Study Material - Physics / Sem. 2 / Diffraction / Dr. T. Kar / Class 2

Zone Plate: A zone plate is an optical device based on the Fresnel's Theory of half-period zones. It consist of a plane parallel glass plate having concentric circles of radii proportional to the square root of the consecutive natural numbers 1, 2, 3 ... etc Then even or odd order of annular spaces between the circles are made completely dark. Such a plate behaves like a convex lens and can produce image of a source of light on a screen placed at a suitable distance.

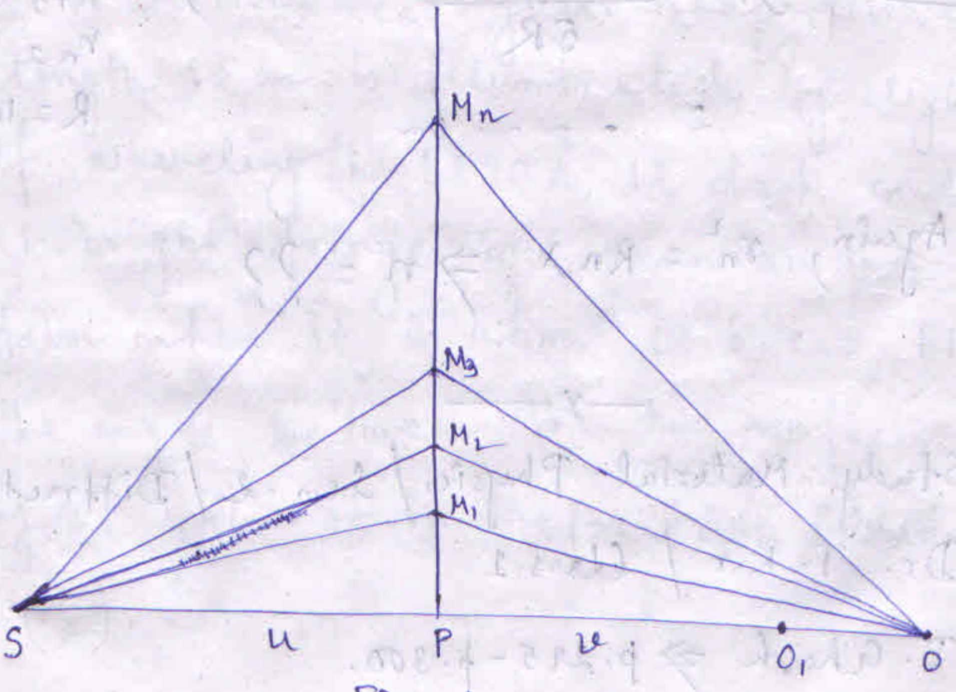

Fig. 1.

Let, $PM_n \rightarrow$ imaginary plane $\perp^r$ to the plane of the paper

$S \rightarrow$ point source of light (having wavelength 'λ') in the plane of paper

$SP = u$, $PO = v$, $PM_n = r_n$ & $r_n << u$

we have to find out resultant disturbance at O due to spherical wavelets emitted by S.

The points $M_1, M_2, M_3 \cdots M_n$ on the plane (Fig. 1) are such that,

$$(SM_1 + M_1O) - SO = \frac{\lambda}{2}$$

$$(SM_2 + M_2O) - SO = \frac{2\lambda}{2}$$

$$- - - - - -$$

$$(SM_n + M_nO) - SO = \frac{n\lambda}{2} \longrightarrow (1)$$

The area of the circle of radius PM_1 on the plane is called first half period zone. The area of the annular space between the circles of radii PM_1 and PM_2 is called the second half-period zone and so on.

Now, $SM_n^2 = r_n^2 + u^2$

$$\therefore SM_n = [r_n^2 + u^2]^{1/2} = u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u\left[1 + \frac{r_n^2}{2u^2}\right] \longrightarrow (2)$$

Similarly, $M_nO^2 = r_n^2 + v^2$

$$\therefore M_nO = [r_n^2 + v^2]^{1/2}$$

$$= v\left[1 + \frac{r_n^2}{v^2}\right]^{1/2} = v\left[1 + \frac{r_n^2}{2v^2}\right] \rightarrow (3)$$

From (1), $(SM_n + M_nO) - SO = \frac{n\lambda}{2}$

$$\therefore u\left[1 + \frac{r_n^2}{2u^2}\right] + v\left[1 + \frac{r_n^2}{2v^2}\right] - (u + v) = \frac{n\lambda}{2}$$

(3)

or, $u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - u - v = \frac{n\lambda}{2}$

or, $\frac{r_n^2}{2}\left[\frac{1}{u} + \frac{1}{v}\right] = \frac{n\lambda}{2}$

or, $r_n^2\left[\frac{1}{u} + \frac{1}{v}\right] = n\lambda \longrightarrow (4)$

From (4), $r_n^2\left[\frac{u+v}{uv}\right] = n\lambda$

or, $r_n^2 = \left(\frac{uv\lambda}{u+v}\right) n$

or, $r_n^2 = (\text{Constant})\, n$

$\therefore r_n \propto \sqrt{n} \longrightarrow (5)$

Putting, $n = 1, 2, 3 \ldots$ etc. we get values of r_1, r_2, r_3 etc. Thus the radii of half period zones are proportional to the square root of natural numbers or zone numbers.

Now, if on a transparent plate circles are drawn with radii proportional to the square root of natural no.s 1, 2, 3 ... etc. and alternate zones are blackened, such a plate is called zone plate. The zone plate such constructed behaves as a convergent lens which will be discussed.

Area of n th. zone $= \pi (r_n^2 - r_{n-1}^2)$

$$= \left[\frac{\pi u v \lambda}{u+v}\right][n-n+1]$$

$$= \frac{\pi u v \lambda}{u+v} = \text{constant.}$$

Thus all the zones are of equal area. The numerical values of resultant amplitude $a_1, a_2 \ldots a_n$ at 'O' due to secondary wavelets from 1st, 2nd etc. zone will have magnitudes in slightly decreasing order due to obliquity factor only. The wavelets from alternate zones differ in phase by π. Thus resultant amplitude at 'O' is given by $R = a_1 - a_2 + a_3 - a_4 + \cdots$

If the 2nd, 4th., 6th. etc. zones are intercepted, the resultant amplitude at 'O' will be $R = a_1 + a_3 + a_5 + \cdots$

This is many times greater than $\frac{a_1}{2}$, which is the resultant amplitude due to the wavelets from all the zones when none of them is made opaque. Thus the point 'O' will be, as a result, a point of maximum illumination.

Now from (4), $r_n^2\left[\frac{1}{u}+\frac{1}{v}\right]=n\lambda$

$$\therefore \frac{1}{u}+\frac{1}{v}=\frac{n\lambda}{r_n^2}=\frac{1}{f} \rightarrow (6)$$

This eq. is similar to the lens formula.

$f=\frac{r_n^2}{n\lambda}$ is called the principal focal length of the zone plate. Thus zone plate acts as a convergent lens with multiple foci for a particular wavelength, depending on the values of n and r_n.

<u>Multiple foci of zone plate</u>

If $u=\infty$, then $v=f=\frac{r_n^2}{n\lambda}$ i.e, image is formed at the principal focus on the axis of the zone plate (point 'O' in fig. 1). Let us consider another point O_1 along the axis of the zone plate, so that each exposed elements on the zone plate will contain three half period elements. The resultant amplitude at O_1 is,

$$R_1=(a_1-a_2+a_3)+(a_7-a_8+a_9)+\cdots$$

$$=\left(\frac{a_1}{2}+\frac{a_1}{2}-a_2+\frac{a_3}{2}+\frac{a_3}{2}\right)+\left(\frac{a_7}{2}+\frac{a_7}{2}-a_8+\frac{a_9}{2}+\frac{a_9}{2}\right)+\cdots$$

$$=\left[\frac{a_1}{2}+\left(\frac{a_1+a_3}{2}-a_2\right)+\frac{a_3}{2}\right]+\left[\frac{a_7}{2}+\left(\frac{a_7+a_9}{2}-a_8\right)+\frac{a_9}{2}\right]+\cdots$$

$$=\frac{1}{2}\left[a_1+a_3+a_7+a_9+\cdots\right]$$

Thus the point O_1 is sufficiently bright.

O_1 represents the second focal point and the second focal length is $f_2 = \frac{r_n^2}{3n\lambda}$. The resultant amplitude at O_1 is less than that at O. So the intensity at O_1 is less than that at O. Similarly other foci occur at $f_3 = \frac{r_n^2}{5n\lambda}$, $f_4 = \frac{r_n^2}{7n\lambda}$ and so on. Intensity of successive foci decreases gradually.

It may be noted that maximum intensity occurs at those points for which the exposed elements of zone plate contain odd number of half period elements. If the clear space is occupied by even number of half period zones, they cancel in pairs and hence the intensity is zero.

Generalising the results, we can write,

$$f_m = \frac{r_n^2}{(2m-1)\,n\lambda} \longrightarrow (7)$$

where, $m = 1, 2, 3, \ldots$

Eq. (7) gives, $f_1 = \frac{r_n^2}{n\lambda}$

$$f_2 = \frac{r_n^2}{3n\lambda} = \frac{f_1}{3}$$

$$f_3 = \frac{r_n^2}{5n\lambda} = \frac{f_1}{5}$$ and so on.

Thus we see that a zone plate has multiple foci.

Comparison of a zone plate with a convex lens

Similarities :

1. The relation connecting the conjugate distances are similar.

2. Focal lengths of both depend upon wavelength of light used. Hence both show chromatic aberration.

3. Both form real image of an object on the side opposite to that of the object.

4. The formulae for linear magnification for a zone plate and convex lens are similar.

Differences :

1. For a particular wavelength, a convex lens has one focus while a zone plate has a number of foci depending on the zones used. Thus for a fixed object a lens produces only one image whereas a zone plate produces a number of images.

2. Light from the consecutive clear zones of the zone plate arrives at the image point after one complete period of the wave. In a lens, the rays reach the image point in the same phase.

3. The focal length of a lens is given by the formula —
$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ whereas the focal length for a zone plate is given by $\frac{1}{f} = \frac{n\lambda}{r_n^2}$.

4. The thickness of a convex lens varies from point to point but that of a zone plate is constant everywhere.

5. For a zone plate focal length decreases as λ increases. Therefore, focal length for red light is shorter than that for violet light ($\because \lambda_r > \lambda_v$). But for a lens, focal length increases as λ increases. So for a lens $f_r > f_v$.

6. In case of a zone plate, the image is formed by the diffraction phenomenon. In case of a convex lens, the image is formed ~~by~~ due to refraction of light.

7. A zone plate can be used over a wide range of wavelengths from microwaves to x-rays. Glass lens cannot be used beyond the visible region.

8. A zone plate acts simultaneously as a convex lens and a concave lens. In addition to a real image, a virtual image is also formed simultaneously.
A convex lens forms only a real image.

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So we can say that zone plate is a practical application of Fresnel's half period zones. According to the construction of half period zones, light from any two successive zones reaches the point under consideration in opposite phases. As a result, the intensity is greatly diminished. If then the alternate zones are made opaque, either the odd or the even numbers will be effective and we get intense illumination in the point under consideration. A transparent plate made on this principle is called a zone plate.