7.2. Faraday's Laws of Electromagnetic Induction

The experimental observations of electromagnetic induction have been summed up in the form of two laws inunciated below which are known as Faraday's laws of electromagnetic induction.

1. *Whenever the magnetic flux linked with a closed circuit changes, an induced e.m.f. is set up in the circuit whose magnitude at any instant is proportional to the rate of change of magnetic flux linked with the circuit.*

If e is the magnetic flux linked with the circuit at any instant t and e is the induced e.m.f., then

$$e \propto \frac{d\phi}{dt}$$

2. *The direction of induced e.m.f. is such that it opposes the change in flux that produces it.* This law is known as **Lenz's law,** because althrough the direction of induced e.m.f. was determined by Faraday, but it was expressed in a law by Lenz.

These laws can be mathematically expressed as

$$e = -\frac{d\phi}{dt} \quad \text{...(1)}$$

where e is the e.m.f. induced in the circuit in volts and ϕ is the instantaneous flux linked with the circuit in webers.

7.3. Faraday's Laws From Lorentz Force.

Consider a circuit shown in fig. 7.5. *ab* and *cd* are parallel horizontal conducting rails and *pq* is a conducting rod of length *l*, which can slide without friction on the rails. A uniform magnetic field of induction *B* exists at the locality of rails. The field *B* is perpendicular to plane of paper and downward.

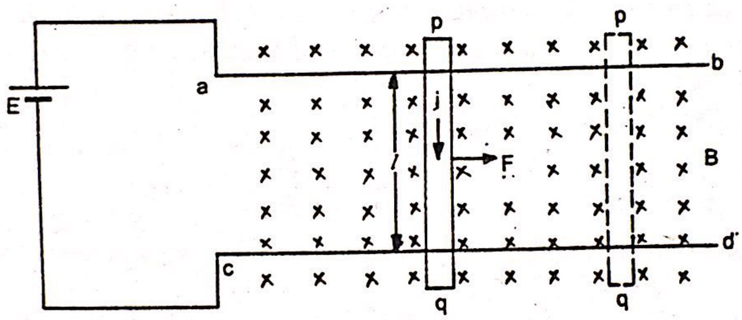

Fig. 7.5

A current *i* amp is allowed to flow throught the rod *pq*.

The force acting on the rod, $\mathbf{F} = i\,\vec{l} \times \mathbf{B}$.

The magnitude of force is *Bil* and its direction is as shown in fig. 7.5.

Due to this force let dx be the displacement of rod in time dt. The work done for this displacement,

$$dW = F\,dx$$
$$= Bil\,dx.$$

If E is the e.m.f. of battery, the energy given by battery in time dt is $E\,i\,dt$.

This energy is used up in two parts :

(i) A part is used overcoming the resistance R of circuit *i.e.*

$$i^2 R\,dt.$$

(ii) The remaining part is used as work for displacement of rod.

∴ Therefore conservation theorem for energy gives

$$E\,i\,dt = i^2 R\,dt + Bil\,dx$$

i.e.

$$i^2 R\,dt = E\,i\,dt - Bil\,dx.$$

Concelling the common factor i (since $i \neq 0$)

$$i\,R\,dt = E\,dt - Bl\,dx$$

This gives

$$i = \frac{E - Bl\frac{dx}{dt}}{R}. \qquad ...(1)$$

In this eqution $Bl\,\frac{dx}{dt}$ must have dimensions of e.m.f.

But $l\,dx$ (= dA) is the area swept by rod pq is time dt. Therefore

$$Bl\,\frac{dx}{dt} = B.\frac{dA}{dt}.$$

As B is constant, we have

$$Bl\,\frac{dx}{dt} = \frac{d}{dt}(BA) = \frac{d\phi}{dt}.$$

Where $d\phi = d\,(BA)$ is the change in magnetic flux in time dt. Therefore

$$\frac{d\phi}{dt} = Bl\,\frac{dx}{dt}$$

gives the rate of change of flux.

From equation (1), the e.m.f. induced in the circuit due to rate of change of flux,

$$e = -\frac{d\phi}{dt} = -Bl\,\frac{dx}{dt}$$
$$= -Blv.$$

The negative sign shows that the e.m.f. induced in the circuit opposes the applied e.m.f. E. The property of induced e.m.f. is same as the force of friction in mechanical motion.

10.5 Self-Inductance

When a current flows in a circuit, the magnetic flux produced by the current depends on the geometry of the circuit and, for nonferromagnetic materials, it is proportional to the current. Thus if I is the current flowing in a circuit the attendant magnetic flux can be written as

$$\Phi = LI, \tag{10.14}$$

where L is the *self-inductance*, or simply, the inductance of the circuit. Equation (10.14) defines the self-inductance as *the total flux linked with the circuit for unit current flowing in the circuit.*

For a rigid stationary circuit, the changes in the flux is caused by changes in the current. Thus

$$\frac{d\Phi}{dt} = \frac{d\Phi}{dI}\frac{dI}{dt}, \tag{10.15}$$

which holds even when Φ is not linearly related to I, as in the case of ferromagnetic materials where the permeability depends on the current. The induced e.m.f. in the circuit is given by

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}, \tag{10.16}$$

where $$L = \frac{d\Phi}{dI}. \tag{10.17}$$

Equation (10.16) defines the self-inductance L as *the e.m.f. induced in the circuit for a unit rate of change of current in it.* If Φ is not linearly related to I, the inductance defined by Eq.(10.17) is termed the *incremental inductance* to distinguish it from the inductance defined by Eq.(10.14). However, when Φ is proportional to I, the two definitions lead to the same result, since $\Phi/I = d\Phi/dI$ in that case.

The negative sign in Eq.(10.16) indicates that the induced e.m.f. opposes the flow of current. Thus *the self-inductance measures the ability of a circuit to oppose the variation of current in it.*

In SI units, $\Phi, I, t, \mathcal{E}$ and L are expressed in weber, ampere, second, volt, and henry, respectively.

$$1 \text{ henry} = \frac{1 \text{ weber}}{1 \text{ ampere}} = \frac{1 \text{ volt}\cdot\text{second}}{1 \text{ ampere}} = 1 \text{ V}\cdot\text{A}^{-1}\cdot\text{s},$$

As Φ has the dimensions $[ML^2T^{-2}I^{-1}]$, the dimensions of L are $[ML^2T^{-2}I^{-2}]$.

10.7 Mutual Inductance

Suppose that there are N number of circuits designated by $1, 2, \cdots, N$. When the circuits carry currents, the flux produced by the current in one circuit may link with another circuit. The flux Φ_i linked with the current i is therefore determined by adding up the contributions from each circuit. Thus

$$\begin{aligned} \Phi_i &= \Phi_{i1} + \Phi_{i2} + \cdots + \Phi_{ii} + \cdots + \Phi_{iN} \\ &= \sum_{j=1}^{N} \Phi_{ij}, \end{aligned} \tag{10.26}$$

where Φ_{ij} is the flux through the ith circuit due to the jth circuit. For non-ferromagnetic materials, Φ_{ij} is proportional to the current I_j in the jth circuit. Hence one can write

$$\Phi_{ij} = M_{ij} I_j \qquad (i \neq j), \tag{10.27}$$

where M_{ij} is referred to as the *mutual inductance* between the circuits i and j. *The mutual inductance between two circuits is therefore defined as the flux linked with one circuit due to unit current in the other.* Observe that Φ_{ii} is the self flux and Φ_{ii}/I_i is the self-inductance L_i of the ith circuit.

The induced *e.m.f.* in the *ith* circuit, $\mathcal{E}_i$, is given by

$$\mathcal{E}_i = -\frac{d\Phi_i}{dt} = -\sum_{j=1}^{N} \frac{d\Phi_{ij}}{dt}. \tag{10.28}$$

For rigid fixed circuits, the changes in Φ_{ij}'s are caused by changes in the currents. Thus

$$\frac{d\Phi_{ij}}{dt} = \frac{d\Phi_{ij}}{dI_j} \cdot \frac{dI_j}{dt} = M_{ij}\frac{dI_j}{dt}, \tag{10.29}$$

where $$M_{ij} = \frac{d\Phi_{ij}}{dI_j} \qquad (i \neq j). \tag{10.30}$$

Therefore the e.m.f. induced in the ith circuit due to the change of current in the jth circuit is

$$\mathcal{E}_{ij} = -M_{ij}\frac{dI_j}{dt}. \tag{10.31}$$

Thus the *mutual inductance between the two circuits may also be defined as the e.m.f. induced in one circuit due to the unit rate of change of current in the other.* The two definitions lead to the same result when the fluxes are linearly related to the currents.

We shall see below that $M_{ij} = M_{ji}$, so that there is no ambiguity in the subscripts. Note that $d\Phi_{ii}/dI$ is the self inductance L_i of the ith circuit. The unit of mutual inductance is the same as that of self-inductance, i.e., henry.