 + p_1^2 - 2p_0p_1\cos\theta \quad \text{--- (5)}$$

From the conservation of energy in the collision, it follows that

$$E_0 + m_0c^2 = E_1 + E$$

$$\Rightarrow \quad E = (E_0 - E_1) + m_0c^2 \quad \text{--- (6)}$$

Again squaring (6)

$$p^2 = (p_0 - p_1)^2 + 2m_0c(p_0 - p_1) \quad \text{--- (7)}$$

Comparing equ^n (5) & (7), we get

$$(p_0 - p_1)^2 + 2m_0c(p_0 - p_1) = p_0^2 + p_1^2 - 2p_0p_1\cos\theta \quad \text{... (8)}$$

$\therefore$ equation (8) reduces to

$$\frac{1}{p_1} - \frac{1}{p_0} = \frac{1}{m_0c}(1 - \cos\theta)$$

Multiplying through by h and applying equn ① & ②, we obtain the Compton equation

$$\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_c(1-\cos\theta)$$

Where λ_c is the compton wavelength defined as

$$\lambda_c \equiv h/m_0 c = 0.0243 A^\circ$$

The relation betn scattering and recoil angle and kinetic energy of the recoiled electron

$$\cot\phi = (1+\alpha)\tan\theta/2 \qquad \left[\alpha = \frac{\lambda_c \nu_0}{c} = \frac{\lambda_c}{\lambda_0}\right]$$

$$K = h\nu_0 \frac{\alpha(\cos\theta - 1)}{1+\alpha(\cos\theta-1)}$$

To explain the presence of peak at unchanged photon wavelength λ_0, we observed that if the electron involved in scattering are ~~parti~~ particularly strongly bound to the atom in the target then the whole atom recoils. Therefore, the electron rest mass m_0 in compton equation ①, has to be replaced by mass of the atom $M \gg m_0$ and hence the Compton shift becomes very too small, $\Delta\lambda \sim 1/M$.