(1)

Study Material - Physics / Sem. 2 / Diffraction / Dr. T. Kar / Class 3

Different classes of Diffraction phenomena

1. Fresnel Diffraction phenomena →
Either the source or the point of observation or both are at finite distance from the diffraction obstacle or opening. Here the incident wavefront is divergent.

2. Fraunhofer Diffraction phenomena →
Both the source and the point of observation are effectively at infinite distance from the diffraction obstacle or opening. Here the incident wavefront is plane.

Fraunhofer Diffraction in a single slit

Let a plane wavefront AB of monochromatic light of wavelength λ propagating normal to the slit S_1S_2 (width 'a'). Each point on the wavefront, acts as a source of secondary wavelets. The secondary wavelets travelling normal to the slit is brought to focus by a convex lens L

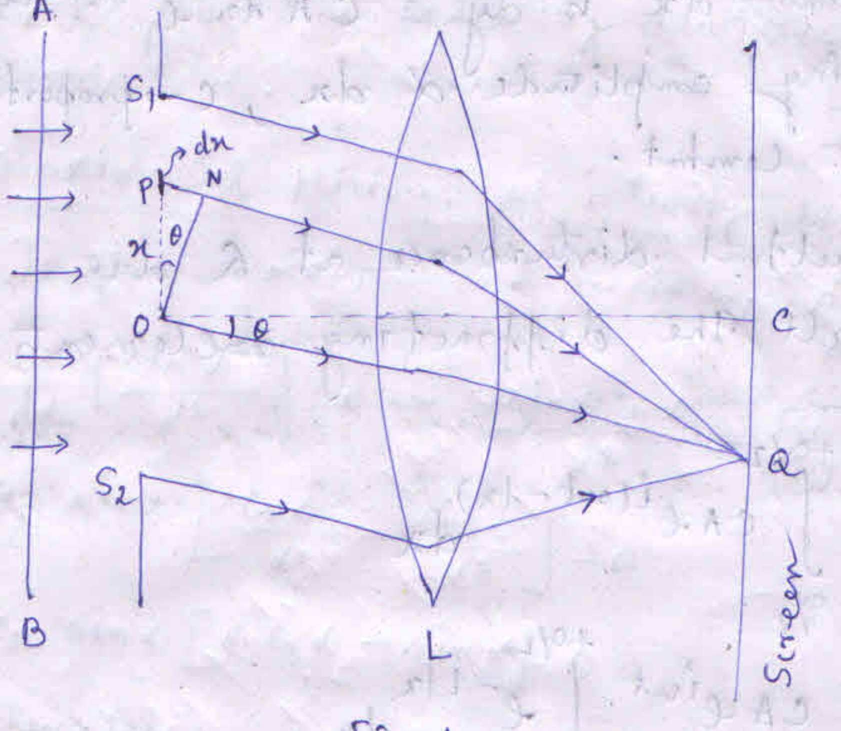

Fig. 1

on The screen at C. The wavelets travelling at an angle θ with The normal are brought to focus at Q. We have to calculate The intensity of light at Q. Let The complex light disturbance at any instant at O is given by $Ae^{i\omega t}$ where $A \rightarrow$ amplitude and $\omega \rightarrow$ circular frequency of wave.

Phase difference between waves at Q coming from The point 'O' and The point 'P' (which is at a distance 'x' from 'O'), is

$\frac{2\pi}{\lambda} PN = \frac{2\pi}{\lambda} x\sin\theta = lx$ where, $l = \frac{2\pi}{\lambda}\sin\theta$

$\therefore$ Disturbance at Q due to waves from P is proportional to $e^{i(\omega t - lx)}$.

$\therefore$ Disturbance at Q due to diffracting

(3)

element dx is $dy = CA\,dx\,e^{i(\omega t - lx)}$ assuming amplitude $\propto dx$, $C \to$ proportionality constant.

$\therefore$ Resultant disturbance at Q due to all the diffracting elements is —

$$y = \int_{-a/2}^{+a/2} CA e^{i(\omega t - lx)} dx$$

$$= CA e^{i\omega t} \int_{-a/2}^{+a/2} e^{-ilx} dx$$

$$= CA e^{i\omega t} \left[\frac{e^{-ilx}}{-il} \right]_{-a/2}^{+a/2}$$

$$= CA e^{i\omega t} \left[\frac{e^{-ila/2} - e^{ila/2}}{-il\left(\frac{2a}{2a}\right)} \right]$$

$$= CaA e^{i\omega t} \left[\frac{e^{ila/2} - e^{-ila/2}}{2i\,(la/2)} \right]$$

$$= CaA e^{i\omega t} \left[\frac{\sin(la/2)}{(la/2)} \right]$$

$\therefore$ Resultant intensity at Q is —

$$I = yy^* = (CaA)^2 \frac{\sin^2(la/2)}{(la/2)^2} = I_0 \frac{\sin^2\alpha}{\alpha^2} \to (1)$$

where, $I_0 = (CaA)^2$, $\alpha = \frac{la}{2} = \frac{2\pi}{\lambda}\sin\theta\left(\frac{a}{2}\right)$
$= \frac{\pi a}{\lambda}\sin\theta$

(4)

Eq.-(1) gives the intensity distribution in a single slit.

Maxima & Minima of intensity distribution

To find maxima and minima, $\frac{dI}{d\alpha} = 0$

$$\frac{d}{d\alpha}\left[\frac{\sin^2\alpha}{\alpha^2}\right] = 0 \quad \text{or,} \quad \frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3} = 0$$

$$\text{or, } \sin\alpha\left[\frac{\cos\alpha}{\alpha^2} - \frac{\sin\alpha}{\alpha^3}\right] = 0$$

$$\text{or, } \sin\alpha\left[\alpha\cos\alpha - \sin\alpha\right] = 0$$

Therefore, either $\boxed{\sin\alpha = 0}$ or $\boxed{\alpha = \tan\alpha}$

Now, $\frac{d^2}{d\alpha^2}\left[\frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3}\right]$

$$= \frac{2\cos^2\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^2} - \frac{4\sin\alpha\cos\alpha}{\alpha^3} - \frac{4\sin\alpha\cos\alpha}{\alpha^3} + \frac{6\sin^2\alpha}{\alpha^4}$$

$$= \frac{2\cos^2\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^2} - \frac{8\sin\alpha\cos\alpha}{\alpha^3} + \frac{6\sin^2\alpha}{\alpha^4}$$

If $\sin\alpha = 0 \Rightarrow \frac{d^2I}{d\alpha^2} = \frac{2\cos^2\alpha}{\alpha^2} = \frac{2}{\alpha^2} \Rightarrow +ve$
$\Rightarrow$ minima

if $\alpha = \tan\alpha \Rightarrow$ maxima

Minima: $\sin\alpha = 0 = \sin m\pi$

or, $\alpha = m\pi$ where $m = \pm 1, \pm 2, \pm 3 \ldots$ except zero.

$\therefore \frac{\pi a}{\lambda}\sin\theta = m\pi$ or, $\boxed{a\sin\theta = m\lambda}$

(5)

If $m = 0 \Rightarrow \alpha = 0 \Rightarrow \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$ which actually gives maxima.

Maxima:

The position of maxima can be obtained by solving the eq. $\alpha = \tan\alpha$ graphically. By plotting $y = \alpha$ and $y = \tan\alpha$ we have to find out the point of intersection.

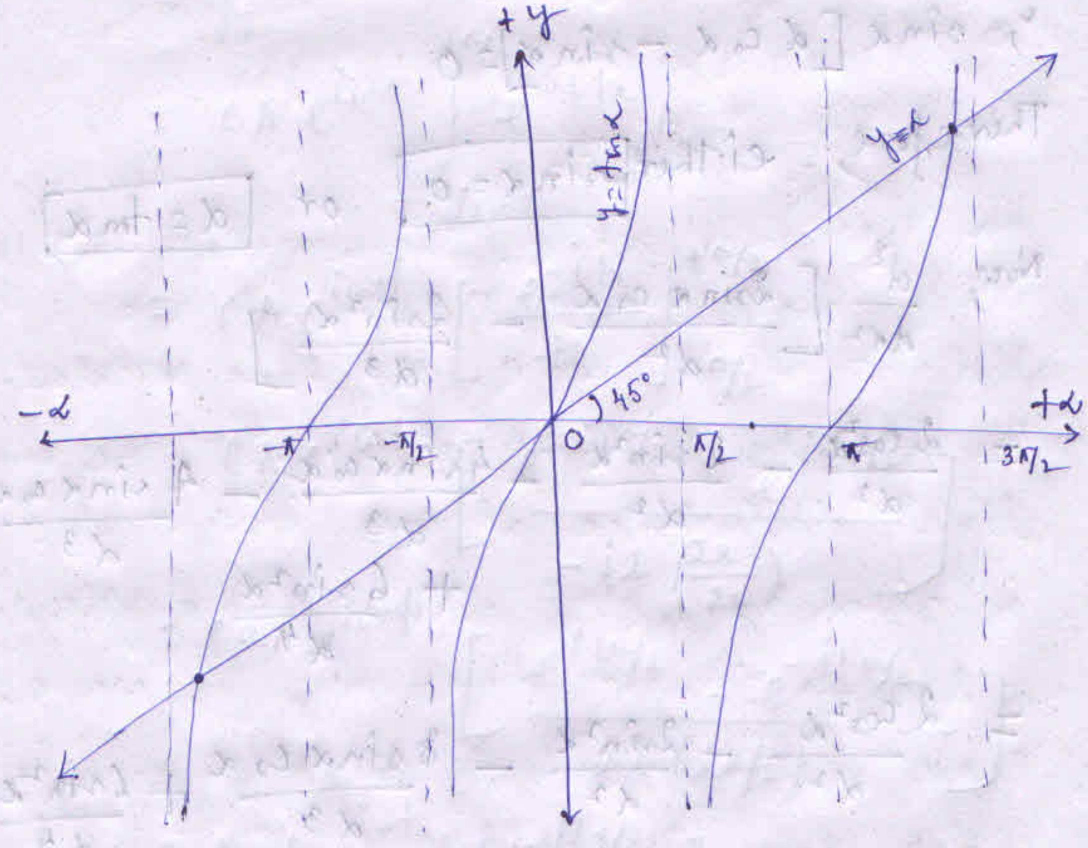

Fig. 2.

From fig. 2, it is found that $\alpha = 0$ and the other values of α which gradually approaching towards $\pm \frac{3\pi}{2}$, $\pm \frac{5\pi}{2}$ etc. give the position of maxima. $\alpha = 0$ gives position of principal maxima and others give secondary maxima.

(6)

The intensity of principal maxima is

$$I_{PM} = \lim_{\alpha \to 0} I_0 \frac{\sin^2\alpha}{\alpha^2} = I_0$$

The intensity of first secondary maxima is —

$$I_1 \simeq I_0 \frac{\sin^2\left(\frac{3\pi}{2}\right)}{\left(\frac{3\pi}{2}\right)^2} = \frac{4 I_0}{9\pi^2}$$

The intensity of second secondary maxima is —

$$I_2 \simeq I_0 \frac{\sin^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2} = \frac{4 I_0}{25\pi^2}$$ and so on

Thus it is found that intensities of secondary maxima are diminishing very rapidly with increasing order number.

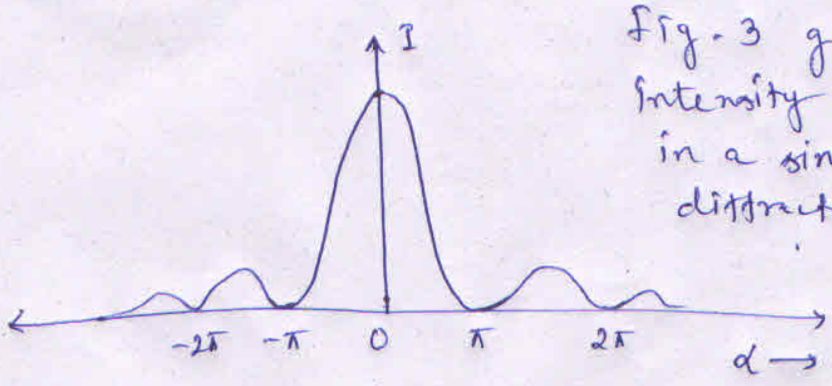

Fig - 3 gives the intensity distribution in a single slit diffraction pattern.

Fig. 3.

If θ_1 be the angle of diffraction of first minima on either side of principal maxima, then,

$a\sin\theta_1 = \lambda$ or, $\sin\theta_1 \simeq \theta_1 = \frac{\lambda}{a}$

$\therefore$ Angular width of principal maxima is $2\theta_1 = \frac{2\lambda}{a}$.