Sequence continued. A. Mondal (8972339491)

Theorem:- If a sequence converges, then it's limit is unique.

A sequence can't converge to two different limits.

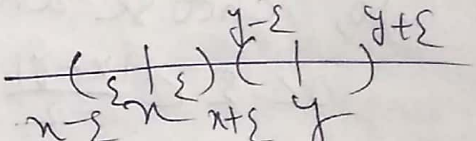
or

Theorem:- Limit of a convergent sequence is unique.

Proof:- To show this you just consider that ~~a~~ the sequence converges to two different limits and then show it is contradiction.

So, suppose $\{x_n\}$ is a convergent sequence and it converges to two limits x & y, where $x \neq y$.

Let $x < y$



So, for every ~~every~~ $\varepsilon > 0$ we can find some n_1 such that for $n \geq n_1$, ~~$x_n \in (x-$~~ $|x_n - x| < \varepsilon$

$\Rightarrow x_n \in (x-\varepsilon, x+\varepsilon) \quad \forall n \geq n_1$

Similarly we can find some n_2 such that for $n \geq n_2$, $|x_n - y| < \varepsilon$

$\Rightarrow x_n \in (y-\varepsilon, y+\varepsilon) \quad \forall n \geq n_2$.

Now ~~you~~ We can always find a n s.t. $n_0 = \max(n_1, n_2)$ and for which $x_n \in (x-\varepsilon, x+\varepsilon)$ & $x_n \in (y-\varepsilon, y+\varepsilon) \quad \forall n \geq n_0$

That means for $n \geq n_0$, x_n lies on ~~$x_n \in (x-\varepsilon$~~ both the interval $(x-\varepsilon, x+\varepsilon)$ & $(y-\varepsilon, y+\varepsilon)$

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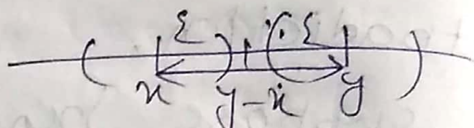
~~Next for $n \geq$~~

Now if we can show that these two intervals $(x-\varepsilon, x+\varepsilon)$ & $(y-\varepsilon, y+\varepsilon)$ are disjoint then $\forall n \geq n_0$ ~~both~~ x_n can't lie on both the intervals. Which will contradict our assumption.

Now, can you find an ε s.t. ~~$(x-\varepsilon)$~~ & $(x-\varepsilon, x+\varepsilon)$ and $(y-\varepsilon, y+\varepsilon)$ are disjoint. Total length of each of the intervals is 2ε.

~~If $2\varepsilon < \frac{y-x}{2}$~~

If $\varepsilon < \frac{y-x}{2}$ then they will be disjoint.



So, choose any positive ε ~~with~~ s.t

$0 < \varepsilon < \frac{|x-y|}{2}$ → mod is taken because that will work for both $x<y$ & $y<x$.

So, if we choose $0<\varepsilon < \frac{|x-y|}{2}$ then ~~x_n can~~ both the intervals will be disjoint and so x_n will not be able to lie on ~~the~~ both the intervals $(x-\varepsilon, x+\varepsilon)$ & $(y-\varepsilon, y+\varepsilon)$ for $n \geq n_0$.

Which is a contradiction.

(As we have considered $\forall \varepsilon > 0$ x_n should lie on $(x-\varepsilon, x+\varepsilon)$ & $(y-\varepsilon, y+\varepsilon)$ $\forall \varepsilon>0$)

Hence the proof.

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Limit pt. of a sequence :-

A number l is said to be a limit pt. of a sequence $\{x_n\}$ if ~~every neighbourhood~~ for any $\varepsilon > 0$, ~~$x_n \in (l-\varepsilon$~~

$x_n \in (l-\varepsilon, l+\varepsilon)$, ~~for infinitely~~ for infinitely many values of $n \in \mathbb{N}$.

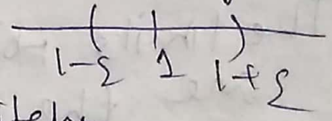
Note :- Limit of a sequence is unique. But limit point of a sequence ~~may~~ can be more than 1.

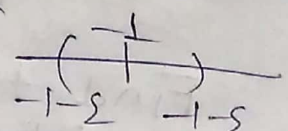
~~Examp~~

Note :- 1. The sequence $\{(-1)^n\}$ has two limit points 1 & -1. As the elements of the sequence are either 1 or -1. So there are infinitely many 1 & ~~and~~ -1.

So, $1 \in (1-\varepsilon, 1+\varepsilon)$ $\forall \varepsilon > 0$, for infinitely many n

Simi $-1 \in (-1-\varepsilon, -1-\varepsilon)$ $\forall \varepsilon > 0$, for infinitely many n





~~But~~ As it has two limit points it is not a convergent sequence. ~~We~~ As we know limit of a convergent sequence is unique.

2. If a sequence has unique limit point then that does not imply the sequence is convergent.

suppose $\left\{\frac{n}{2}\left(1+(-1)^n\right)\right\}$

$= \{0, 1, 0, 2, 0, 3, 0, 4, 0, 5, \cdots\}$

The sequence has unique limit point 0. But it is not a convergent sequence as it is not bounded ~~above~~.

(3) Consider the sequence $\{x_n\}$

$$x_n = \begin{cases} 1 & \text{for } n=1 \\ n - \frac{k(k+1)}{2} & \text{for } \frac{k(k+1)}{2} < n \le \frac{(k+1)(k+2)}{2} \end{cases}$$

$$\{x_n\} = \{1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, \dots\}$$

We see $\{x_n\}$ is a sequence of natural numbers.

The limit points of the sequence are the set $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ of natural numbers.

(5) $$x_n = \begin{cases} \cancel{1/2} & \cancel{\text{for } n=1} \\ \frac{n}{k+2} - \frac{k(k+1)}{2(k+2)} & \text{for } \frac{k(k+1)}{2} < n \le \frac{(k+1)(k+2)}{2} \end{cases}$$

Suppose $\{x_n\} = \{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}, \frac{1}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \dots\}$

The limits point of the sequence are the rational numbers on $(0,1)$.

Find the limit points of the following if exists:

(1) $\{n^2\}$, (2) $\{1, 2, 1, 4, 1, 6, 1, 8, \dots\}$

(3) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$

(4) ~~{1, 3, 1,}~~ $\{2 + (-1)^n\}$

(5) $\{1, 2, 3, 1, 2, 3, 1, 2, 3, \dots\}$ (6) $\{2 - \frac{1}{2^{n-1}}\}$

~~(7) $\{x_n\}$, $x_n = \{2$ when~~ n is even

(7) $\{x_n = \frac{1}{m} + \frac{1}{n}\}_{n\ge 1}$ (8) $\{x_n = m + \frac{1}{n}\}_{n \ge 1}$

Some Algebric Properties of Limit of a sequence :-

Theorem :- Let $\{x_n\}$ and $\{y_n\}$ be two convergent sequences such that $\{x_n\}$ converges to x and $\{y_n\}$ converges to y.

Then (i) $\{x_n + y_n\}$ converges to $x+y$.

(ii) $\{x_n - y_n\}$ converges to $x-y$.

(iii) $\lim_{n\to\infty} (x_n y_n) = x \cdot y = \lim_{n\to\infty} x_n \cdot \lim_{n\to\infty} y_n$

(iv) If $y_n \neq 0$ & $y \neq 0$

$$\lim_{n\to\infty} \frac{x_n}{y_n} = \frac{x}{y} = \frac{\lim_{n\to\infty} x_n}{\lim_{n\to\infty} y_n}.$$

(v) $k \in \mathbb{R}$, $\{k x_n\}$ converges to kx.

(vi) If $x_n \leq y_n$, then $x \leq y$.

PROOF: (i) $\lim_{n\to\infty} x_n = x$ & $\lim_{n\to\infty} y_n = y$

Therefore for any $\varepsilon > 0$, $\exists$ N_1, N_2 in $\mathbb{N}$ such that

$\forall n \geq N_1$; $|x_n - x| < \varepsilon/2$,

$\forall n \geq N_2$, $|y_n - y| < \varepsilon/2$.

Let $N = \max\{N_1, N_2\}$.

Then $\forall n \geq N$, $|x_n - x| < \varepsilon/2$ & $|y_n - y| < \varepsilon/2$

Now, $|(x_n + y_n) - (x+y)| \leq |x_n - x| + |y_n - y| < \varepsilon$ $\forall n \geq N$

$\Rightarrow \lim_{n\to\infty} (x_n + y_n) = (x+y) = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n$

The proof of (2) is trivial ~~and its imaginary agrtagt~~.

Try yourself.

(iii) We use the identity

$$(x_n y_n - xy) = (x_n - x)(y_n - y) + x(y_n - y) + y(x_n - x) \quad \text{--(i)}$$

Given any $\varepsilon > 0$, there are integers N_1, N_2 such that

$\forall n \geq N_1,\ |x_n - x| < \sqrt{\varepsilon}$, --(ii)

$\forall n \geq N_2,\ |y_n - y| < \sqrt{\varepsilon}$ --(iii)

Let $N = \max\{N_1, N_2\} \Rightarrow N \geq N_1$ & $N \geq N_2$

Then, $|x_n - x| < \sqrt{\varepsilon}$ & $|y_n - y| < \sqrt{\varepsilon}$ $\forall n \geq N$

[As (ii) & (iii) holds $\forall n \geq N_1$ & $n \geq N_2$ respectively, so (ii) & (iii) holds for $\forall n \geq N$ as $N \geq N_1$ & $N \geq N_2$]

~~Now~~, so, $\forall n \geq N$, $|(x_n - x)(y_n - y)| < \varepsilon\ \forall n$

$$\Rightarrow \lim_{n \to \infty} (x_n - x)(y_n - y) = 0$$

From (i)

$$\lim_{n \to \infty} (x_n y_n - xy) = \lim_{n \to \infty} (x_n - x)(y_n - y) + \lim_{n \to \infty} x(y_n - y) + \lim_{n \to \infty} y\,\cancel{(y_n - y)}(x_n - x)$$

[As we known $\lim_{n \to \infty} (a_n + b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$]

[as we know $\lim_{n \to \infty} (x_n + y_n) = \lim_{n \to \infty} x_n + \ldots$

or, $\cancel{\lim_{n\to\infty} x_n y_n - \lim}$

or, $\lim_{n\to\infty} x_n y_n - xy = 0+0+0$

$\Rightarrow \lim_{n\to\infty} x_n y_n = xy$.

[$\because \lim_{n\to\infty} k x_n = k \lim_{n\to\infty} x_n$ we do this proof later]

(IV) ~~Since~~

Since $\lim_{n\to\infty} y_n = y$,

for any $\varepsilon>0$ $\exists$ $N_1 \in \mathbb{N}$ s.t.

$|y_n - y| < \varepsilon/2$ $\forall n \geq N_1$ — (a)

Let us take $\varepsilon = |y|>0$ [as (a) is true for all $\varepsilon>0$]

so, $\forall n \geq N_1$, $|y_n - y| < \frac{|y|}{2}$

Hence $\forall n \geq N_1$

$\frac{|y|}{2} > |y_n - y| = |y - y_n| \geq |y| - |y_n|$

$\Rightarrow |y_n| > |y| - \frac{1}{2}|y| = \frac{1}{2}|y|$, $\forall n \geq N_1$ — (b)

We have for every value of n

$\left|\frac{x_n}{y_n} - \frac{x}{y}\right| = \left|\frac{y x_n - x y_n}{y_n y}\right|$

$= \left|\frac{y(x_n - x) - x(y_n - y)}{y_n y}\right|$

$\leq \frac{|y||x_n - x| + |x||y_n - y|}{|y_n||y|}$

$|y_n| > \frac{1}{2}|y| \Rightarrow \frac{1}{|y_n|} < \frac{2}{|y|}$

Now for $\forall n \geq N_1$

$\left|\frac{x_n}{y_n} - \frac{x}{y}\right| \leq \frac{|y||x_n - x| + |x||y_n - y|}{|y|} \cdot \frac{2}{|y|}$

$\forall\, n \geq N_1$

$$= \frac{2}{|y|} |x_n - x| + \frac{2|x|}{|y|^2} |y_n - y| \quad \text{---(c)}$$

Now since $\lim_{n\to\infty} x_n = x$ & $\lim_{n\to\infty} y_n = y$

for every $\varepsilon > 0$ $\exists N_2, N_3$ in $\mathbb{N}$ such that

$|x_n - x| < \frac{|y|}{4}\varepsilon$, —(d) $\forall\, n \geq N_2$

$|y_n - y| < \frac{|y|^2}{4(|x|+1)}\varepsilon$ —(e) $\forall\, n \geq N_3$

we are taking this type of ($\varepsilon > 0$) as we want to make (c) as ε only

let $N = \max\{N_1, N_2, N_3\}$

Then $\forall\, n \geq N_0$, (c), (d), (e) holds

so, $\left| \frac{x_n}{y_n} \right.$

So, $\forall\, n \geq N_0$

$$\left| \frac{x_n}{y_n} - \frac{x}{y} \right| < \frac{2}{|y|} \frac{|y|}{4}\varepsilon + \frac{2|x|}{|y|^2} \frac{|y|^2\varepsilon}{4(|x|+1)}$$

$$= \varepsilon/2 + \frac{\varepsilon}{2} \frac{|x|}{|x|+1}$$

$$< \varepsilon/2 + \varepsilon/2 \quad \text{as } \frac{|x|}{|x|+1} < 1$$

$$= \varepsilon$$

$$\Rightarrow \lim_{n\to\infty} \frac{x_n}{y_n} = \frac{x}{y}$$

(v) is left for you.

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(VI) We have to show that

if $x_n \le y_n \ \forall n$ then $x \le y$.

We will show by contradiction.

Suppose $x \not\le y$.

Let $x > y$.

as $\lim_{n\to\infty} x_n = x$ & $\lim_{n\to\infty} y_n = y$.

Then for every $\varepsilon > 0 \ \exists \ N_1, N_2 \in \mathbb{N}$ such than

$x_n \in (x-\varepsilon, x+\varepsilon) \quad \forall n \ge N_1$

$y_n \in (y-\varepsilon, y+\varepsilon) \quad \forall n \ge N_2$

Let $N = \max\{N_1, N_2\}$

Then $x_n \in (x-\varepsilon, x+\varepsilon) \quad \forall n \ge N$

$y_n \in (y-\varepsilon, y+\varepsilon)$

$\Rightarrow \quad x_n > y_n \quad \forall n \ge N$

But this is false as we are given

$x_n \le y_n \ \forall n$.

Which is a contradiction.

Hence the proof.

Note:- Even if $x_n < y_n \ \forall n$ we can't say that $x < y$. [$x_n < y_n$ does not imply $x < y$]

as an example, $\{x_n\} = \{-1/n^2\}$

$\{y_n\} = 1/n$

$x_n < y_n$

but $x = 0 = y$

$\Rightarrow x \not< y$.

$\{x_n\} \quad \{y_n\}$

$\ \ \| \qquad\quad \|$

$\{0\} \quad \{1/n\}$

$x_n < y_n$ but $x \not< y$

Examples

These are the theorems that can be used very often.

We have seen that the limit of $\{1/n\}$ & $\{c\}$ ~~const~~ converge to 0 & c.

$\{c\}$ = constant sequence

But using these theorems one can decide convergence or divergence of various sequences and also decide what their limits are.

Let $x_n = \frac{3n+5}{7n-1} = \frac{3+5/n}{7-1/n} \to \frac{3+0}{7-0}$

$\to 3/7$ as $n \to \infty$

For the following ~~exercises~~ using the above theorems at first find the limit and then apply the def^n to ~~check~~ verify

(1) $\lim\limits_{n\to\infty} \frac{3n+5}{7n-1} = ?$

Sol^n:- $= \lim\limits_{n\to\infty} \frac{3+5/n}{7-1/n} = \frac{\lim\limits_{n\to\infty}(3+5/n)}{\lim\limits_{n\to\infty}(7-1/n)}$ $\left[\because \lim\limits_{n\to\infty} \frac{x_n}{y_n} = \frac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n}\right.$

$= \frac{3+0}{7-0}$ $\left[\because \lim\limits_{n\to\infty}(x_n \pm y_n) = \lim\limits_{n\to\infty} x_n \pm \lim\limits_{n\to\infty} y_n\right.$

$= 3/7$

So, to show $\lim\limits_{n\to\infty} \frac{3n+5}{7n-1} = 3/7$

~~we have~~ for any $\varepsilon > 0$, we have to find a natural number n_0 s.t.

~~$\forall n \geq n_0, \left|\frac{3n+5}{7n-1}\right| = 3/7 \quad \forall n$~~

$$\forall n \geq n_0, \quad \left|\frac{3n+5}{7n-1} - 3/7\right| < \varepsilon$$

$$\Leftrightarrow \left|\frac{(3n+5)7 - 3(7n-1)}{7(7n-1)}\right| < \varepsilon$$

$$\Leftrightarrow \left|\frac{38}{7(7n-1)}\right| < \varepsilon$$

$$\Leftrightarrow \left|\frac{38}{49(n-1/7)}\right| < \varepsilon \qquad \text{as } n \geq 1,\ n - 1/7 \geq 0.$$

$$\Leftrightarrow (n - 1/7) > \frac{38}{49\varepsilon}$$

$$\Leftrightarrow n > \frac{38}{49\varepsilon} + 1/7.$$

So, take $n_0 = \left[1/7 + \frac{38}{49\varepsilon}\right] + 1$

Therefore $\forall n \geq n_0, \left|\frac{3n+5}{7n-1} - 3/7\right| < \varepsilon$.

Find the limits using the theorems and then verify it by using definition —

(1) $\lim_{n\to}$ ~~Simil~~

(2) $\lim_{n\to\infty} \frac{2n+1}{n+1}$ (3) $\lim_{n\to\infty} \frac{n+1}{2n}$

(4) $\lim_{n\to\infty} \frac{3n-1}{n+2}$ (5) $\lim_{n\to\infty} \frac{2n+5}{6n-11}$

(6) $\lim_{n\to\infty} \frac{3n}{n+5\sqrt{n}}$ (7) $\lim_{n\to\infty} \frac{n^2+n+1}{3n^2+1}$

(8) $\lim_{n\to\infty} \frac{n^2-1}{n^2+n+1}$ (9) $\lim_{n\to\infty} \frac{2n^2+1}{n^2+3n}$

(10) $\lim_{n\to\infty} \frac{n^2+n+1}{n^2-5n-3}$ (11) $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})$.

(12) $\lim_{n\to\infty} \frac{n^2+3n}{2n^2+n-1}$ (13) $\lim_{n\to\infty} \frac{5n^2+2}{6n^2+4n+7}$

(13) $\lim_{n\to\infty}$

(8) $\lim_{n\to\infty} \frac{n^2-1}{n^2+n+1}$

$= \lim_{n\to\infty} \frac{1-1/n^2}{1+1/n+1/n^2}$

$= \frac{1-0}{1+0+0} = 1$

Sol, Now,

$\left|\frac{n^2-1}{n^2+n+1} - 1\right| < \varepsilon$.

$\Leftrightarrow \left|\frac{\cancel{n^2}-1-\cancel{n^2}-n-1}{n^2+n+1}\right| < \varepsilon$

$\Rightarrow \left|\frac{-(n+2)}{n^2+n+1}\right| < \varepsilon$

$\Rightarrow \left\{\frac{n+2}{n^2+n+1}\right\} < \varepsilon$.

Now, for $\underline{n>2}$ $n+2 < n+n$
$\leq 2n$

$n^2+n+1 > n^2 \ \forall n$

i.e
So, $\frac{n+2}{n^2+n^2+1} < \frac{2n}{n^2} = 2/n < \varepsilon$

$\Rightarrow \underline{n > 2/\varepsilon}$

So, choose $n_0 = \max\{2, [2/\varepsilon]\}$

~~So,~~ Therefore $\forall n \geq n_0$

$\left|\frac{n^2-1}{n^2+n+1} - 1\right| < \varepsilon$

(9) $\lim\limits_{n\to\infty} \dfrac{2n^2+1}{n^2+3n}$

$= \lim\limits_{n\to\infty} \dfrac{2+1/n^2}{1+3/n} = 2$

$\left|\dfrac{2n^2+1}{n^2+3n} - 2\right| = \left|\dfrac{1-6n}{n^2+3n}\right| = \left|\dfrac{6(1/6-n)}{n^2+3n}\right|$ — (*)

$\cancel{|1/6-n| \le |1/6|+|n|} = n + 1/6$

$\forall\ n \ge 1$

$|1/6 - n| \le \frac{1}{6} + n \le n + n = 2n$

$n^2+3n > n^2$

$\Rightarrow \dfrac{1}{n^2+3n} < \dfrac{1}{n^2}$

So, $\left|\dfrac{2n^2+1}{n^2+3n} - 2\right| \le \dfrac{2n}{n^2} = \dfrac{2}{n} < \varepsilon$

$\Rightarrow n > 2/\varepsilon$

Let us take $n_0 = [2/\varepsilon] + 1$

$\exists\ n_0$ s.t.

So, $\forall\ n \ge n_0$, $\left|\dfrac{2n^2+1}{n^2+3n} - 2\right| < \varepsilon$.

(10) $\lim\limits_{n\to\infty} \dfrac{n^2+n+1}{n^2-5n-3}$.

$= \lim\limits_{n\to\infty} \dfrac{1+1/n+1/n^2}{1-5/n-3/n^2} = 1$

$\left|\dfrac{n^2+n+1}{n^2-5n-3} - 1\right| = \left|\dfrac{6n+4}{n^2-5n-3}\right| = \left|\dfrac{6(n+4/6)}{n^2-5n-3}\right|$ — (1)

$\forall\ n \ge 1$, $n + 4/6 = n + \frac{2}{3} < n + n = 2n$ — (2)

$|n^2-5n-3| \ge n^2 - (5n+3)$ $[\because |a-b| \ge |a| - |b|]$

$\ge n^2 - \frac{1}{2}n^2$ if $5n+3 < \frac{n^2}{2}$

$= \frac{1}{2}n^2$ if "

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$\cancel{n+3 < \frac{n^2}{2}}$

$\cancel{\Rightarrow n^2 - 6n - 6 > 0}$

$\cancel{\Rightarrow n^2 - 2.3.n + 9 - 6 - 9 > 0}$

$\cancel{\Rightarrow (n-3)^2 - 15 > 0}$

$\cancel{\Rightarrow (n-3)^2 > 15}$

$\cancel{n^2} > \quad 5n+3 < \frac{n^2}{2}$

$\Rightarrow 10n + 6 < n^2$

$\Rightarrow n^2 - 10n - 6 > 0$

$\Rightarrow (n^2 - 2.5.n + 25) - 6 - 25 > 0$

$\Rightarrow (n-5)^2 - 31 > 0$

$\Rightarrow (n-5) > \pm\sqrt{31}$

$\Rightarrow n > 5 \pm \sqrt{31}$

$\Rightarrow n > 5 + \sqrt{31}$ $\left[\because 5 - \sqrt{31} < 0 \text{ \& our } n \geq 1\right]$

So, we take $n_0 = [5+\sqrt{31}] + 1$

So, for $n \geq n_0$

$|n^2 - 5n - 3| \geq \frac{n^2}{2}$

$\Rightarrow \frac{1}{|n^2 - 5n - 3|} \leq \frac{2}{n^2}$ — (3)

So from (1) applying (2) & (3) we get

$\left|\frac{n^2+n+1}{n^2-5n-3} - 1\right| \leq \frac{2n \times 2}{n^2} = \frac{4}{n} < \varepsilon$ whenever $\forall n \geq n_0$

$\Rightarrow n > 4/\varepsilon$

So, we take

$N = \max\{[5+\sqrt{31}]+1, [4/\varepsilon]+1\}$

~~So,~~ Therefore $\forall n \geq N$

$\left|\frac{n^2+n+1}{n^2-5n-3} - 1\right| < \varepsilon$.

(11) $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})$

$= \lim_{n\to\infty} \left(\frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} \right)$

$= \lim_{n\to\infty} \left[\frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} \right] = \lim_{n\to\infty} \frac{1}{\sqrt{n}(1+\sqrt{1+1/n})}$

$= 0$

Now, $|\sqrt{n+1} - \sqrt{n} - 0|$

$= \left| \frac{1}{\sqrt{n} + \sqrt{n+1}} \right| < \frac{1}{2\sqrt{n}} \quad [\because \sqrt{n+1} > \sqrt{n}]$

$< \varepsilon$

$\Rightarrow n > \frac{1}{4\varepsilon^2}$

So, we take $n_0 = \left[\frac{1}{4\varepsilon^2}\right] + 1$

(14) $\lim_{n\to\infty} \frac{6n^3+2n+1}{n^3+n^2}$

(15) $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})\sqrt{n+1/2}$

(16) $\lim_{n\to\infty} (\sqrt[3]{n+1} - \sqrt[3]{n})$

(17) $\lim_{n\to\infty} (\sqrt{n^2+n} - n)$

(18) $\lim_{n\to\infty} \frac{3+2\sqrt{n+1}}{\sqrt{n+1}}$

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