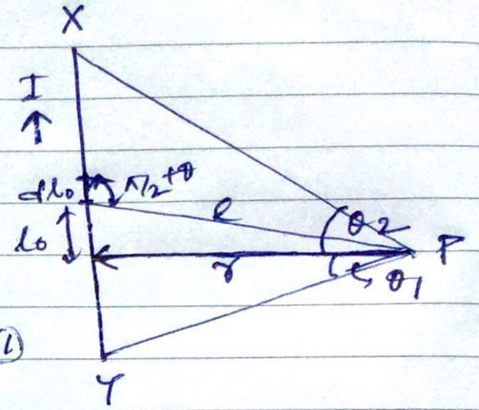
Applications of Biot-Savart Law:

Consider a straight wire segment XY carrying a current I & P is point at a distance r from XY. The value of magnetic field at P is to be determined.



According to Biot-Savart law, the magnetic field at P due to elementary length dl_0 of the wire is

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl_0}{l^2} \sin(\pi/2 + \theta) \quad \text{---(1)}$$

'l' is the distance dl_0 from P.

$\therefore \quad l_0 = r\tan\theta$

$dl_0 = r\sec^2\theta\, d\theta$ and $l = r\sec\theta$.

Substituting these values in equation (1)

$$dB = \frac{\mu_0 I}{4\pi r} \cos\theta\, d\theta$$

$\therefore$ total field at P due to entire wire is

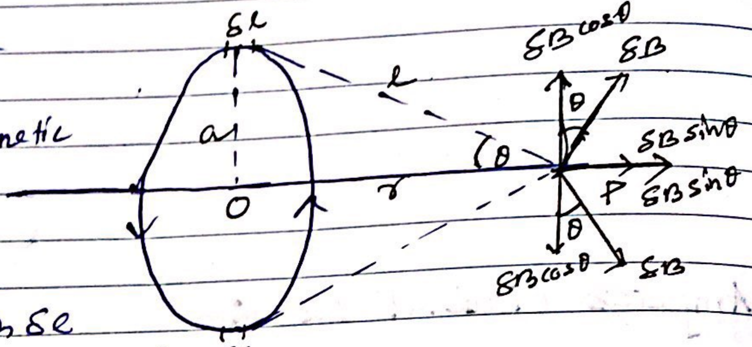
$$B = \frac{\mu_0 I}{4\pi r}\int_{-\theta_1}^{\theta_2} \cos\theta\, d\theta = \frac{\mu_0 I}{4\pi r}\left(\sin\theta_2 + \sin\theta_1\right)$$

For an infinitely long wire, $\theta_1 = \theta_2 = \pi/2$

$$\therefore \quad B = \mu_0 I / 2\pi r$$

Magnetic field at a point on the axis of a circular conductor carrying current:

Let P be a point on the axis of a circular loop carrying current I. The magnetic field at P due to I is to be determined.



Consider an elementary length δl on the loop. According to Biot-Savart law the magnetic field at P due to δl is

$$\delta\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\delta\vec{l} \times \vec{l}}{l^3}$$

'l' is the distance of P from dl. Since the angle betn δl & l is $\pi/2$.

$$\therefore \quad \delta B = \frac{\mu_0 I}{4\pi} \frac{\delta l}{l^2}$$

This field acts perpendicularly to the plane containing $\delta\vec{l}$ & $\vec{l}$. Let this field is represented by the vector $\delta\vec{B}$. It has two components: $\delta B \sin\theta$ along the axis of loop & $\delta B \cos\theta$ ~~along~~ perpendicular to axis. As symmetry, when the entire loop is considered, the components perpendicular to the axis cancel out and only the components along the axis contribute.

$$\therefore \quad \delta B \sin\theta = \delta B (a/l)$$

$\therefore$ the component of $\delta\vec{B}$ along the loop axis is

$$\frac{\mu_0 I}{4\pi} \frac{\delta l}{l^2} \left(\frac{a}{l}\right) = \frac{\mu_0 I a}{4\pi l^3} \delta l$$

$\therefore$ The magnetic field due to entire loop is

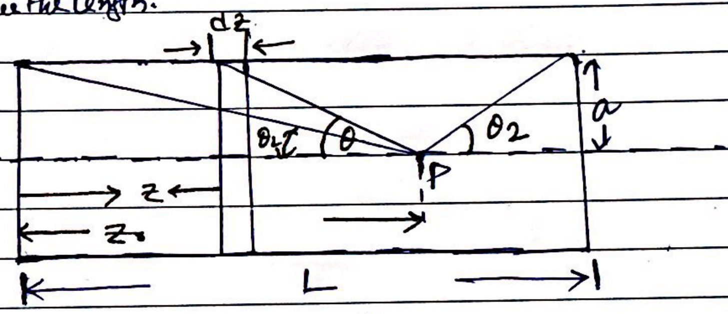
$$B = \frac{\mu_0 I a}{4\pi r^2}\int dl = \frac{\mu_0 I a}{4\pi r^3}(2\pi a) = \frac{\mu_0 I a^2}{2r^3} = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}$$

if the coil has N number of closely wound turns, then

$$B = \frac{\mu_0 I N a^2}{2(z^2+a^2)^{3/2}}$$

Axial magnetic field of a solenoid :

A solenoid is a device containing a number of turns wound uniformly. Let N be the number of turns, L be the length. 'a' be the radius of the solenoid carrying current I. The magnetic field at P is to be determined.



Let z_0 be the distance of P from the left end of the solenoid. We divide the length 'L' into the small elements of width dz. Now the magnetic induction at P due to this small element

$$dB = \frac{\mu_0 N I a^2 dz}{2L[a^2+(z_0-z)^2]^{3/2}}$$

the no. of turns in the element dz is $N dz/L$.

$$\text{as } \frac{a}{(z_0-z)} = \tan\theta \quad \& \quad dz = \frac{a\,d\theta}{\sin^2\theta}$$

$$\therefore \sqrt{a^2+(z_0-z)^2} = a/\sin\theta$$

$$\therefore dB = \frac{\mu_0 N I}{2L}\sin\theta\, d\theta$$

∴ for all elements, the total magnetic induction at P is

$$B = \frac{\mu_0 N I}{2L} \int_{\theta_1}^{\pi-\theta_2} \sin\theta \, d\theta = \frac{\mu_0 N I}{2L} (\cos\theta_1 + \cos\theta_2)$$

At midpoint $\theta_1 = \theta_2$, the magnetic field is

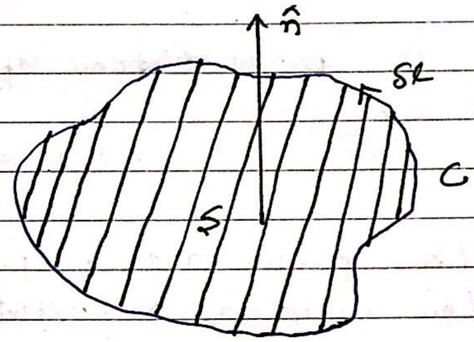
$$B_{mid} = \frac{\mu_0 N I}{L} \cos\theta_1$$

if the solenoid is very long compared to the radius, $\theta_1 \to 0$, thus

$$B = \frac{\mu_0 N I}{L}$$

Ampere's Circuital Law :

It states that the line integral of the magnetic induction around a closed path is the total current enclosed by the path multiplied by the permeability of free space.



We know $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

if we take the scalar surface integral through any surface S, we get

$$\int_s (\vec{\nabla} \times \vec{B}) \cdot \hat{n}\, ds = \mu_0 \int_s \vec{J} \cdot \hat{n}\, ds$$

Using Stokes' theorem, we can write

$$\int_s (\vec{\nabla} \times \vec{B}) \cdot \hat{n}\, ds = \oint_c \vec{B} \cdot d\vec{l}$$

where C is a closed path bounding S.

$$\therefore \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \int_s \vec{J} \cdot \hat{n}\, ds$$

This equation is known as Ampere's circuital law.

Magnetic Induction or flux density $\vec{B}$:

If a charge moving through a point experiences a side way force, a magnetic field is said to exist at that point. The field is described by means a vector called magnetic induction or flux density $\vec{B}$.

If a positive charge q_0 moving with velocity 'v' through a point in a magnetic field experiences a force $\vec{F}$, then the magnetic induction $\vec{B}$ at that point is defined by $\vec{F} = q_0(\vec{v} \times \vec{B})$. The magnitude of magnetic induction

$$B = F/q_0 v \sin\theta$$

where 'θ' is the angle betn $\vec{B}$ & $\vec{v}$.

Magnetization (I) :

When a magnetic material is placed in a magnetic field, the elementary current loop in the material become aligned parallel to the field. The material is then magnetized and acquired a magnetic moment. The magnetic moment per unit volm of the material is defined as the magnetization I of the material.

S.I unit of I is amp/meter

Magnetic Intensity $\vec{H}$:

When a magnetic material is placed in a magnetic field, it becomes magnetized. The capacity of the magnetic field to magnetize the material, is expressed by means of a magnetic vector $\vec{H}$ called the magnetic intensity of the field.