Avarage value of emf or current in A.C Circuit :-

The avarage value of alternating emf or current over one complete cycle is zero, because there are equal negative or positive half cycle. So average value of alternating emf or current over half cycle is taken therefore

$$E_{ave} = \frac{\int_0^{T/2} E_0 \sin\omega t\,dt}{\int_0^{T/2} dt}$$

$$= \frac{2}{T} E_0 \int_0^{T/2} \sin\omega t\,dt$$

$$= \frac{2E_0}{T\omega}\left[-\cos\omega t\right]_0^{T/2}$$

$$= \frac{2E_0}{T\omega}\left[-\cos \omega T/2 + \cos 0\right]$$

$$= \frac{2E_0}{T\omega}\cdot 2$$

$$= \frac{2E_0}{\pi} = \frac{2}{\pi}\cdot \text{peak value.}$$

Similarly average value of the current passing through R over half cycle is ~~Emon~~ $I_{ave} = \frac{2I_0}{\pi}$

$$= \frac{2}{\pi}\cdot \text{peak value.}$$

R.M.S value or effective value of virtual value of emf :-

We know the instantenious value of alternating emf is

$$E = E_0 \sin\omega t$$

$$E^2 = E_0^2 \sin^2\omega t$$

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$$\overline{E^2} = \frac{\int_0^T E_0^2 \sin^2\omega t\,dt}{\int_0^T dt}$$

$$= \frac{E_0^2}{2T}\int_0^T 2\sin^2\omega t\,dt$$

$$\frac{E_0^2}{\ } \int^T (1-\cos 2\omega t)\,dt$$

$$= \frac{E_0^2}{2T}[T-0]$$

$$= \frac{E_0^2}{2}$$

Therefore root mean square value of emf

$$E_{R.M.S} = \sqrt{\overline{E^2}}$$

$$= \frac{E_0}{\sqrt{2}}$$

Similarly RMS or virtual value of current through R

$$I_{RMS} = \frac{I_0}{\sqrt{2}}$$

Therefore virtual current or RMS current which is defined as that steady current which passing through a particular resistance produce heat at the same rate as that produce by an ~~a~~ alternating current flowing through the same resistance.

Form Factor

The ratio of the RMS value to the average value over a half cycle of a periodic function is defined to be the form factor of the periodic wave form.

For a sinusoidal current the Form Factor is given by

$$\text{Form Factor} = \frac{I_0}{\sqrt{2}} \Big/ \frac{2I_0}{\pi}$$

$$= \pi/2\sqrt{2} = 1.11$$

Impedence

Alternating current in a circuit in controlled by resistance, inductance and capacitance while direct current is controlled by resistance only. In alternating current circuit the ratio of potential drop across the circuit and of the current passing through it is known as impedance. Physically the resistance offered by the alternating current circuit to flow of alternating current is term as impedance of the circuit. Generally the impedance is represented by Z. The impedance

$$Z = \frac{E_v}{I_v} = \frac{\text{Virtual Voltage}}{\text{Virtual current}}.$$

Reactance:-

Resistance offered by the inductance and capacitance to the ~~flow of~~ alternating current in an alternating ~~current~~ circuit is known as Reactance. The Reactance produce due to the inductance is known as inductive Reactance and that produce due to the capacitance is known as capacitive Reactance. They are represented by X_L and X_C respective by.

$$\text{The Reactance} = \frac{\text{Aplied virtual emf}}{\text{Virtual current through inductance and capacitance.}}$$

Admittance:-

The inverse of the impedance of an alternating current circuit is known as admittance of the circuit and it is represented by Y

$$Y = 1/Z$$

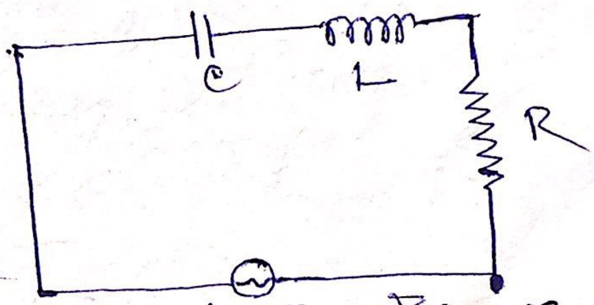
Susceptance:-

The inverse of the Reactance of an alternating current circuit is known as susceptance of the circuit.

$$\text{Susceptance} = \frac{1}{\text{Reactance}}$$

L-C-R circuit :-

Let an alternating emf E(t) is applied to a circuit containing a capacitor of capacitance, a pure resistance R and a coil of inductance L in series.



Let i be the instantaneous current when q is the charge store in the condenser, then the instantaneous voltage across R, L and C are respectively iR, $L\,di/dt$, q/c.

Now applying ~~KVL~~ KVL we get

$$E(t) = Ri + L\,di/dt + q/c$$

or, $Ri + L\,di/dt + q/c = E(t)$ --- (1)

if $E(t) = E_0 \cos \omega t$

Suppose, i_r be the corresponding solution for the current.

and $E(t) = E_0 \sin \omega t$ then i_i be the solution for the current and the complex current

$$i = i_r + j i_i$$

and the complex emf $E(t) = E_0 e^{j\omega t}$

Therefore equation (1) takes form.

$$Ri + L\,di/dt + q/c = E_0 e^{j\omega t}$$

Differenciating with respect to time t

$$L\,d^2i/dt^2 + R\,di/dt + 1/c\, i = j\omega e^{j\omega t} E_0 \quad \left[\because \frac{dq}{dt} = i\right]$$

(2)

Let at the steady state

The solution of these equation is given by

$$i = Ae^{j\omega t}$$

$$di/dt = Aj\omega e^{j\omega t}$$

$$d^2i/dt^2 = A(j\omega)^2 e^{j\omega t} = -A\omega^2 e^{j\omega t}$$

Now putting these value in equation (2)

we get

$$-LA\omega^2 e^{j\omega t} + RAj\omega e^{j\omega t} + 1/c\, A e^{j\omega t} = j\omega e^{j\omega t} E_0$$

$$\Rightarrow -LA\omega^2 + RAj\omega + 1/c\, A = j\omega E_0$$

$$\therefore A = \frac{E_0 j\omega}{-L\omega^2 + Rj\omega + 1/c}$$

$$= \frac{E_0}{R + j\omega L + \frac{1}{j\omega c}}$$

$$= \frac{E_0}{R + j\omega L - j/c\omega}$$

$$= \frac{E_0}{R + j(\omega L - 1/C\omega)}$$

Therefore $i = \frac{E_0 e^{j\omega t}}{R + j(L\omega - 1/C\omega)}$

$$= \frac{E_0 e^{j\omega t}}{Z} \quad \text{--- (3)}$$

Where $Z = R + j(L\omega - 1/C\omega)$ is called of the impedence of the circuit.

Let us put $R = |Z|\cos\phi$

$L\omega - 1/C\omega = |Z|\sin\phi$.

$$|Z| = \sqrt{R^2 + (L\omega - 1/C\omega)^2}$$

$$\tan\phi = \frac{L\omega - 1/C\omega}{R}$$

Therefore $Z = |Z|\cos\phi + j|Z|\sin\phi$

$$= |Z| e^{j\phi}$$

$$\therefore i = \frac{E_0 e^{j\omega t}}{|Z| e^{j\phi}}$$

$$= \frac{E_0}{|Z|} e^{j(\omega t - \phi)}$$

$$= \frac{E_0 e^{j(\omega t - \phi)}}{\sqrt{R^2 + (L\omega - 1/C\omega)^2}} \quad \text{--- (4)}$$

If $E(t) = E_0 \cos\omega t$

$=$ real part of $E_0 e^{j\omega t}$

The physical ~~notat~~ notation of current is $i_r(t) = \frac{E_0}{|Z|} \cos(\omega t - \phi)$ --- (5)

Now if $E(t) = E_0 \sin\omega t$ = imaginary of $E_0 e^{j\omega t}$

Then the corresponding physical notation for the current is

$$i_i(t) = \frac{E_0}{|Z|} \sin(\omega t - \phi) \quad \text{--- (6)}$$

The phasor diagram are shown in the figure for the three possible cases

Case-I

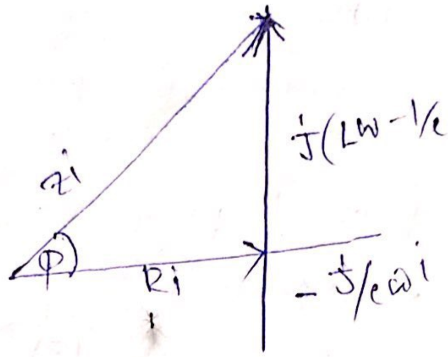
$X_L > X_C$ ϕ is positive

so that the current ~~lag~~ lags behind the applied emf.

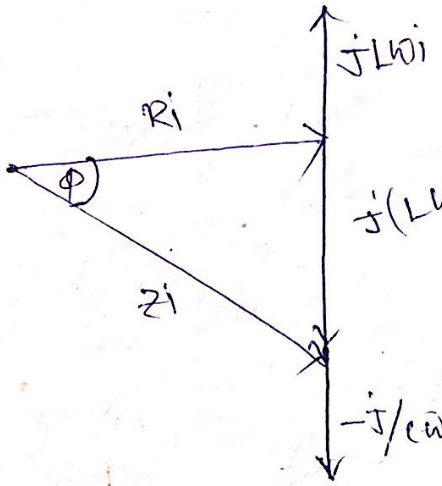
Case —(ii)
$X_L < X_C$ ϕ is negative. so that the current leads the emf.

Case —(iii)
$X_L = X_C$. ϕ is zero. The current is in phase with the emf.

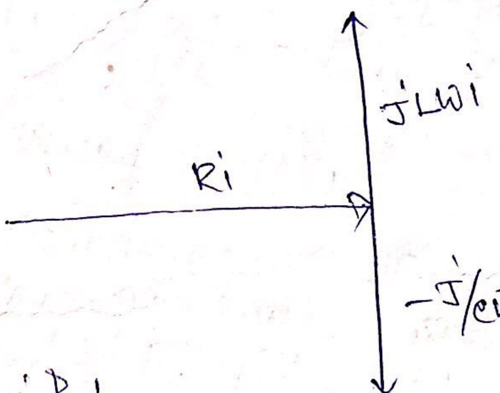
Case —(I)



Case —(II)



Case —(iii)



Power in AC circuit :-

The power in an electrical circuit is the rate at which electrical energy is consumed in the circuit and is equal to the product of the voltage and current. Now in alternating current circuit the power absorbed equal to $\cancel{Ei = E_0 \sin\omega t}\, i$

$Ei = E_0 \sin\omega t \; i_0 \sin(\omega t - \phi)$

$= E_0 i_0 \sin\omega t \sin(\omega t - \phi)$

$= E_0 i_0 \sin\omega t [\sin\omega t \cos\phi - \cos\omega t \sin\phi]$

$= E_0 i_0 [\sin^2\omega t \cos\phi - \sin\omega t \cos\omega t \sin\phi]$

The average power absorbed in circuit

$$P = i_0 E_0 \cos\phi \frac{\int_0^T \sin^2\omega t \, dt}{\int_0^T dt} - \frac{1}{2} i_0 E_0 \sin\phi \frac{\int_0^T 2\sin\omega t \cos\omega t \, dt}{\int_0^T dt}$$

$$= \frac{i_0 E_0 \cos\phi}{2T} \int_0^T (1 - \cos 2\omega t) \, dt - \frac{i_0 E_0 \sin\phi}{2T} \int_0^T \sin 2\omega t \, dt$$

$$= \frac{i_0 E_0}{2T} \cos\phi [T - 0] - \frac{i_0 E_0 \sin\phi}{2T} \cdot 0$$

$$= \frac{1}{2} i_0 E_0 \cos\phi = \frac{i_0}{\sqrt{2}} \cdot \frac{E_0}{\sqrt{2}} \cos\phi$$

$$= i_{r.m.s} \cdot E_{r.m.s} \cos\phi$$

Therefore average power

= Virtual volt. Virtual amp. $\cos\phi$

This extra term $\cos\phi$ is known as power factor.

From above expression of instantaneous power, it is seen that the average value of second term is zero. So this part of power that is $\frac{E_0 i_0}{2} \sin 2\omega t \cdot \sin\phi$ is called wattless or ideal component.

The first part of the power that is $\frac{1}{2} E_0 i_0 \cos\phi$ is ~~called~~ called the in phase component of the circuit non ohmic resistance i.e. no

$R = 0$ $\therefore$ $\cos\phi = R/|Z| = 0 = \cos \pi/2$

$\therefore \phi = \pi/2$

The first part of the power will be zero and the circuit will be entirely wattless.

Now $P = i_{r.m.s} \cdot E_{r.m.s} \cos\phi$

$$= i_{r.m.s} \cdot E_{r.m.s} \cdot R/|Z|$$

$$= i_{r.m.s} \cdot \frac{E_{r.m.s}}{|Z|} \cdot R$$

$$= i_{r.m.s} \cdot i_{r.m.s} \cdot R$$

$$= (i_{r.m.s})^2 \cdot R$$

This formula involves R only. So that the mean power in the refractive part of the circuit is zero when R = 0.

It shows that an inductor or capacitor or both in an AC circuit are store house of energy and the power consumed is due to ohmic resistance only.

Resonance:- In L-C-R series circuit has a very large inductive reactance at high frequencies and a very large capacitive reactance at low frequencies. At some particular frequency the total reactance in the circuit is zero and the resultant impedance is a ~~maximum~~ minimum. Hence at this frequency the series circuit allows the maximum current to flow through it. Then this phenomena is known as electrical series Resonance.

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