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Study Material - Physics / Sem. 2 / Diffraction - Problems / Class 6 / Dr. T. Kar

Zone Plate & Half - Period Zone

1. Calculate the inner and outer radii of the 10 th. half period zone for a plane wavefront with respect to a point at a distance of 50 cm from the wavefront. Make the calculations for the light of wavelength 5000 Å and also for microwave of wavelength 3 cm.

Ans. For the m th. half period zone, the outer radius is $r_m = \sqrt{\left(b+\frac{m\lambda}{2}\right)^2 - b^2}$

$$= \sqrt{b^2 + bm\lambda + \frac{m^2\lambda^2}{4} - b^2}$$

$$= \sqrt{bm\lambda} \quad \text{as } b >> \lambda$$

and the inner radius is $r_{m-1} = \sqrt{\left(b + \overline{m-1}\frac{\lambda}{2}\right)^2 - b^2}$

$$= \sqrt{(m-1)\, b\lambda} \quad \text{as } b >> \lambda$$

Now, $b = 50$ cm, $m = 10$, $\lambda = 5000$ Å & 3 cm

Then $r_{10} = ?$, $r_9 = ?$

2. Consider a circular aperture of radius 0·01 cm illuminated by a plane wave of wavelength 6×10^{-5} cm. Calculate the positions of the brightest and the darkest points on the axis.

Ans. If the distance 'p_1' of the axial point be such that the aperture exposes only the first half period zone, we shall get the brightest point.

$$\therefore p_1^2 + r^2 = \left(p_1 + \frac{\lambda}{2}\right)^2$$

$$\text{or, } p_1^2 + r^2 = p_1^2 + p_1\lambda + \frac{\lambda^2}{4} \simeq p_1^2 + p_1\lambda$$

$$\text{or, } p_1 = \frac{r^2}{\lambda} = ? \quad \text{where, } r_1 = 0{\cdot}01 \text{ cm}, \ \lambda = 6\times10^{-5} \text{ cm}$$

for the darkest point on the axis, the distance 'p_2' be such that the aperture exposes only two half period zones.

$$\therefore p_2^2 + r^2 = \left(p_2 + \frac{2\lambda}{2}\right)^2 = (p_2+\lambda)^2$$

$$= p_2^2 + 2p_2\lambda + \lambda^2 \simeq p_2^2 + 2p_2\lambda$$

$$\therefore p_2 = \frac{r^2}{2\lambda} = ? \quad \text{where, } r_1 = 0{\cdot}01 \text{ cm}, \ \lambda = 6\times10^{-5} \text{ cm}$$

3. A zone plate has the diameter of the first ring 1 mm. If plane waves ($\lambda = 5000$ Å) fall on the plate, where should the screen be placed so that the light is focused to a brightest spot.

Ans. The focal length of the zone plate is given by, $f = \frac{r_n^2}{n\lambda}$

Given: radius of the first ring $= r_1 = \frac{1mm}{2}$
$= 0.5mm$

$n = 1$ & $\lambda = 5000$ Å.

$\therefore f = ?$

4. A zone plate is made by arranging that the radii of the circles which define the zones are the same as the radii of dark Newton's rings formed between a plane surface and the surface having radius of curvature 2m. Find the principal focal length of the zone plate.

Ans. In Newton's ring experiment, the diameter of n th dark ring is —
$D_n^2 = 4n\lambda R$, $R \rightarrow$ radius of curvature of the curved surface.

If r_n be the radius of nth. dark ring, Then –

$r_n^2 = n\lambda R$

The principal focal length of the zone plate is –

$f = \frac{r_n^2}{n\lambda}$ where $r_n \rightarrow$ radius of n th zone

According to the problem, radii of the circles which define the zones are the same as the radii of dark Newton's rings, Therefore,

$$f = \frac{r_n^2}{n\lambda} = \frac{n\lambda R}{n\lambda} = R = 2m$$

Single Slit

1. Light of wavelength 5500 Å falls normally on a slit of width 2.2×10^{-4} cm. Calculate the angular position of the first two minima on either side of the central maximum.

Ans. $a\sin\theta = m\lambda$ or, $\sin\theta = m\left(\frac{\lambda}{a}\right)$

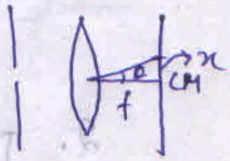
Given: $a = 2.2 \times 10^{-4}$ cm, $\lambda = 5500$ Å, $m = 1$ & 2.

$\sin\theta_1 = \frac{\lambda}{a}$; $\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = ?$

$\sin\theta_2 = \frac{2\lambda}{a}$ or, $\theta_2 = \sin^{-1}\left(\frac{2\lambda}{a}\right) = ?$

2. The Fraunhofer diffraction pattern of a single slit is observed in the focal plane of a lens of focal length 100 cm. The width of the slit is 0·04 cm. The incident light contains two wavelengths λ_1 and λ_2. It is found that the fourth minima corresponding to λ_1 and the 5 th. minima corresponding to λ_2 occur at the same point, 0·5 cm from the central maximum. Find the values of λ_1 and λ_2.

Ans. Given: $a = 0{\cdot}04$ cm, $f = 100$ cm, $x = 0 \cdot 5$ cm



$a \sin\theta \simeq a\,\theta = m_1\lambda_1 = m_2\lambda_2$

; $\frac{ax}{f} = 4\lambda_1 = 5\lambda_2$

$\tan\theta = \frac{x}{f}$; $\theta = \frac{x}{f}$

$\therefore \lambda_1 = ?$, $\lambda_2 = ?$

Double Slit

1. Fraunhofer double slit diffraction pattern is observed in the focal plane of a lens of focal length 0·5m. The wavelength of incident light is 500 nm. The distance between two maxima adjacent to the maxima of zero order is 5 mm and the 4th. order maximum is missing. Find the

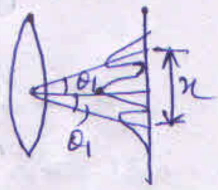
width of each slit and the width of the opaque space.

Ans. $\frac{a+b}{a} = 4$, or, $b = 3a$

For 1st order maxima,

$$(a+b)\sin\theta_1 = \lambda$$

or, $\sin\theta_1 = \frac{\lambda}{a+b}$



or, $\theta_1 \simeq \frac{\lambda}{a+b} = \frac{\lambda}{4a}$

$\therefore x = f(2\theta_1) = \frac{f\lambda}{2a}$ or, $a = \frac{f\lambda}{2x}$

$\therefore a = ?$ & $b = 3a = ?$

2. Deduce the missing orders for a double slit diffraction pattern, if the slit widths are 0.16 mm and they are 0.8 mm apart.

Ans. Given: $a = 0.16\,mm$, $b = 0.8\,mm$

Interference maxima: $(a+b)\sin\theta = n\lambda$

Diffraction minima: $a\sin\theta = p\lambda$

$\therefore \frac{a+b}{a} = \frac{n}{p} = 6$ or, $n = 6p$

For $p = 1, 2, 3 \cdots$ etc, $n = 6, 12, 18 \cdots$ etc.

$\therefore$ 6th, 12th, 18th etc. orders of interference maxima will be missing from the diffraction pattern.

Grating

1. How many orders would be visible, if the wavelength of incident light is 589 nm and the number of lines in the grating is 104 per mm?

Ans. For principal maxima,

$$(a+b)\sin\theta = m\lambda \quad \text{or,} \quad \sin\theta = pm\lambda$$

Here, $\theta = 90°$, $\lambda = 589 \times 10^{-9}\,m$, $p = 104\,/mm$

$$\therefore m = \frac{a+b}{\lambda} = ?$$

2. Sodium light of wavelengths 589nm and 589·6nm are made incident normally on a grating having 500 lines per mm. Calculate the angular separation of these lines in the spectrum of first order.

Ans. For principal maxima,

$$(a+b)\sin\theta = m\lambda \quad \text{or,} \quad \sin\theta = pm\lambda$$

Here, $m = 1$, $p = 500$ lines/mm

$\sin\theta_1 = \ldots$ or, $\theta_1 = ?$

$\sin\theta_2 = \ldots$ or, $\theta_2 = ?$

$\therefore$ Angular separation $= \theta_2 - \theta_1 = ?$