II Superposition of two co-linear oscillations having different freqⁿ: Beats.

When two simple harmonic oscillations of slightly different frequencies along the same straight line we get beats.

Let the two motions have the angular frequencies ω and $\omega - \Delta\omega$ respectively. If α_1 & α_2 are the initial phases, the two motions can be expressed as.

$$x_1 = a_1 \cos(\omega t + \alpha_1)$$

$$x_2 = a_2 \cos[(\omega - \Delta\omega)t + \alpha_2]$$

Putting $\alpha_2 - (\Delta\omega)t = \alpha_2'$ we can get

$$x_2 = a_2(\omega t + \alpha_2')$$

Now the resultant displacement will be

$$x = x_1 + x_2$$

$$= a_1 \cos(\omega t + \alpha_1) + a_2 \cos(\omega t + \alpha_2')$$

$$= A \cos(\omega t + \phi) \quad \text{——— } (1)$$

where

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2') \quad \text{— } (2)$$

and

$$\tan\phi = \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2'}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2'} \quad \text{— } (3)$$

Now if $\alpha_1 - \alpha_2' = \alpha_1 - \alpha_2 + (\Delta\omega)t = (2p+1)\pi$

$A^2 = a_1^2 + a_2^2 - 2a_1 a_2$ when $p = 0, 1, 2, 3,$ etc

$A = a_1 - a_2$

On the other hand if

if $\alpha_1 - \alpha_2 + (\Delta\omega)t = 2p\pi$

$A = a_1 + a_2$.

Equation (1) shows that the resultant oscilation has the angular ~~ω~~ frequency ω.

The amplitude A of the resultant osilation is not a constant. It changes with time between $(a_1 - a_2)$ to $(a_1 + a_2)$ with one angular frequency. $\Delta\omega$.

If $a_1 = a_2 = a$

Then, A varies periodically between 0 to 2a.

The frequency of amplitude variation is the difference between the two component frequencies.

<u>Beats</u>: When two simple harmonic motions of slightly different frequencies superimpose, the ~~resultant~~ amplitude of resultant oscilation changes regularly with time between a maximum and a minimum. This phenomenon is reffered to as ~~a~~ beats.

For clear realization of beats, the frequency difference $\Delta\omega$ should be small and amplitudes a_1 & a_2 should be equal.

The time between two successive maxima or minima of amplitude is $2\pi/\Delta\omega$ or $\frac{1}{\Delta f}$ sec.

So no. of beats/sec $= \Delta f$

Δf is freq^n diff. in Hz of the component oscilation

~~Find~~ Superposition of n number of simple harmonic motions having the same amplitude a and the same angular frequency ω, but equal successive phase displacement δ.

Ans.

$$x_1 = a e^{j\omega t}$$

$$x_2 = a e^{j(\omega t + \delta)}$$

$$x_3 = a e^{j(\omega t + 2\delta)}$$

$$\cdots \quad \cdots$$

$$x_n = a e^{j[\omega t + (n-1)\delta]}$$

The resultant motion is

$$x = x_1 + x_2 + x_3 + \cdots + x_n$$

$$= a e^{j\omega t}\left[1 + e^{j\delta} + e^{j2\delta} + \cdots + e^{j(n-1)\delta}\right]$$

$$= a . e^{j\omega t} . \frac{1 - e^{jn\delta}}{1 - e^{j\delta}}$$

$$= a e^{j\omega t} . \frac{e^{jn\delta/2}\left(e^{-jn\delta/2} - e^{jn\delta/2}\right)}{e^{j\delta/2}\left(e^{-j\delta/2} - e^{j\delta/2}\right)}$$

$$= a e^{j\omega t} . e^{j(n-1)\delta/2} . \frac{\sin(n\delta/2)}{\sin \delta/2}$$

$$= a . \frac{\sin n\delta/2}{\sin \delta/2} . e^{j[\omega t + (n-1)\delta/2]}.$$

Taking the real part as the physical solution,

$$x = a . \frac{\sin n\delta/2}{\sin \delta/2} . \cos\left[\omega t + (n-1)\delta/2\right]$$

The resultant motion is simple harmonic having the amplitude $\frac{a \sin n\delta/2}{\sin \delta/2}$. and the phase angle $(n-1)\delta/2$.

This result is as same as the diffraction pattern due to a plane diffraction grating in optics.

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