12.2 FRAUNHOFER DIFFRACTION IN A DOUBLE SLIT :

Let a parallel beam of light of monochromatic light of wavelength λ be incident normally on a surface containing two slits, each of width a and separated by an opaque space of width b. The distance between

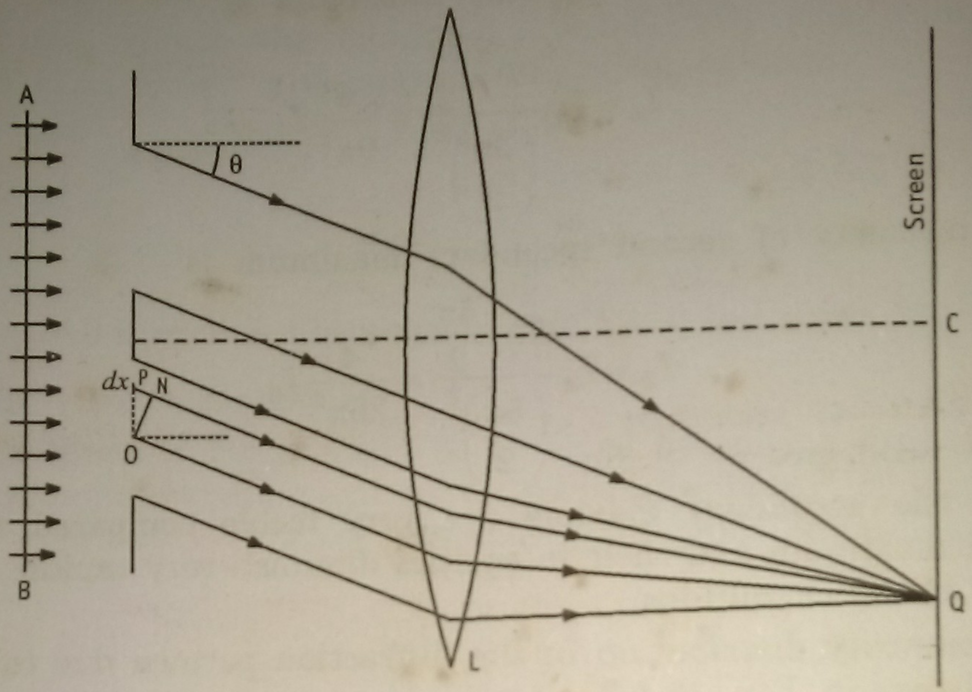


Fig. 12.2-1

any pair of corresponding points of the two slits is $d = a + b$. According to Huygens-Fresnel principle each point of the incident wavefront in the plane of the two slits may be regarded as the source of secondary spherical wavelets. The secondary wavelets travelling normal to the slits are brought to focus by a convex lens L on the screen at C. The wavelets travelling at an angle θ with the normal are brought to focus at Q. Let us calculate the intensity of light at Q. Let the complex light disturbance at any instant due to secondary waves from the origin O (mid point of the first slit) be represented by $Ae^{i\omega t}$ where A is the amplitude and ω is the circular frequency of the wave. The ‡phase difference between the waves at Q coming from O and from a point P at a distance x from O is given by

$$\frac{2\pi}{\lambda} \times PN = \frac{2\pi}{\lambda} \times x\sin\theta = lx; \quad \text{where } l = \frac{2\pi\sin\theta}{\lambda} \qquad \text{...(12.2-1)}$$

Hence the disturbance at Q due to secondary waves from P will be proportional to $e^{i(\omega t - lx)}$. The disturbance at Q due to the diffracting element dx can be written as

$$dy = CAdxe^{i(\omega t - lx)}$$

where we assume the amplitude to be proportional to the width dx; C is a proportionality constant.

‡ N.B. See footnote of page 330

The resultant complex disturbance at Q due to both the slits will be

$$y = \int_{-a/2}^{+a/2} CA.e^{i(\omega t - lx)}dx + \int_{d-a/2}^{d+a/2} CA.e^{i(\omega t - lx)}dx$$

$$= CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{-a/2}^{+a/2} + CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{d-a/2}^{d+a/2}$$

$$= CAa \cdot \frac{\sin\frac{al}{2}}{\frac{al}{2}}\left[1 + e^{-ild}\right].e^{i\omega t} \qquad ...(12.2\text{-}2)$$

The resultant intensity I at Q is obtained by multiplying y by its complex conjugate y^*. Thus

$$I = yy^* = (CAa)^2 \cdot \frac{\sin^2\frac{al}{2}}{\left(\frac{al}{2}\right)^2}\left(1 + e^{-ild}\right)\left(1 + e^{ild}\right)$$

$$= (CAa)^2 \cdot \frac{\sin^2\frac{al}{2}}{\left(\frac{al}{2}\right)^2}\left(2 + e^{ild} + e^{-ild}\right)$$

$$= (CAa)^2 \cdot \frac{\sin^2\frac{al}{2}}{\left(\frac{al}{2}\right)^2}(2 + 2\cos ld)$$

$$\left[\begin{array}{l} \because\ e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{array}\right]$$

$$= 4I_0\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\beta \qquad ...(12.2\text{-}3)$$

where $I_0 = (CAa)^2$, $\alpha = \frac{al}{2} = \frac{\pi}{\lambda}\cdot a\sin\theta$ and $\beta = \frac{ld}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta$

...(12.2-4)

The resultant intensity thus depends on two factors : (i) $I_0 \frac{\sin^2\alpha}{\alpha^2}$, which gives the diffraction pattern of a single slit and (ii) $\cos^2\beta$, which gives interference pattern of the diffracted light beams from the two slits.

Condition for minima :

The resultant intensity would be zero when either factor of Eq. (12.2-3) becomes zero. The factor $\frac{\sin^2\alpha}{\alpha^2} = 0$ when $\sin\alpha = 0$ but $\alpha \neq 0$. Therefore, $\alpha = m\pi$; $m = \pm 1, \pm 2, \ldots$

or, $$a \sin\theta = m\lambda \qquad \ldots(12.2\text{-}4)$$

These minima are known as *diffraction minima.*

The positions of minima due to the factor $\cos^2\beta$ are given by $\beta = \pm(2s+1)\pi/2$; $s = 0, 1, 2, 3\ldots$

or, $$(a+b)\sin\theta = \pm(2s+1)\lambda/2 \qquad \ldots(12.2\text{-}5)$$

These minima are known as the *interference minima.*

Condition of maxima :

The positions of maxima due to the factor $\frac{\sin^2\alpha}{\alpha^2}$ are at $\alpha = 0$ and at values approaching $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$ etc.

The positions of maxima due to the factor $\cos^2\beta$ are given by $\beta = p\pi$; $p = 0, \pm 1, \pm 2, \pm 3, \ldots$

or, $$(a+b)\sin\theta = p\lambda \qquad \ldots(12.2\text{-}6)$$

Missing order :

If the conditions for maxima of interference pattern (12.2-6) and minima of diffraction pattern (12.2-4) are simultaneously satisfied for a given value of θ then the corresponding interference maxima will be missing. If the slit width a is kept constant and separation b is varied then it is obvious from (12.2-6) the distance between consecutive interference maxima changes but the diffraction pattern due to single slit remains constant. Let for some value of θ conditions (12.2-4) and (12.2-6) are satisfied simultaneously. Then,

$$\frac{a+b}{a} = \frac{p}{m} \qquad \ldots(12.2\text{-}7)$$

Thus for missing order the ratio $\frac{a+b}{a}$ should be the ratio of two integers. For example, if $\frac{a+b}{a}=\frac{2}{1}$; or, $a=b$ then $p=2m$. Hence 2, 4, 6 etc. orders of interference maxima are absent which corresponds to 1, 2, 3 etc. orders of diffraction dark bands. There will be 3 interference maxima (corresponding to $p=0, \pm 1$) in the central diffraction maximum.

Similarly if $\frac{a+b}{a}=3$; or, $b=2a$ then $p=3m$. In this case 3, 6, 9 etc. orders of interference maxima are absent which corresponds to 1, 2, 3 etc. order of diffraction dark bands. There will be 5 interference maxima (corresponding to $p=0, \pm 1, \pm 2$) within the central diffraction maximum.

Complete double-slit pattern solved graphically :

In Fig. 12.2-2(a) $\cos^2\beta$ is plotted against β and in Fig. 12.2-2(b), $\sin^2\alpha/\alpha^2$ is plotted against α. The curves of Fig. 12.2-2(c) can be obtained by multiplying the ordinates of the curve (*a*) with those of the curve (*b*) and the contant $4I_0$. As the result depends on the relative values

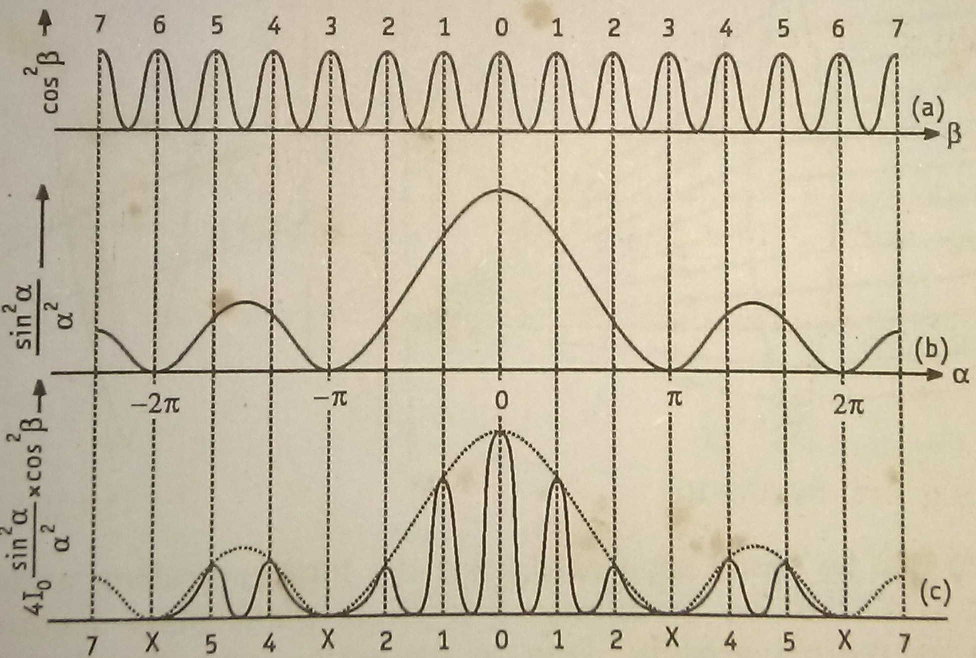
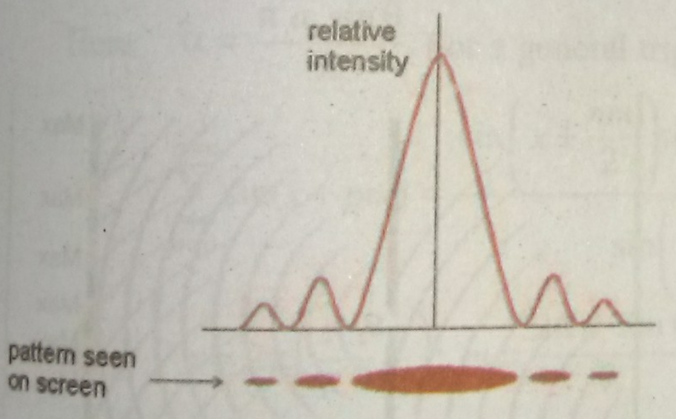


Fig. 12.2-2

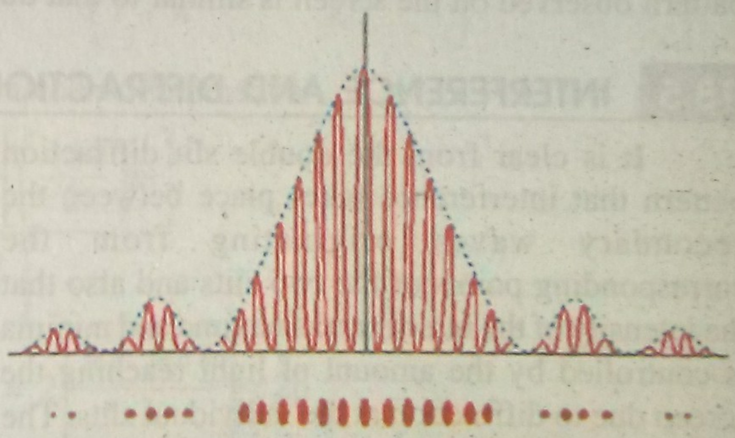
of β and α as well as on d and a as is given in Eq. (12.2-7) the curve (*c*) is drawn when $\beta=3\alpha$; or, $\frac{a+b}{a}=3$. The figure 12.2-2(c) shows that 3, 6, etc. orders of interference maxima are absent which are marked by X in the Fig. 12.2-2 (c).

18.4.2. DISTINCTION BETWEEN SINGLE SLIT AND DOUBLE SLIT DIFFRACTION PATTERNS

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima with in the central maximum. The intensity of the central maximum in diffraction pattern due to a double slit is four times that of the central maximum in the diffraction pattern due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with opaque screen, the pattern observed is similar to the one observed with a single slit.



Single slit diffraction patterns.


Double slit diffraction patterns

The spacing of diffraction maxima and minima depends on a, the width of the slit and the spacing of the interference maxima and minima depends on the value of a and b where b is opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

17.6. DISTINCTION BETWEEN INTERFERENCE AND DIFFRACTION

The main differences between interference and diffraction are as follows:

INTERFERENCE	DIFFRACTION
1. Interference is the result of interaction of light coming from different wave fronts originating from the source.	1. Diffraction is the result of interaction of light coming from different parts of the same wavefront.
2. Interference fringes may or may not be of the same width.	2. Diffraction fringes are not of the same width.
3. Regions of minimum intensity are perfectly dark.	3. Regions of minimum intensity are not perfectly dark.
4. All bright bands are of same intensity.	4. The different maxima are of varying intensities with maximum intensity for central maximum.