The relation (iv) can be used to find $T_{\frac{1}{2}}$.

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{2R(\ln 2)}{v} \exp\left[\frac{4\pi^2 Zze^2}{hv} - \frac{16\pi Zze^2}{hv}\sqrt{\frac{E}{B}}\right]$$

which shows that for higher energy of $\alpha$-particles, $T_{\frac{1}{2}}$ decreases.

## 13.9 Beta decay : Determination of β-energy : beta ray spectrum

Both positive and negative electrons (positrons and negatrons) are emitted spontaneously from radioactive nuclei. This phenomenon is called *β-decay*. The reverse process is the *electron capture* where the nucleus absorbs one of its own orbital electrons. This also comes under *β-decay*.

Now, the $\beta$-particles ejected from a radioactive source possess a range of velocities and hence a range of energies. The distribution or spread of these energies of $\beta$-particles is called the *β-ray spectrum* of the given nuclide. This distribution can be studied experimentally by deflecting them in a magnetic field. Such an arrangement is known as *β-ray spectrometer* and is described below.

**β-ray spectrometer** — This apparatus, as conceived and used by Rutherford and Robinson, is shown schematically in Fig. 13.14. The source of $\beta$-rays — a thin wire coated with radioactive material — is placed at $A$ under a slit $S$, in the same horizontal plane as that of the supported photographic plate $P$ on a lead block. The chamber housing these parts is highly evacuated. A uniform magnetic field is maintained over the region and is at right angles to the plane of the paper. Under its influence, the $\beta$-particles emitted from $A$ would describe circular tracks, to be received finally on the photographic plate $P$.

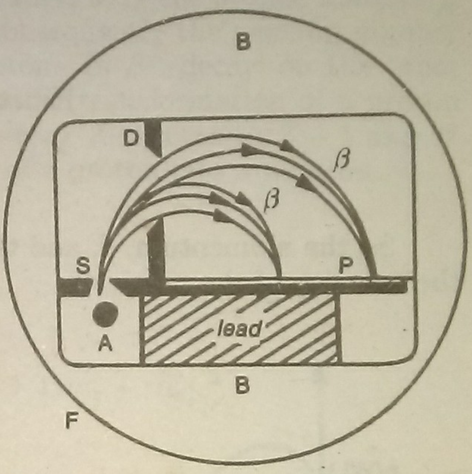

**Fig.13.14** β-ray spectrometer

**Velocity determination** — Let $v$ be the velocity of a given $\beta$-particle, $B$ the magnetic flux density, $m$ the relativistic mass of the $\beta$-particle and $r$ the radius of the track. Then we have

$$\frac{mv^2}{r} = Bev \tag{i}$$

$$\therefore \quad r = \frac{mv}{Be} = \frac{m_0}{\sqrt{1 - v^2/c^2}} \cdot \frac{v}{Be} \tag{ii}$$

where $m_0$ is the rest mass of the $\beta$-particle, $c$ the velocity of light in vacuo and $e$ the electronic charge.

Eq. (ii) shows that the $\beta$-particles having the same velocity will move in circles intersecting at the plate $P$. For a wide slit, a given velocity of particles corresponds to a line formed at $P$; particles with higher energy (velocity) will have arcs of higher radii of curvature.

If $x$ be the distance between the slit and the point of intersection $P$, and if $AS$ be equal to $h$, we may write

$$(2r)^2 = h^2 + x^2$$

$$\text{or,} \quad r^2 = \frac{h^2 + x^2}{4} \tag{iii}$$

Thus $r$ can be easily measured and $v$ could then be calculated from eq. (ii) using known values of $e, m_0, B$ and $c$.

**Discussion** — On developing the photographic plate, it is seen that a region from the plate-end nearest the source up to a certain maximum distance is blackened *continuously*. The density of blackening however is not the same at all parts. This proves that the $\beta$-particles from various emitters have velocities (hence energies) varying *continuously* between zero and a certain maximum value.

Instead of using the photographic plate, a GM-counter could as well be used to count the emitted $\beta$-particles. From eq. (ii), we have,

$$\therefore \quad \text{momentum, } p = mv = Ber$$

Since $p \propto r$, the particles with higher velocities have larger radii, that is, they are focussed at a greater distance from the source. The number of $\beta$-particles of a given velocity can thus be counted by shifting the positions of the GM-counter. The counting confirms that the blackening of plate is from zero up to a certain maximum distance.

If $E$ be the total energy of the $\beta$-particles,

$$E^2 = p^2c^2 + m_0^2c^4$$

$$\therefore \quad \text{Kinetic energy, } W_k = E - m_0c^2$$

$$= \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$$

So the momentum, $p$, and the kinetic energy, $W_k$, become known from a determination of the velocity of the particles.

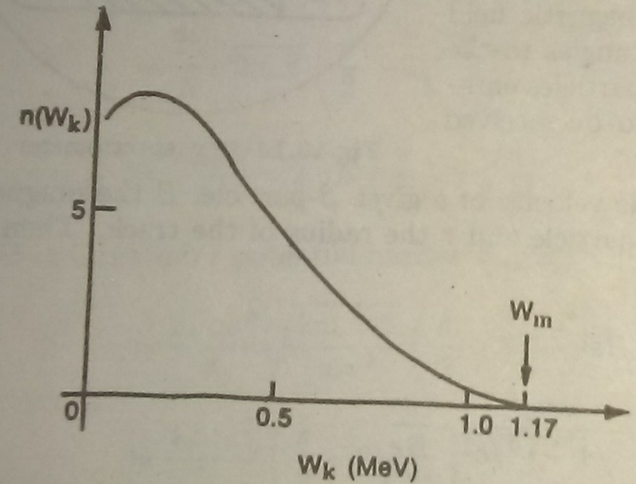

**Fig.13.15a** Energy distribution of $\beta$-particles

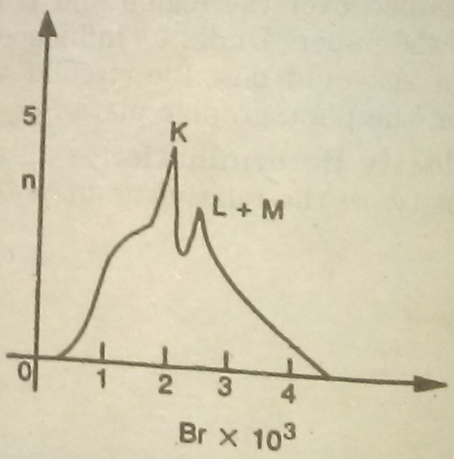

**Fig.13.15b** Momentum distribution of $\beta$-particles

The distribution of the kinetic energy, $W_k$, of the $\beta$-particles with the number of particles, $n(W_k)$, is shown in Fig.13.15a. This is called the *continuous $\beta$-spectrum*. In Fig.13.15b, instead of $W_k$, a plot of $Br$ (which depends on $W_k$) has been made along the abscissa. It shows a number of sharp peaks $K, L$, etc. superimposed on the continuous background — the so-called *line spectrum* of the $\beta$-rays. The existence of discrete well-defined lines is also found in the photographic plate.

**Nature of spectrum** — The process of $\beta$-decay differs from $\alpha$-decay in *two* important respects. *First*, the $\alpha$-particle is composed of nucleons already present in the initial nucleus; in contrast, the electron is not present in the nucleus and must be created in the decay process itself. *Secondly*, unlike $\alpha$-decay, the energy spectrum of the emitted electrons is not discrete but is found to be continuous. The magnetic deflection experiments with various $\beta$-emitters show that a single source produces $\beta$-particles with all energies (velocities) from zero up to a definite maximum $W_m$, characteristic of the nuclide, the so-called *end-point energy*. This is the *continuous $\beta$-spectrum*, the shape of which is generally the same for all nucleus. Superposed on the continuous background, however, there is a number of sharp lines (peaks) which are found to be very prominent on the photographic plate. This is the *line spectrum* of the $\beta$-rays and it indicates that there are definite energy levels in the nucleus. A typical $\beta$-ray spectrogram is shown in Fig.13.16.

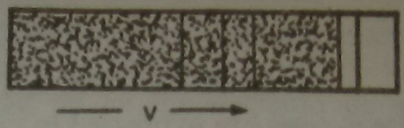

**Fig.13.16** A typical $\beta$-ray spectrogram

In interpreting the $\beta$-spectrum, therefore, we must have to reconcile *two* facts : (i) the existence of *discrete energy levels* and (ii) the existence of the *continuous energy spectrum*.

### 13.9.1 Energetics of β-decay

In all the *three* processes of $\beta$-decay, namely $\beta^-$ decay, $\beta^+$ decay and orbital electron capture, the mass number $A$ of the parent nucleus does not change, only the atomic number $Z$ changes by one unit. In $\beta^-$ decay, $Z$ increases to $Z+1$ and consequently the neutron number $N$ decreases to $N-1$ since a neutron transforms into a proton. In $\beta^+$ decay, on the other hand, $Z$ decreases to $Z-1$ and $N$ increases to $N+1$ due to the transformation of a proton into a neutron. In orbital electron capture, however, like $\beta^+$ decay $Z$ reduces to $Z-1$ and $N$ increases to $N+1$ as the process involves the transformation of a proton into a neutron.

So, for $\beta^-$ decay, we write : ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$

$\therefore$ The disintegration energy in $\beta^-$ decay is

$$\begin{aligned}Q_{\beta^-} &= [M_n(A,Z) - M_n(A,Z+1) - m_e]c^2\\ &= [M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e]c^2\end{aligned}$$

in terms of atomic mass.

$$\begin{aligned}&= [M(A,Z) - M(A,Z+1)]c^2,\\ &= M(A,Z) - M(A,Z+1), \text{ in energy unit} \qquad \text{(i)}\end{aligned}$$

where $M_n$ is the nuclear mass, $M$ the atomic mass and $m_e$ the mass of electron.

From (i), $Q_{\beta^-} > 0$, if $M(A,Z) > M(A,Z+1)$ implying that $\beta^-$ decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

Similarly, $\beta^+$ decay is represented as: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e$.

$$\begin{aligned}\therefore\ Q_{\beta^+} &= [M_n(A,Z) - M_n(A,Z-1) - m_e]c^2\\ &= [M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e]c^2\\ &= [M(A,Z) - M(A,Z-1) - 2m_e]c^2\\ &= M(A,Z) - M(A,Z-1) - 2m_e, \qquad \text{(ii)}\end{aligned}$$

when masses are expressed in energy units.

$$\therefore\ Q_{\beta^+} > 0, \text{ if } M(A,Z) > M(A,Z-1) + 2m_e$$

which implies that $\beta^+$ decay is possible if the mass of the parent atom is greater than the daughter atom by at least twice the electronic mass, i.e. 1.02 MeV.

Finally, the orbital electron capture may be represented as

$${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y.$$

∴ Disintegration energy, $Q_e = [M_n(A,Z) + m_e - M_n(A,Z-1)]c^2 - B_e$ where $B_e$ is the binding energy of the electron to the orbit.

$$\begin{aligned} \therefore\ Q_e &= [M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e]c^2 - B_e \\ &= [M(A,Z) - M(A,Z-1)]c^2 - B_e \\ &= M(A,Z) - M(A,Z-1) - B_e \end{aligned}$$

if the masses are expressed in energy unit.

∴ In electron capture, $Q_e > 0$, if $M(A,Z) > M(A,Z-1) + B_e$ which implies that electron capture is possible if the mass of the parent atom is greater than that of the daughter atom by at least the binding energy of the electron.

### 13.9.2 Difficulties in interpretation of β-spectrum

The various difficulties in interpreting the $\beta$-spectrum are the following.

(i) The nuclear energy states are discrete. So the continuous $\beta$-spectrum appears to be a violation of the *conservation of energy principle*.

(ii) When a $\beta$-particle is emitted with an energy $W_k$, the difference $W_m - W_k$ cannot be accounted for.

(iii) By placing the decaying nuclei in a delicate calorimeter with thick wall that stops the $\beta$-particles, the heat produced is related to the average energy and *not* to the maximum energy. The missing energy is thus not absorbed in matter immediately surrounding the emitter.

(iv) Attempts to explain the observed continuous distribution by way of loss of different amounts of energy due to scattering of $\beta$-particles in the source also failed.

(v) The emitted $\beta$-particle also does not travel in the direction opposite to the recoil velocity of the daughter product. It thus appears to violate the *principle of conservation of linear momentum* also.

(vi) The $\beta$-emission from a radioactive nuclide is supposed to be the result of the transformation of either (a) or (b) below :

$$\text{(a)} \quad p \rightarrow n + \beta^+$$

$$\text{(b)} \quad n \rightarrow p + \beta^-$$

Since all the particles in the above two transformations are known to have half-integral spin, the law of *conservation of angular momentum* is also violated.

**Neutrino hypothesis of Pauli** — The two conservation laws however were saved by Pauli by a daring postulate put forward by him in 1927. His idea was developed subsequently into a consistent *theory of β-decay* by Enrico Fermi. Pauli's idea was simply that in the $\beta$-decay process a second *new particle* was also simultaneously emitted.

To conserve the charge in the process, the new particle should be electrically *neutral*. Further, the $\beta$-particle itself is capable, on occasions, of carrying off all the available energy. So the new particle should carry *little kinetic energy* and, in fact, would have to have exceedingly small rest mass. Pauli postulated that the new particle had *zero rest mass* as well as *zero charge*.

To conserve the momentum, the new particle had to be endowed with a *spin* equal to $\frac{1}{2}(h/2\pi)$. Further, the new particle must *interact very weakly with matter*. For, if it were not so, they would have been stopped in the calorimeter experiment and their energy would have been absorbed by the calorimeter, giving a rise in temperature corresponding to the maximum available energy.

The new particle was labelled *neutrino* and symbolised by $\nu$ and the hypothesis was called the *neutrino hypothesis* of Pauli.

**The neutrino hypothesis explains well the emission of $\beta$-particles by transition of nucleon from neutron to the proton state with the *simultaneous creation* of an *electron-neutrino pair*. These two particles escape with a constant total energy, the maximum energy $W_m$ available, being equal to the difference between the energies of the original and the final nucleus.**

**The *continuous energy distribution* arises from the *variable* manner in which the total energy is shared between the electron and the neutrino.** The upper limit corresponds to the case where the neutrino gets no energy, the whole being carried off by the electron; in the low energy portion, the neutrino gets the greater share. Thus the neutrino carries the missing energy which is equal to the difference between the maximum or end-point energy, $W_m$, in $\beta$-ray spectrum and the energy carried by the $\beta$-particle.

Subsequently, it was seen that there are, in fact, *two* kinds of neutrino involved in $\beta$-decay – the *neutrino* ($\nu$) and the *antineutrino* ($\bar{\nu}$). The subtle difference between neutrino and antineutrino will be examined later. The neutrinos involved in $\beta$-decay are termed *electron neutrinos*— there are two other types: *muon neutrinos* and *tau neutrinos*, to be discussed in the Chapter: *Elementary particles*. In an ordinary $\beta$-decay, it is the antineutrino which is emitted so that the decay process can be represented as :

$$n \rightarrow p + e^- + \bar{\nu} \qquad \text{(iv)}$$

**Example :** ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + e^- + \bar{\nu}$

In the positron decay, however, a neutrino is emitted and the process may be interpreted as a transformation of a proton in the nucleus into a neutron :

$$p \rightarrow n + e^+ + \nu \qquad \text{(v)}$$

**Example :** ${}^{11}_6\text{C} \rightarrow {}^{11}_5\text{B} + e^+ + \nu$

**Properties of a neutrino** — The properties of a neutrino may be summarised as under.

(i) It has *no charge*. Like the neutron, it is a *neutral particle*.
(ii) It has an *extremely small mass*.
(iii) It is a *spin $\frac{1}{2}$ particle*.
(iv) It can have only a very small magnetic moment, much smaller than that of electron.
(v) It *interacts extremely feebly with matter*.

It is the last property that made the detection of neutrinos extremely difficult. As they are both chargeless and massless and also not electromagnetic in nature, unlike photons, they can pass unimpeded through unbelievably vast amount of matter. A neutrino may pass through, without interaction, 1000 *light-years* of solid iron on the average ! A process called *inverse $\beta$-decay* (see later) is the only interaction a neutrino can have with matter. Its existence was experimentally established by F. Reines and C.L. Cowan in 1956 in the immense flux of neutrinos from $\beta$-decays in a reactor.

## 13.10 Fermi's theory of β-decay : Outlines

Based on Pauli's neutrino hypothesis, Enrico Fermi developed a theory for $\beta$-decay to obtain the continuous energy spectrum as well as the decay constant for $\beta$-emissions. The theory is rather involved and we shall rest content by giving a bare outline of it.

Fermi obtained an expression for the decay constant $\lambda$ from the initial state $i$ of the parent nucleus to the final state $f$ of the daughter nucleus. From the time-dependent perturbation theory, the transition probability per sec from state $i$ to state $f$ is given by

$$P = \lambda = \frac{2\pi}{\hbar} |H_{if}|^2 \rho(E) \qquad \text{(i)}$$