Blackbody Radiation:

JAM 2017

Q.23 In the radiation emitted by a black body, the ratio of the spectral densities at frequencies 2ν and ν will vary with ν as:

(A) $\left[e^{h\nu/k_BT} - 1\right]^{-1}$

(B) $\left[e^{h\nu/k_BT} + 1\right]^{-1}$

(C) $\left[e^{h\nu/k_BT} - 1\right]$

(D) $\left[e^{h\nu/k_BT} + 1\right]$

JAM 2014

Q.42 According to Wien's theory of black body radiation, the spectral energy density in a blackbody cavity at temperature T is given as

$$u_T(\lambda)\,d\lambda = \frac{\alpha}{c^3\lambda^5} e^{-\beta/\lambda T}\, d\lambda$$

where α and β are constants and c is the speed of light. Further, the intensity of radiation coming out of the cavity is $\frac{u_T c}{4}$, where $u_T = \int_0^\infty u_T(\lambda)\,d\lambda$ is the total energy density of radiation. Given that Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ Wm^{-2}K^{-4} and $\lambda_{\max}T = 2.90\times10^{-3}$ m.K, find the values of α and β. The value of integral $\int_0^\infty x^3 e^{-x}\,dx = 6$.

JAM 2013

Q.6 A blackbody at temperature T emits radiation at a peak wavelength λ. If the temperature of the blackbody becomes $4T$, the new peak wavelength is

(A) $\frac{1}{256}\lambda$ (B) $\frac{1}{64}\lambda$ (C) $\frac{1}{16}\lambda$ (D) $\frac{1}{4}\lambda$

JAM 2012

Q.8 When the temperature of a blackbody is doubled, the maximum value of its spectral energy density, with respect to that at initial temperature, would become

(A) $\frac{1}{16}$ times (B) 8 times (C) 16 times (D) 32 times

JAM 2007:

7. The black body spectrum of an object O_1 is such that its radiant intensity (i.e., intensity per unit wavelength interval) is maximum at a wavelength of 200 nm. Another object O_2 has the maximum radiant intensity at 600 nm. The ratio of power emitted per unit area by O_1 to that of O_2 is

(A) $\frac{1}{81}$

(B) $\frac{1}{9}$

(C) 9

(D) 81

Photoelectric Effect:

JAM 2014

Q.8 In a photoelectric effect experiment, ultraviolet light of wavelength 320 nm falls on the photocathode with work function of 2.1 eV. The stopping potential should be close to

(A) 1.8 V (B) 1.6 V (C) 2.2 V (D) 2.4 V

JAM 2011

Q.6 Light described by the equation E= (90 V/m)[sin(6.28×10^{15} s^{-1}) t + sin(12.56×10^{15} s^{-1}) t] is incident on a metal surface. The work function of the metal is 2.0 eV. Maximum kinetic energy of the photoelectrons will be

(A)) 2.14 eV (B) 4.28 eV (C) 6.28 eV (D) 12.56 eV

JAM 2007:

20. A beam of light of wavelength 400 nm and power 1.55 mW is directed at the cathode of a photoelectric cell. (given: hc = 1240 eV nm, e = 1.6×10^{-19} C). If only 10% of the incident photons effectively produce photoelectrons, find the current due to these electrons. If the wavelength of light is now reduced to 200 nm, keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. What are the values of the stopping potentials for the two wavelengths? [21]

Compton Effect:

JAM 2017

Q.31 A photon of frequency ν strikes an electron of mass m initially at rest. After scattering at an angle ϕ, the photon loses half of its energy. If the electron recoils at an angle θ, which of the following is (are) true?

(A) $\cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

(B) $\sin\theta = \left(1 - \frac{mc^2}{h\nu}\right)$

(C) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$.

(D) Change in photon wavelength is $\frac{h}{mc}(1 - 2\cos\phi)$.

JAM 2016

Q.57 X-rays of 20 keV energy is scattered inelastically from a carbon target. The kinetic energy transferred to the recoiling electron by photons scattered at 90° with respect to the incident beam is ________keV.
(Planck constant = 6.6×10^{-34} Js, Speed of light = 3×10^{8} m/s, electron mass = 9.1×10^{-31} kg, Electronic charge = 1.6×10^{-19}C)

JAM 2015: Section C

Q.8 X-rays of wavelength 0.24 nm are Compton scattered and the scattered beam is observed at an angle of 60° relative to the incident beam. The Compton wavelength of the electron is 0.00243 nm. The kinetic energy of scattered electrons in eV is ___________.

JAM 2013

Q.18 A beam of X-rays of wavelength 0.2 nm is incident on a free electron and gets scattered in a direction with respect to the direction of the incident radiation resulting in maximum wavelength shift. The percentage energy loss of the incident radiation is

JAM 2010

Q.19 (a) A photon of initial momentum p_0 collides with an electron of rest mass m_0 moving with relativistic momentum P and energy E. The change in wavelength of the photon after scattering by an angle θ is given by, $\Delta\lambda = 2c\lambda_0 \frac{p_0 + P}{E - cP}\sin^2\frac{\theta}{2}$, where c is the speed of light and λ_0 is the wavelength of the incident photon before scattering. What will be the value of $\Delta\lambda$ when the electron is moving in a direction opposite to that of the incident photon with momentum P and energy E? Show that the value of $\Delta\lambda$ becomes independent of the wavelength of the incident photon when the electron is at rest before collision. (12)

(b) In a Compton experiment, the ultraviolet light of wavelength 2000 Å is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the Compton shift, if the observation is made at 90° with respect to the direction of the incident light? (9)

JAM 2008

Q.11 A photon of wavelength λ is incident on a free electron at rest and is scattered in the backward direction. The fractional shift in its wavelength in terms of the Compton wavelength λ_c of the electron is

(A) $\frac{\lambda_c}{2\lambda}$ (B) $\frac{2\lambda_c}{3\lambda}$ (C) $\frac{3\lambda_c}{2\lambda}$ (D) $\frac{2\lambda_c}{\lambda}$

JAM 2006:

23. A photon of energy E_{ph} collides with an electron at rest and gets scattered at an angle 60° with respect to the direction of the incident photon. The ratio of the relativistic kinetic energy T of the recoiled electron and the incident photon energy E_{ph} is 0.05.

(a) Determine the wavelength of the incident photon in terms of the Compton wavelength $\lambda_c\left(=\frac{h}{m_e c}\right)$, where h, m_e, c are Planck's constant, electron rest mass and velocity of light respectively. [12]

(b) What is the total energy E_e of the recoiled electron in units of its rest mass ? [9]

Waveparticle Duality, Uncertainty Principle, Two Slit Experiment:

JAM 2017

Q.57 In an electron microscope, electrons are accelerated through a potential difference of 200 kV. What is the best possible resolution of the microscope?
(Specify your answer in picometers to two digits after the decimal point.)

JAM 2016

Q.17 Consider a free electron (e) and a photon (ph) both having 10 eV of energy. If λ and P represent wavelength and momentum respectively, then
(mass of electron = 9.1×10^{-31} kg; speed of light = 3×10^{8} m/s)

(A) $\lambda_e = \lambda_{ph}$ and $P_e = P_{ph}$.

(B) $\lambda_e < \lambda_{ph}$ and $P_e > P_{ph}$.

(C) $\lambda_e > \lambda_{ph}$ and $P_e < P_{ph}$.

(D) $\lambda_e < \lambda_{ph}$ and $P_e < P_{ph}$.

JAM 2016

Q.39 A slit has width 'd' along the x-direction. If a beam of electrons, accelerated in y-direction to a particular velocity by applying a potential difference of 100 ± 0.1 kV passes through the slit, then, which of the following statement(s) is(are) correct?

(A) The uncertainty in the position of electrons in x-direction before passing the slit is zero.

(B) The momentum of electrons in x-direction is $\sim \hbar/d$ immediately after passing the slit.

(C) The uncertainty in the position of electrons in y-direction before passing the slit is zero.

(D) The presence of the slit does not affect the uncertainty in momentum of electrons in y-direction.

JAM 2016

Q.55 The *de* Broglie wavelength of a relativistic electron having 1 MeV of energy is ________ $\times 10^{-12}$ m. (Take the rest mass energy of the electron to be 0.5 MeV. Planck constant = 6.63×10^{-34} Js, Speed of light= 3×10^{8} m/s, Electronic charge = 1.6×10^{-19}C)

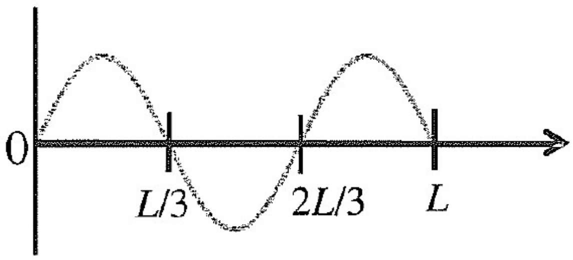
JAM 2015: Section A

Q.26 A nucleus has a size of 10^{-15} m. Consider an electron bound within a nucleus. The estimated energy of this electron is of the order of

(A) 1 MeV
(B) 10^2 MeV
(C) 10^4 MeV
(D) 10^6 MeV

JAM 2013

Q.19 A free particle of mass m is confined to a region of length L. The de Broglie wave associated with the particle is sinusoidal in nature as given in the figure. The energy of the particle is



JAM 2006:

8. A neutron of mass $m_n = 10^{-27}$ kg is moving inside a nucleus. Assume the nucleus to be a cubical box of size 10^{-14} m with impenetrable walls. Take $\hbar \approx 10^{-34}$ Js and $1\text{MeV} \approx 10^{-13}$ J. An estimate of the energy in MeV of the neutron is

(A) 80 MeV
(B) $\frac{1}{8}$MeV
(C) 8 MeV
(D) $\frac{1}{80}$ MeV

COLLECTED:

Q1. Zero point energy of harmonic oscillator from uncertainty principle:

If Δp_x is the standard deviation of measurement of p_x then: $(\Delta p_x)^2 = \langle {p_x}^2\rangle - \langle p_x\rangle^2 = \langle {p_x}^2\rangle$

Similarly if Δx is the standard deviation of measurement of x then: $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \langle x^2 \rangle$

Now $\quad E = \frac{p_x{}^2}{2m} + \frac{1}{2}kx^2$

So $\quad \langle E \rangle = \frac{\langle p_x{}^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle = \frac{(\Delta p_x)^2}{2m} + \frac{1}{2}k(\Delta x)^2 = \frac{(\Delta p_x)^2}{2m} + \frac{m\omega^2}{2}(\Delta x)^2$

Uncertainty Principle: $(\Delta x)(\Delta p_x) \geq \frac{\hbar}{2}$

$\Rightarrow (\Delta x)^2(\Delta p_x)^2 \geq \frac{\hbar^2}{4}$

$\Rightarrow (\Delta p_x)^2 \geq \frac{\hbar^2}{4(\Delta x)^2}$

Thus $\quad \langle E \rangle \geq \frac{\hbar^2}{8m(\Delta x)^2} + \frac{m\omega^2}{2}(\Delta x)^2$

The r.h.s will be minimum if $\frac{\partial(r.h.s)}{\partial[(\Delta x)^2]} = 0;\ i.e.\ if -\frac{\hbar^2}{8m(\Delta x)^4} + \frac{m\omega^2}{2} = 0;\ i.e.\ if\ (\Delta x)^2 = \frac{\hbar}{2m\omega}$

So $\langle E \rangle_{min} = \frac{\hbar^2}{8m}\frac{2m\omega}{\hbar} + \frac{m\omega^2}{2}\frac{\hbar}{2m\omega}$

$= \boldsymbol{\frac{1}{2}\hbar\omega = \textbf{zero point energy of quantum harmonic oscillator}}$.

Infinite Potential Well:

JAM 2018

Q.28 A particle of mass m is in a one dimensional potential $V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$

At some instant its wave function is given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_1(x) + i\sqrt{\frac{2}{3}}\psi_2(x)$, where $\psi_1(x)$and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

(A) $\langle x \rangle = \frac{L}{2}$; $\langle E \rangle = \frac{\hbar^2}{2m}\frac{3\pi^2}{L^2}$

(B) $\langle x \rangle = \frac{2L}{3}$; $\langle E \rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$

(C) $\langle x \rangle = \frac{L}{2}$; $\langle E \rangle = \frac{\hbar^2}{2m}\frac{8\pi^2}{L^2}$

(D) $\langle x \rangle = \frac{2L}{3}$; $\langle E \rangle = \frac{\hbar^2}{2m}\frac{4\pi^2}{3L^2}$

JAM 2017

Q.45 A particle of mass m is placed in a three-dimensional cubic box of side a. What is the degeneracy of its energy level with energy $14\left(\frac{\hbar^2\pi^2}{2ma^2}\right)$?
(Express your answer as an integer.)

JAM 2015: Section B

Q.5 A particle is moving in a two-dimensional potential well

$$V(x, y) = 0, \qquad 0 \le x \le L, 0 \le y \le 2L$$
$$= \infty, \qquad \text{elsewhere.}$$

Which of the following statements about the ground state energy E_1 and ground state eigenfunction φ_0 are true?

(A) $E_1 = \frac{\hbar^2\pi^2}{mL^2}$

(B) $E_1 = \frac{5\hbar^2\pi^2}{8mL^2}$

(C) $\varphi_0 = \frac{\sqrt{2}}{L}\sin\frac{\pi x}{L}\sin\frac{\pi y}{2L}$

(D) $\varphi_0 = \frac{\sqrt{2}}{L}\cos\frac{\pi x}{L}\cos\frac{\pi y}{2L}$

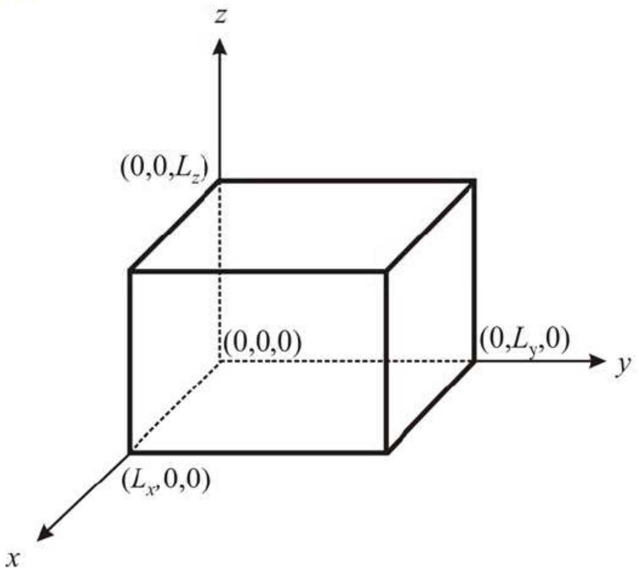
JAM 2014

Q.34 Four particles of mass m each are inside a two dimensional square box of side L. If each state obtained from the solution of the Schrodinger equation is occupied by only one particle, the minimum energy of the system in units of $\frac{h^2}{mL^2}$ is

(A) 2 (B) $\frac{5}{2}$ (C) $\frac{11}{2}$ (D) $\frac{25}{4}$

JAM 2012

Q.22 A particle of mass m is confined in a potential-box of sides L_x, L_y, and L_z, as shown in the figure. By solving the Schrödinger equation of the particle, find its eigenfunctions and energy eigenvalues.



JAM 2010

Q.12 A particle of mass m is confined in a two-dimensional infinite square well potential of side a. The eigen-energy of the particle in a given state is $E = \frac{25\pi^2\hbar^2}{ma^2}$. The state is

(A) 4-fold degenerate

(B) 3-fold degenerate

(C) 2-fold degenerate

(D) Non-degenerate

JAM 2010

Q.14 Three identical non-interacting particles, each of spin ½ and mass m, are moving in a one-dimensional infinite potential well given by,

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a \\ \infty & \text{for } x \le 0 \text{ and } x \ge a \end{cases}$$

The energy of the lowest energy state of the system is

(A) $\frac{\pi^2\hbar^2}{ma^2}$ (B) $\frac{2\pi^2\hbar^2}{ma^2}$ (C) $\frac{3\pi^2\hbar^2}{ma^2}$ (D) $\frac{5\pi^2\hbar^2}{2ma^2}$

JAM 2008

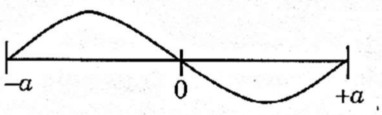
Q.19 The wave function $\Psi_n(x)$ of a particle confined to a one-dimensional box of length L with rigid walls is given by $\Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$, $n = 1, 2, 3, \ldots$.

(a) Determine the energy eigenvalues. Also, determine the eigenvalues and the eigenfunctions of the momentum operator.

(b) Show that the energy eigenfunctions are not the eigenfunctions of the momentum operator.

JAM 2007

3. A particle is confined in a one dimensional box with impenetrable walls at $x = \pm a$. Its energy eigenvalue is 2 eV and the corresponding eigenfunction is as shown below.



The lowest possible energy of the particle is

(A) 4 eV

(B) 2 eV

(C) 1 eV

(D) 0.5 eV

Eigen Function, Eigen Value:

JAM 2011

Q.10 The wave function of a quantum mechanical particle is given by

$$\psi(x) = \frac{3}{5}\varphi_1(x) + \frac{4}{5}\varphi_2(x),$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are eigenfunctions with corresponding energy eigenvalues -1 eV and -2 eV, respectively. The energy of the particle in the state ψ is

(A) $-\frac{41}{25}$ eV (B) $-\frac{11}{5}$ eV (C) $-\frac{36}{25}$ eV (D) $-\frac{7}{5}$ eV

Wave Packet:

JAM 2009

13. A wav packet in a certain medium is constructed by superposing waves of frequency ω around $\omega_0 = 100$ and the corresponding wave-number k with $k_0 = 10$ as given in the table below

ω	k
81.00	9.0
90.25	9.5
100.00	10.0
110.25	10.5
121.00	11.0

Find the ratio v_g / g_p of the group velocity v_g and the phase velocity v_p.

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

JAM 2009

20. A particle of mass 'm' is confined in a one dimensional box of unit length. At time t = 0 the wavefunction of the particle is $\psi(x,0) = A\sin 2\pi x \cos \pi x$ where A is the normalization constant.

(a) Write the wavefunction $\psi(x,t)$ at a later time t.

(b) Find the expectation values of momentum and energy at t = 0.

JAM 2006:

7. The relation between angular frequency ω and wave number k for given type of waves is $\omega^2 = \alpha k + \beta k^3$. The wave number k_0 for which the phase velocity equals the group velocity is

(A) $3\sqrt{\frac{\alpha}{\beta}}$

(B) $\left(\frac{1}{3}\right)\sqrt{\frac{\alpha}{\beta}}$

(C) $\sqrt{\frac{\alpha}{\beta}}$

(D) $\left(\frac{1}{2}\right)\sqrt{\frac{\alpha}{\beta}}$

Finite Potential Well, Step Potential, Potential Barrier:

JAM 2016

Q.40 A free particle of energy E collides with a one-dimensional square potential barrier of height V and width W. Which one of the following statement(s) is(are) correct?

(A) For $E > V$, the transmission coefficient for the particle across the barrier will always be unity.

(B) For $E < V$, the transmission coefficient changes more rapidly with W than with V.

(C) For $E < V$, if V is doubled, the transmission coefficient will also be doubled.

(D) Sum of the reflection and the transmission coefficients is always one.

JAM 2015: Section A

Q.3 A particle with energy E is incident on a potential given by

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x \geq 0 \end{cases}.$$

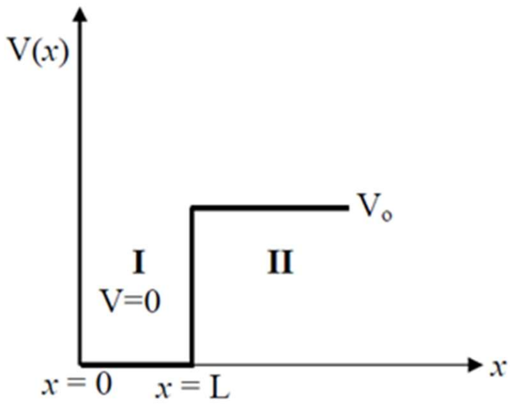
The wave function of the particle for $E < V_0$, in the region $x > 0$ (in terms of positive constants A, B and k) is

(A) $Ae^{kx} + Be^{-kx}$
(B) Ae^{-kx}
(C) $Ae^{ikx} + Be^{-ikx}$
(D) Zero

JAM 2011

Q.23 A particle of mass m moves in a potential given by

$V(x) = \infty$ for $x < 0$
$= 0$ for $0 < x < L$
$= V_o$ for $x > L$

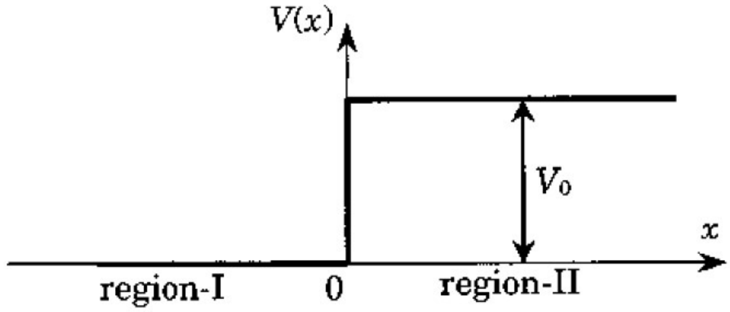


(a) Write down the general solutions for wave functions in regions **I** and **II**, if the energy of the particle $E < V_o$. Using appropriate boundary conditions, find the equation that relates E to V_o, m and L.

(b) Now, set $V_o = 0$ and assume that a beam of particles is incident on the infinite step potential (from $x > 0$) with energy $E(> 0)$. Using the general solution for the wave function, calculate the reflection coefficient.

JAM 2010

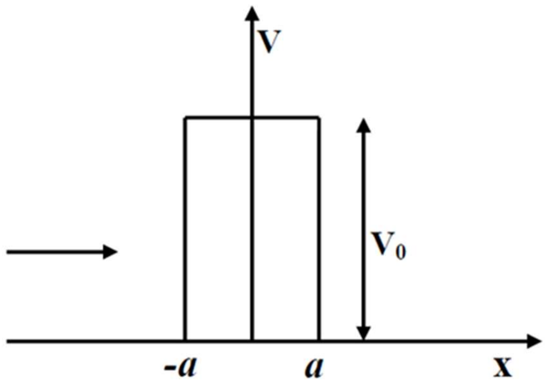
Q.22 A free particle of mass m with energy $V_0/2$ is incident from left on a step potential of height V_0 as shown in the figure below.



Writing down the time independent Schrödinger equation in both the regions, obtain the corresponding general solutions. Apply the boundary conditions to find the wave functions in both the regions. Determine the reflection coefficient R. What is the transmission coefficient T? (21)

JAM 2006

13. Electrons of energy E coming in from $x = -\infty$ impinge upon a potential barrier of width $2a$ and height V_0 centered at the origin with $V_0 > E$, as shown in the figure below. Let $k = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$. In the region $-a \le x \le a$, the wave function for the electrons is a linear combination of



(A) e^{kx} and e^{-kx}

(B) e^{ikx} and e^{-kx}

(C) e^{ikx} and e^{-ikx}

(D) e^{-ikx} and e^{kx}

JAM 2005:

20. A particle of mass m and energy E, moving in the positive x-direction, encounters a one-dimensional potential barrier at $x = 0$. The barrier is defined by

$V = 0$ for $x < 0$

$V = V_0$ for $x \geq 0$ (V_0 is positive and $E > V_0$)

If the wave function of the particle in the region $x < 0$ is given as $A\,e^{ikx} + B\,e^{-ikx}$,

(a) Find the ratio $\frac{B}{A}$.

(b) If $\frac{B}{A} = 0.4$, find $\frac{E}{V_0}$, and the transmission and reflection coefficients.

Harmonic Oscillator:

JAM 2017

Q.33 Consider a one-dimensional harmonic oscillator of angular frequency ω. If 5 identical particles occupy the energy levels of this oscillator at zero temperature, which of the following statement(s) about their ground state energy E_0 is (are) correct?

(A) If the particles are electrons, $E_0 = \frac{13}{2}\hbar\omega$.
(B) If the particles are protons, $E_0 = \frac{25}{2}\hbar\omega$.
(C) If the particles are spin-less fermions, $E_0 = \frac{25}{2}\hbar\omega$.
(D) If the particles are bosons, $E_0 = \frac{5}{2}\hbar\omega$.

JAM 2015: Section C

Q.2 A particle is in a state which is a superposition of the ground state φ_0 and the first excited state φ_1 of a one-dimensional quantum harmonic oscillator. The state is given by $\Phi = \frac{1}{\sqrt{5}}\varphi_0 + \frac{2}{\sqrt{5}}\varphi_1$. The expectation value of the energy of the particle in this state (in units of $\hbar\omega$, ω being the frequency of the oscillator) is ___________.

JAM 2013

Q.25 A particle of mass m is subjected to a potential $V(x) = ax^2$, $-\infty < x < \infty$, where a is a positive constant of appropriate dimensions. Using the relation $\Delta x\,\Delta p \approx \hbar/2$, estimate the minimum energy of the particle.

END