4.4. Isoquant Curve and Economic Region

'ISO' means equal and 'QUANT' for quantities. Therefore, ISO-QUANT (IQ) curve means a curve along which the production or output must be constant or equal.

From the previous discussion, we know that the Law of Variable Proportion can be developed from the short-run production function where one of the factors is variable and the other must be fixed, where as Returns to Scale is a concept of long-run production function where all the existing factors are variable. The law of variable proportion can be expressed with the help of the total production curve, as discussed and returns to scale can be explained with the help of Isoquant analysis.

Let the long-run production function is, $Q = f(L, K)$

Where labour and capital both are variable. Now isoquant (IQ) means constant output, Q. Therefore the above mentioned production function can be written as :

$$\bar{Q} = f(L, K) \quad \text{.......................................} (i)$$

From equation (i), it is easy to conclude that the increase in L definitely means decrease in K to maintain constant output, $\bar{Q}$. Hence L and K both are inversely related.

One interesting relationship can be developed from this behaviour. Let labour (L) is increasing, that means increase in production where as capital (K) is decreasing means decrease in production. Now constant output implies those two components must be equal in magnitude and opposite in sign.

Let, labour is increasing from L_1 to L_2, i.e. $L_2 > L_1$

$\Rightarrow \Delta L = (L_2 - L_1) > 0$

Now, MP_L means addition to the total production due to a change in labour by extra one unit. Therefore MP_L is the 'last unit' labour productivity. So for the increase in L by ΔL unit, **addition to production** will be, $(\Delta L.MP_L)$.

Similarly, capital is let decreasing from K_1 to K_2, i.e. $K_2 < K_1$

$\Rightarrow \Delta K = (K_2 - K_1) < 0$

So, for the reduction in K by ΔK unit, **reduction in production** will be, $(\Delta K.MP_K)$

Hence, along the isoquant, constant output with change in labour as well as capital means, $(+)\ \Delta L.MP_L = (-)\ \Delta K.MP_K \Rightarrow \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$

If capital is measured on the vertical axis and labour is on the horizontal axis,

then, $\frac{\Delta K}{\Delta L}$ = Slope of IQ Curve

$$= -\frac{MP_L}{MP_K} \quad \text{.......................................} (ii)$$

Now from the Section 4.3.5, we know that a rational producer will operate in Stage II of production,

where, $MP_L > 0$, $MP_K > 0$

Hence, Slope of IQ Curve $= \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K} < 0$

So, the IQ Curve must be negatively sloped.

One alternative explanation of the isoquant curve can be developed with the help of Economic and Non-Economic Region of production. For this discussion, the technical shape of IQ is necessary.

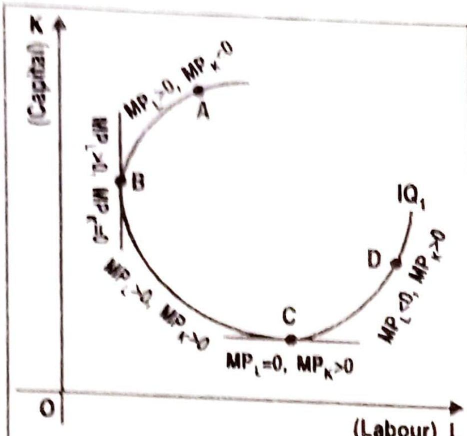

Fig. 4.5

Now the equation of IQ is, $\bar{Q} = f(L, K)$ so different combinations of L and K can be obtained, which are technically possible. Joining these combinations the technical shape of IQ can be derived. It is to be noted that this technical shape is the summation of economic shape and non-economic shape. So the main target is to identify the economic shape and economic region or the elimination of non-economic shape.

In Fig. 4.5, the technical shape (ABCD) of isoquant curve is drawn which is the locus of all technically possible input combinations.

At point B one tangent is drawn which is parallel to the vertical axis and one another tangent at point C which is parallel to horizontal axis. Following conclusions can be drawn from the diagram;

1. ***At the range BC*** : The IQ Curve is negatively sloped

$$\Rightarrow \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K} < 0 \Rightarrow MP_L > 0, MP_K > 0.$$

2. ***At point B*** : The tangent is vertical

$$\Rightarrow \text{slope of IQ} = \alpha \Rightarrow -\frac{MP_L}{MP_K} = \alpha \Rightarrow MP_L > 0, MP_K = 0.$$

3. ***At point C*** : The tangent is horizontal

$$\Rightarrow \text{slope of IQ} = 0 \Rightarrow -\frac{MP_L}{MP_K} = 0 \Rightarrow MP_L = 0, MP_K > 0.$$

4. ***At the range AB*** : The IQ curve is positively sloped

$$\Rightarrow \text{slope of IQ} > 0 \Rightarrow -\frac{MP_L}{MP_K} > 0 \Rightarrow MP_L > 0, MP_K < 0.$$

5. ***At the range CD*** : The IQ curve is positively sloped

$$\Rightarrow \text{slope of IQ} > 0 \Rightarrow -\frac{MP_L}{MP_K} > 0 \Rightarrow MP_L < 0, MP_K > 0.$$

Hence from the technical shape ABCD of IQ, the range 'BC' represents the economic shape where marginal productivities of both the factors are positive. The other two ranges like AB and CD are non-economic because their marginal productivity of one factor is positive but the other is negative. Similarly at points B and C, marginal productivity of one factor is positive but the other factor is zero. Therefore, the economic shape BC actually the Stage–II production of the Law of Variable Proportion.

Now, one Isoquant Map can be drawn which will be the set or collection of different isoquant curves with their technical shapes. Different isoquants will represent different output levels. From this diagram the 'economic region' of production can be identified.

Let we have three isoquant curves in the isoquant map. All the isoquants are in their technical shapes.

By the diagram B_1C_1, B_2C_2, B_3C_3 are the economic shapes or most feasible ranges respectively for the isoquant IQ_1, IQ_2, IQ_3. The outer boundaries are the ***Ridge Lines***. These two lines are very important in economic analysis. They are as follows :

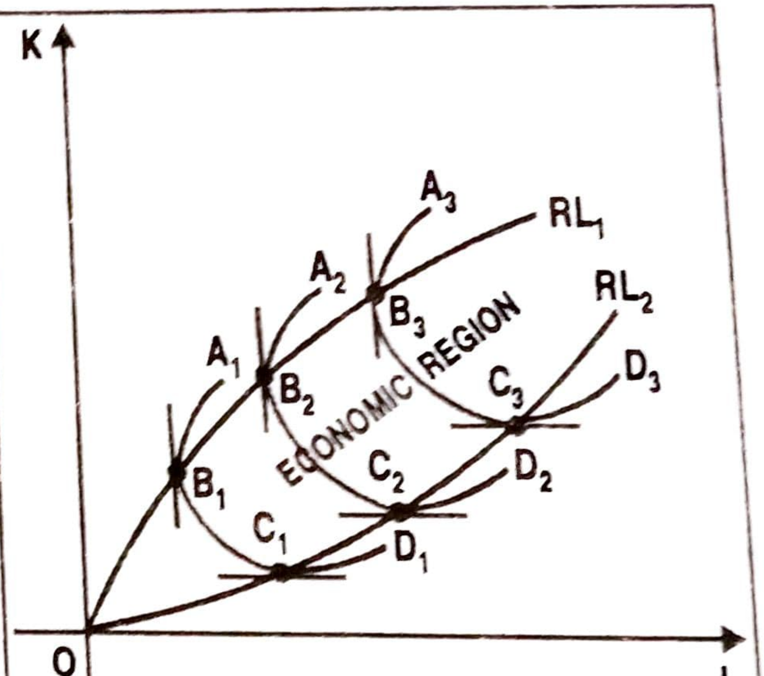

Fig. 4.6

(i) RL_1 is the locus of the points B_1, B_2, B_3 where $MP_k = 0$. Therefore the **'Upper Ridge Line'** is the boundary where marginal productivity of that factor is Zero, which is measured in the vertical axis. Hence, the equation of upper ridge line is $MP_K = 0$.

(ii) RL_2 is the locus of the points C_1, C_2, C_3 where $MP_L = 0$. Therefore, the **'Lower Ridge Line'** is the boundary where marginal productivity of that factor is Zero, which is measured in the horizontal axis. Hence the equation of lower ridge line is $MP_L = 0$.

Mathematical Extension

Let, the production function is,

equally productive, therefore movement of the producer is restricted along the IQ and a rational producer must be 'indifferent' to these available combinations.

◈ 4.5.3. Properties of an Iso-Quant Curve

The isoquant curve has the same properties possessed the indifferent curve. The properties are as follows :

1. ***IQ Curve is negatively Sloped*** : From the discussion, mentioned above, three explanations can be developed,

   (a) *Logical* : The equation of IQ is $\overline{Q} = f(L, K)$ where both the inputs are substitutes to maintain constant output, these two must be inversely related, i.e. increase (decrease) in L means decrease (increase) in K. Due to this inverse relationship, IQ must be negatively sloped.

   (b) *Mathematical* : From equation (ii) in Section 4.4, Slope of IQ Curve $= -\frac{MP_L}{MP_K}$. Since by the assumption $MP_L > 0$, $MP_K > 0$.

   ∴ IQ curve must be negatively sloped,

   (c) *Graphical* : From Fig. 4.8 (extension of Fig. 4.7) represents $Q_A = Q_B = Q_C$

   For $L_1 < L_2 < L_3$

   $K_1 > K_2 > K_3$ i.e. the inverse relation between two inputs.

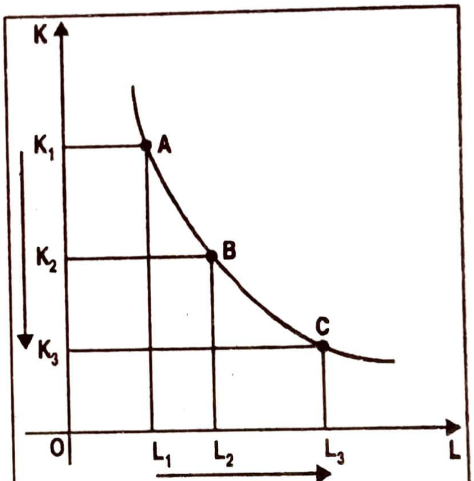

Fig. 4.8

One alternative explanation can be developed to derive the shape of an IQ. We can draw three conclusions :

(i) **IQ cannot be horizontal,** because it means, Slope of IQ Curve = 0

$$\Rightarrow -\frac{MP_L}{MP_K} = 0 \Rightarrow MP_L = 0 \text{ but } MP_K > 0, \text{ which is not possible.}$$

(ii) **IQ cannot be vertical,** because it means, Slope of IQ curve = α

$$\Rightarrow -\frac{MP_L}{MP_K} = \alpha \Rightarrow MP_L > 0 \text{ but } MP_K = 0, \text{ which is not possible.}$$

(iii) **IQ cannot be positively sloped,** because it means, slope of IQ Curve > 0

$$\Rightarrow -\frac{MP_L}{MP_K} > 0 \Rightarrow \text{either } MP_L > 0, MP_K < 0$$

or, $MP_L < 0$, $MP_K > 0$ these two are not possible.

Since we have a single possibility, i.e. $MP_L > 0$, $MP_K > 0$ therefore the IQ curve must be negatively sloped, i.e. the economic shape.

2. ***IQ Curve must be convex to the origin*** : It is to be noted that the absolute value of the slope of an IQ represents the Marginal Rate of technical Substitution (MRTS), i.e. $MRTS_{LK} = \left|\frac{\Delta K}{\Delta L}\right| = \frac{MP_L}{MP_K}$.

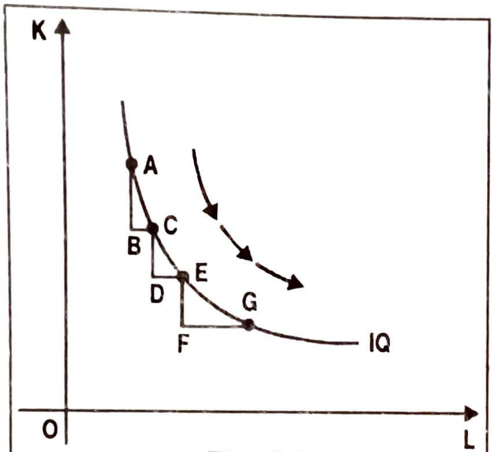

Fig. 4.9

Now, MRTS measures the reduction in one input per unit increase in the other to maintain a constant output level. So as we move from A to C along the IQ in Fig. 4.7, then labour is substituted for capital. But they are assumed to be imperfect substitutes not perfect, which means as labour is substituted for capital, then MP_L declines and MP_K increases. Hence the MRTS declines as labour is substituted for capital. Now MRTS represents the slope. Therefore, diminishing MRTS means diminishing slope and diminishing slope means the curve must be convex to the origin.

By the diagram : $\frac{AB}{BC} > \frac{CD}{DE} > \frac{EF}{FG} \Rightarrow$ MRTS must be diminishing.

So the convexity of one isoquant is based on the assumption (ii), mentioned in 4.5.1.

Now, Isoquant may be concave to the origin due to increasing MRTS and may be a straight line for constant MRTS.

3. ***Higher (lower) IQ means higher (lower) Output :*** A single IQ represents a specific value of output or constant value of output with all the possible input combinations physically capable. Therefore, different isoquants can be drawn for different values of output. That will be called the 'Isoquant Map'. In such isoquant map, higher position of one IQ will represent higher level of output and lower one will be for lower level of output.

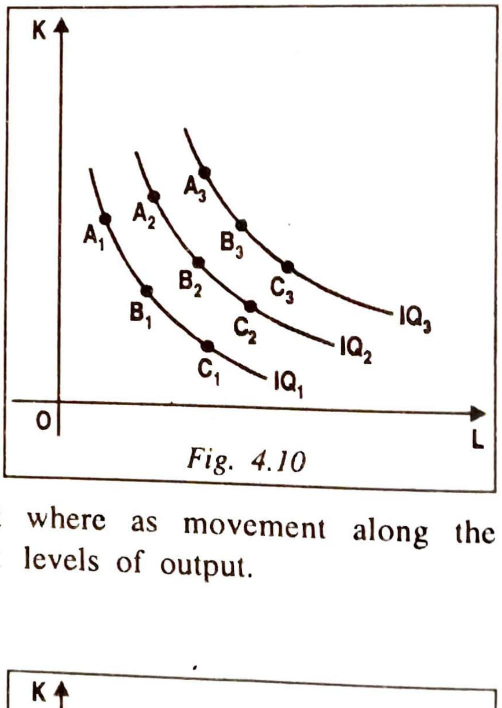

Fig. 4.10

Therefore, movement along the isoquant curve represents constant output where as movement along the isoquant map means variable or different levels of output.

Fig. 4.10 represents,

$Q_{A1} = Q_{B1} = Q_{C1}$ but $Q_{A3} > Q_{A2} > Q_{A1}$

$Q_{A2} = Q_{B2} = Q_{C2}$ $\quad Q_{B3} > Q_{B2} > Q_{B1}$

$Q_{A3} = Q_{B3} = Q_{C3}$ $\quad Q_{C3} > Q_{C2} > Q_{C1}$

4. ***Two Isoquants never intersect each other :***

Along IQ_1 : $Q_A = Q_C$

Again, along IQ_2 : $Q_A = Q_B$

$\therefore \; Q_B = Q_C$ .... *(iii)*

But B and C, these two points lie on different IQ. From the definition of IQ, we know that, different points lie on

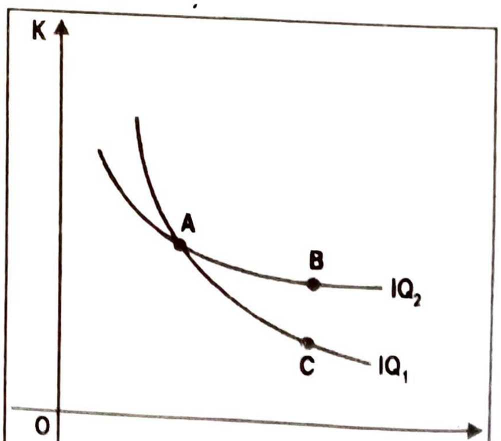

Fig. 4.11

same IQ only can represent equal or constant output. So by the definition,

$$Q_B \neq Q_C \quad \text{.......................................} \quad (iv)$$

These two are contradictory to each other, which means two isoquants never intersect each other.

MATHEMATICAL PROBLEM

Problem 5.

Show that for the production function $Q = .75 L^{.63} K^{.37}$ the **isoquants are nega-**