Piezoelectric coefficients

Because of the anisotropic nature of piezoelectric ceramics, piezoelectric effects are dependent on direction. To identify directions, the axes 1, 2, and 3 are introduced, corresponding to X, Y and Z of the classical right-handed orthogonal axis set. The axes 4, 5 and 6 identify rotations (shear)—23, 31, 12. Figure 5.14 illustrates them.

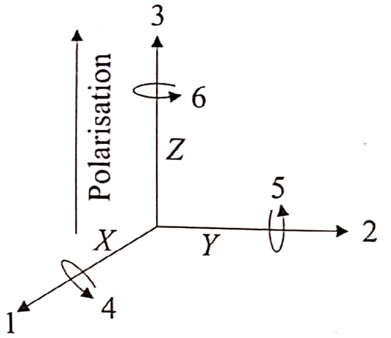

Fig. 5.14 Orthogonal system describing the properties of a poled piezoelectric ceramic. Axis 3 is the poling direction.

The direction of polarisation (axis 3) is established during manufacturing process by a strong dc field applied between two electrodes. For linear actuators[19] which involve translation, the piezo properties along the poling axis, along which the largest deflection generally takes place, are the most important.

Piezoelectric materials are characterised by d, g, h, e coefficients as well as a coupling parameter k. We will discuss them after we talk about the notations used in defining them.

Apart from the piezoelectric coefficients, piezoelectricity is also affected by

1. Electric properties like permittivity and pyroelectricity[20]
2. Elastic property like the Young's modulus
3. Thermal property like the Curie temperature

Notations. Piezoelectric constants are generally expressed with double subscripts. The subscripts link electrical and mechanical quantities. The first subscript indicates the direction of the stimulus while the second, the direction of the reaction of the system.

For example, d_{33} applies when the electric field is along the polarisation axis (direction 3) and the strain (deflection) is along the same axis. d_{31} applies if the electric field is in the same direction as before, but the deflection of interest is that along axis 1 (perpendicular to the polarisation axis).

In addition, piezoceramic material constants may be written with a *superscript* which specifies either a mechanical or electrical boundary condition. The superscripts are T, E, D and S are explained in Table 5.8.

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[19] See at page 146.

[20] Pyroelectric materials are those which produce electric charge as they undergo a temperature change. Piezoelectric materials are also pyroelectric. When their temperature is increased, they develop a voltage that has the same orientation as the polarisation voltage. When their temperature is decreased, they develop a voltage having an orientation opposite to the polarisation voltage. This creates a depolarising field with the potential to degrade the state of polarisation of the part.

Table 5.8 Significance of superscripts used to specify piezoelectric material constants

Superscript	*Implication*	*Meaning*
T	Stress = constant	Mechanically free
E	Electric field = 0	Short circuited
D	Charge displacement (i.e. current) = 0	Open circuit
S	Strain = constant	Mechanically clamped

Note:

1. We use here S for strain and T for stress rather than the conventional symbols ε and σ only to avoid confusion with the permittivity symbol ε.
2. In a dielectric material the presence of an electric field $\mathbf{E}$ causes the bound charges in the material (atomic nuclei and their electrons) to slightly separate, inducing a local electric dipole moment. The electric displacement field $\mathbf{D}$ is defined as
$$\mathbf{D} \equiv \varepsilon_0 \mathbf{E} + \mathbf{P}$$
where ε_0 is the vacuum permittivity (also called permittivity of free space), and $\mathbf{P}$ is the (macroscopic) density of the permanent and induced electric dipole moments in the material, called the *polarisation density.*
3. In a linear, homogeneous, isotropic dielectric with instantaneous response to changes in the electric field, $\mathbf{P}$ depends linearly on the electric field, giving rise to the relation
$$\mathbf{P} = \chi \varepsilon_0 \mathbf{E}$$
where the constant of proportionality χ is called the *electric susceptibility* of the material. Thus
$$\mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} = \varepsilon \mathbf{E}$$
where ε $(= \varepsilon_0 \varepsilon_r)$ is the permittivity and ε_r $(= 1 + \chi)$ is the relative permittivity of the material.
4. In linear, homogeneous, isotropic media ε is a constant. However, in linear anisotropic media it is a matrix.

Now, let us define the different coefficients we have talked about.

d coefficient. The piezoelectric charge coefficient (aka *charge constant*), d_{ij}, is defined as follows:

Direct effect

$$d_{ij} = \left.\frac{\text{Charge density developed in } i\text{-direction}}{\text{Applied stress in } j\text{-direction}}\right|_{E=0} \quad \text{C/N} \quad \left[\text{from } \frac{\text{C/m}^2}{\text{N/m}^2}\right] \tag{5.20}$$

Inverse effect

$$d_{ij} = \left.\frac{\text{Developed strain in } j\text{-direction}}{\text{Applied electric field in } i\text{-direction}}\right|_{T=\text{const.}} \quad \text{m/V} \quad \left[\text{from } \frac{\text{m/m}}{\text{V/m}}\right] \tag{5.21}$$

Note: The directions i and j are inverted in the inverse effect—the j-direction is in the numerator in this case.

Equations (5.20) and (5.21) may be written in the following form:

$$d_{ij} = \left(\frac{\partial D_i}{\partial T_j}\right)^E = \left(\frac{\partial S_j}{\partial E_i}\right)^T$$

Because for the inverse piezoelectric effect, the strain induced in a piezoelectric material by an applied electric field is the product of the value for the electric field and the value for d_{ij}, it is an important indicator of a material's suitability for strain-dependent (actuator) applications. The larger the value of d_{ij}, the larger the mechanical displacement which is usually sought in motional transducer devices.

***g* coefficient.** The piezoelectric voltage coefficient (aka *voltage constant* or *voltage sensitivity*), g_{ij}, is defined as

Direct effect

$$g_{ij} = -\left.\frac{\text{Developed electric field in } i\text{-direction}}{\text{Applied stress in } j\text{-direction}}\right|_{D=0} \quad \text{Vm/N} \quad \left[\text{from } \frac{\text{V/m}}{\text{N/m}^2}\right] \tag{5.22}$$

Inverse effect

$$g_{ij} = \left.\frac{\text{Strain developed in } j\text{-direction}}{\text{Applied charge density in } i\text{-direction}}\right|_{T=\text{constant}} \quad \text{m}^2\text{/C} \quad \left[\text{from } \frac{\text{m/m}}{\text{C/m}^2}\right] \tag{5.23}$$

Combining Eqs. (5.22) and (5.23), we can write

$$g_{ij} = -\left(\frac{\partial E_i}{\partial T_j}\right)^D = \left(\frac{\partial S_j}{\partial D_i}\right)^T \tag{5.24}$$

Because the strength of the induced electric field produced by a piezoelectric material in response to an applied physical stress is the product of the value for the applied stress and the value for g_{ij}, high g_{ij} constants favour large voltage output, and therefore, are sought after for sensor applications.

***h* coefficient.** The third coefficient h_{ij} is defined as

$$h_{ij} = -\left(\frac{\partial E_i}{\partial S_j}\right)^D = -\left(\frac{\partial T_j}{\partial D_i}\right)^S$$

It can be interpreted as negative of the voltage gradient per unit strain when the displacement is constant for the direct effect, or negative of the stress gradient per unit charge displacement when the strain is constant for the inverse effect.

***e* coefficient.** The fourth coefficient e_{ij} is defined as

$$e_{ij} = \left(\frac{\partial D_i}{\partial S_j}\right)^T = -\left(\frac{\partial T_j}{\partial E_i}\right)^S$$