

5.21. Magnetic Materials : Magnetic Susceptibility and Permeability.

All materials, with respect to their bulk magnetic properties, may be classified as :

(i) diamagnetic, (ii) paramagnetic, (iii) ferromagnetic.

For distinguishing above classes of magnetic materials, let us first define *magnetic permeability* and *susceptibility*. As already pointed out that the *magnetic polarisation* or *magnetisation* $\mathbf{M}$ is defined as magnetic moment per unit volume

i.e.
$$\mathbf{M} = \frac{d\mathbf{m}}{dv} = \frac{\mathbf{m}}{V} \quad \dots(1)$$

and *permeability* μ is defined as

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} \quad \text{i.e.} \quad \mathbf{B} = \mu \mathbf{H}. \quad \dots(2)$$

Also the *magnetic field intensity* $\mathbf{H}$ is defined as

$$\mathbf{H} = \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) \quad \dots(3a)$$

i.e.
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}). \quad \dots(3b)$$

For simple isotropic materials $\mathbf{M}$ is found to be proportional to $\mathbf{H}$ and parallel to it i.e.

$$\mathbf{M} \propto \mathbf{H}$$

or

$$\mathbf{M} = \chi_m \mathbf{H}, \quad \dots(4)$$

where χ_m is a dimensionless constant known as *magnetic susceptibility* and is a characteristic of the medium.

In most hand books on physical data χ_m is not listed directly but instead is given as the *mass susceptibility* $\chi_{m, mass}$ and *molar susceptibility* $\chi_{m, molar}$ defined by

mass susceptibility
$$\chi_{m, mass} = \frac{\chi_m}{\rho}, \quad \dots(5)$$

where ρ is the mass density.

molar susceptibility

$$\chi_{m, \text{molar}} = \frac{\chi_m \rho}{A}, \quad \dots(6)$$

where A is the molecular weight.

Since $\mathbf{M}$ and $\mathbf{m}$ both have the dimensions of magnetic moment per unit volume,, it is evident that $(\chi_{m, \text{mass}} \mathbf{H})$ and $(\chi_{m, \text{molar}} \mathbf{H})$ give magnetic moment per unit mass and magnetic moment per mole respectively.

Important relations. Let us now find important relations between various magnetic quantities.

Substituting values of $\mathbf{B}$ and $\mathbf{M}$ from equations (2) and (4) in equation (3), we get

$$\mathbf{H} = \left(\frac{\mu \mathbf{H}}{\mu_0} - \chi_m \mathbf{H} \right)$$

This equation yields

$$\mu = \mu_0 (1 + \chi_m) \quad \dots(7)$$

or

$$\frac{\mu}{\mu_0} = (1 + \chi_m). \quad \dots(8)$$

But

$$\frac{\mu}{\mu_0} = K_m = \text{relative permeability of medium.}$$

Hence equation (8) may be expressed as

$$K_m = 1 + \chi_m.$$

Now let us distinguish between diamagnetic, paramagnetic and ferromagnetic substances.

Diamagnetic Substances. Diamagnetic substances have negative values of magnetic susceptibility which is practically independent of temperature and consequently possess permeability [refer equation (7)] that is less than free space. When placed in a strong magnetic field they display magnetic moments that are antiparallel to the field. By the presence of such materials the magnetic induction $\mathbf{B}$ is weakened than the free space value. In a non-uniform field they experience a force towards the weaker portions of the field. Most of the simple insulators and approximately half the simple metals are diamagnetic. The examples are copper, silver, gold, bismuth, diamond, sodium, water, carbon dioxide (1 atm), hydrogen (1 atm) and nitrogen (1 atm) etc.

Paramagnetic Substances. Paramagnetic substances are characterised by small positive values of magnetic susceptibility. When placed in a strong magnetic field they display magnetic moments that are parallel to the applied field. The magnetic induction is increased by the presence of such materials. In a non-uniform field they experience a force towards the stronger portion of the field. For paramagnetic substance χ_m usually increases with decrease of temperature and is about 10^{-3} at room temperature.

When the temperature is not very small, the paramagnetic substances obey **Curie law**, according to which the magnetic susceptibility of paramagnetic substances is inversely proportional to the absolute temperature *i.e.*

$$\chi_m \propto \frac{1}{T}$$

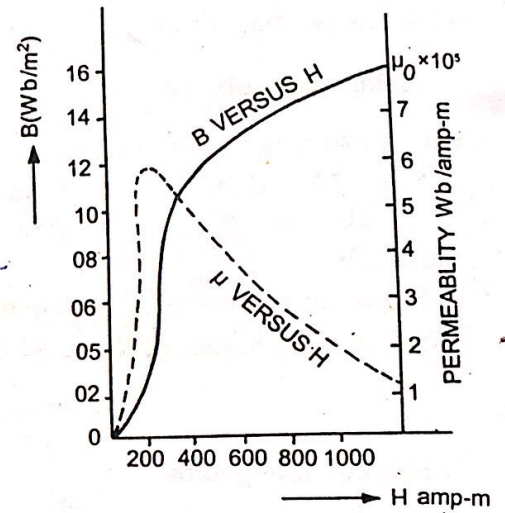
or

$$\chi_m = \frac{\lambda}{T}$$

where λ is a constant, called **Curie constant**.

For these substances permeability is greater than that of free space, thus for air it is $\mu_{\text{air}} = (1 + 0.38 \times 10^{-6}) \mu_0$ and for aluminium $\mu_{\text{Al}} = (1 + 0.25 \times 10^{-4}) \mu_0$. The examples of paramagnetic substances are aluminium, magnesium, titanium, tungsten, oxygen (1 atm) and air etc.

Ferromagnetic Substances. Ferromagnetic substances are characterised by large positive values of magnetic susceptibility. The relative permeability of these substances is very high of the order of hundred or even several thousands. In general the flux density B is not a linear function of magnetic intensity H or in other words the permeability μ is not constant but depends upon the strength of the magnetic field and upon their past history. Thus the equation $\mathbf{B} = \mu \mathbf{H}$ and $\mathbf{M} = \chi_m \mathbf{H}$ are not applicable to these substance. It is customary to use $\mathbf{B} = \mu \mathbf{H}$ as defining equation for μ , but this practice can lead to a difficulty to certain situations. If the μ of a ferromagnetic substance is defined by $\mathbf{B} = \mu \mathbf{H}$, then depending on the value of $\mathbf{H}$, μ goes through an entire range of values from infinity to zero and they may be either positive or negative. Iron, nickel, cobalt and godolinium are the only ferromagnetic elements at room temperatue, but a number of elements at low temperatures and Heusler alloys show these effects.



(Fig. 6.44)

The susceptibility of ferromagnetic substances decreases with rise of temperature. Above a certain temperature, called the Curie temperature, ferromagnetic substances change to paramagnetics. It is advisable to consider each problem involving ferromagnetism separately and try to determine which region of B - H curve is important for the particular problem and make approximations appropriate to this region.

Let us consider an unmagnetised sample of ferromagnetic material say *annealed iron*. If the magnetic intensity initially zero is increased *monotonically*, then B - H curve obtained is shown in fig. 6-44. This is called the magnetisation curve of the material. It is obvious that the value of μ taken from magnetisation curve using the relation $\mu = B/H$ is not a constant, the dotted curve represents a graph of μ as a function of H . The maximum permeability occurs at the *knee* of the curve, in some materials the maximum permeability is $10^5 \mu_0$; in others it is much lower. The reason for the *knee* in the curve is that the magnetisation $\mathbf{M}$ reaches a maximum value in the material and $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$ continues to increase at large values of $\mathbf{H}$ due to the term $\mu_0 \mathbf{H}$ only. The maximum value of $\mathbf{M}$ is called the *saturation magnetisation* of the material.

Hysteresis in Ferromagnetic Materials:

Consider an unmagnetized specimen of a ferromagnetic material.

When the magnetic intensity H is zero, the magnetic induction B is also zero initially. If the magnetic intensity is now monotonically increased from zero, B will increase along the curve 1, as shown in fig 9-8. This curve is known as the magnetization curve of the material. With an increase in H from zero, the magnetization M increases from zero to its saturation value M_s . Accordingly B increases following the relationship $B = \mu_0(\vec{H} + \vec{M})$. When M saturates at high values of H , B will slowly increase due to the $\mu_0 H$ term.

will slowly increase due to the $\mu_0 \vec{H}$ term. This accounts for the nonlinear nature of the curve in Fig.9.8. The relative permeability, calculated from the relationship $K_m = B/(\mu_0 H)$, varies with H as shown in Fig.9.9, when B traces curve 1 in Fig.9.8. The maximum permeability is seen to occur at the 'knee' of curve 1 in Fig.9.8. This point of maximum K_m is the point of contact of the tangent drawn from the origin to curve 1. In some materials, the maximum relative permeability may be as high as 10^5 .

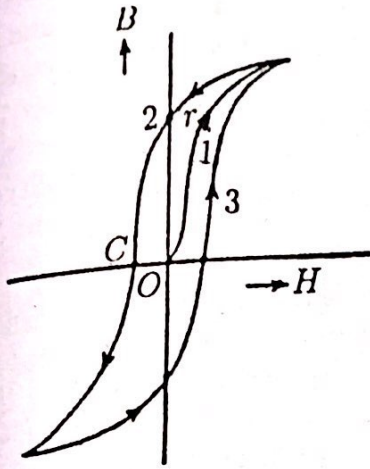


Fig.9.8. Magnetization curve and hysteresis loop of a ferromagnetic material

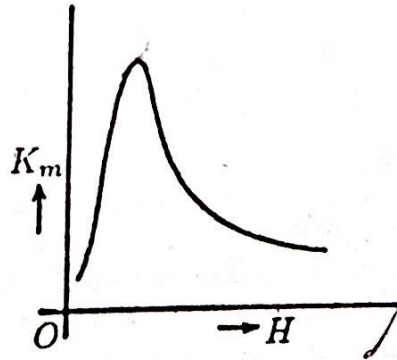


Fig.9.9. Plot of relative permeability against magnetic intensity

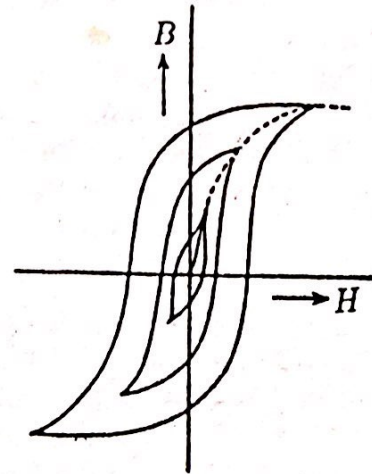


Fig.9.10. A few hysteresis loops of a ferromagnetic material

After reaching the value M_s , the saturation magnetization, the value of H is gradually decreased. The $\vec{B}-\vec{H}$ relationship does not follow back down the curve 1 of Fig.9.8. Instead, the B values trace the curve 2 which shows that even when H is zero, B is not so. That is, the material has been permanently magnetized. The value of B at the point r , denoted by B_r , is called the remanent induction. The magnetic intensity is to be reversed and increased in magnitude to reduce B to zero. The magnitude of H at point C , denoted by H_c , is referred to as the coercive force of the material. As H is increased in the reverse direction, curve 2 is traced till the magnetization saturates in the reverse direction. As H is reduced back to zero, the $B-H$ relationship follows curve 3. Again, when H is zero a residual negative B value exists. As H is reversed and increased, curve 3 is followed until the magnetization saturates. Curves 2 and 3 in Fig.9.8 are symmetrically positioned and enclose a loop in the $B-H$ plane. This shows that in a cyclic variation of the magnetic intensity, the magnetic induction B lags the exciting intensity H . This phenomenon is referred to as hysteresis, from the Greek word meaning "to lag". The loop traced in the $B-H$ plane is known as the hysteresis loop or the $B-H$ loop of the material.

From Fig.9.8 we find that the permeability given by $\mu = B/H$ is negative in the second and fourth quadrants of the diagram. The shape of the hysteresis loop depends on the nature of the material. It also depends on the maximum value of $\vec{H}$, as shown in Fig.9.10. However, if the maximum value of $\vec{H}$ produces magnetic saturation, the shape of the $B-H$ loop remains unchanged as the maximum value of $\vec{H}$ is increased further.

When a small alternating H -field is superimposed on a large constant value, one can define an incremental permeability as follows :

$$\mu_{in} = \frac{\delta B}{\delta H}, \quad (9.34)$$

where δB is the change in B caused by a change δH in H . μ_{in} is given approximately by the slope of the $B-H$ curve at the operating point. It is a maximum at the point where the $B-H$ curve is the steepest. In the saturation region, $\mu_m = \mu_0$. At the point of the magnetization curve where K_m is a maximum, we have $\mu_0 K_m = \mu_m$, since there $\frac{B}{H} = \frac{\delta B}{\delta H}$.

Ferromagnetic materials like soft iron are used to increase the magnetic flux for a given exciting field as in electromagnets. We shall see in Sec. 11.5 that as the magnetic field goes through a complete cycle, an energy proportional to the area of the hysteresis loop is dissipated as heat. This loss of energy is referred to as the *hysteresis loss*. To minimise hysteresis losses, the materials having a small B - H loop area are used in electromagnets and transformer cores. Such materials are known as the *soft magnetic materials*. Such materials have a low remanent induction and a low coercive force, but a high permeability. Another source of energy loss in soft magnetic materials is the eddy current loss, which can be reduced by increasing the electrical resistivity of the medium. Iron with 4 % silicon, iron-nickel alloys (such as, permalloy), ferrites and garnets are suitable for transformer core materials. The values of $\mu_0 M_s$ and H_c for iron with 4 % silicon are 1.95 tesla and 60 A/m, respectively. Its maximum relative permeability is 8000. Ferrites having a nearly rectangular hysteresis loop find applications as memory cores in computers.

Ferromagnetics are also used to produce permanent magnets. Such materials are called *hard magnetic materials*. To produce a permanent magnet, the specimen is first magnetized to saturation with the help of an exciting field. The exciting field is then removed. Clearly the materials for permanent magnets must have a large remanent induction and a large coercive force. Steel and Al-Ni-Co (alnico) alloy are common materials for permanent magnets. The values of $\mu_0 M_s$, B_r and H_c for alnico are 1.2 tesla, 0.7 tesla and 5×10^4 A/m, respectively. Pure Fe_2O_3 is used in magnetic tapes to store information. It is evident that the study of B - H curves is extremely useful in the choice of ferromagnetic materials for specific applications.

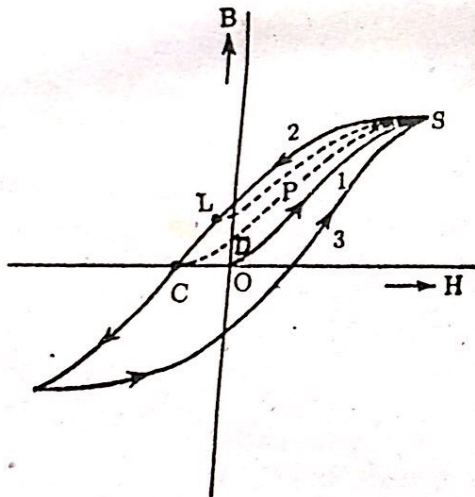


Fig.9.10A. Different magnetization curves

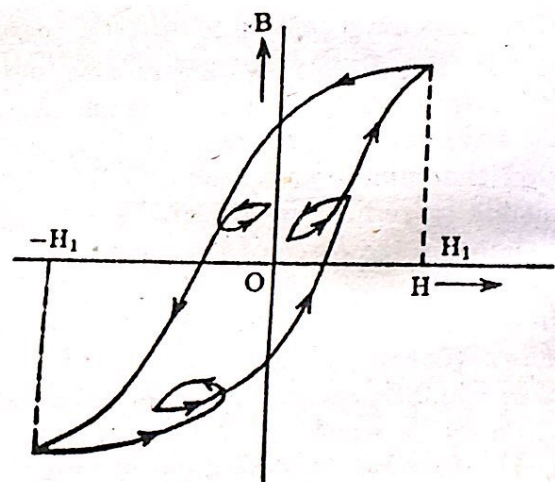


Fig.9.10B. Hysteresis loop with small cyclic changes of H superimposed

Observations : If the coercive force H_c is removed, the magnetic induction B follows the dashed curve CD (Fig.9.10A), so that at zero magnetic intensity, the magnetic induction has the value OD . If H is now increased in the forward direction, B increases along the dashed curve DS . If, from any point L on curve 2, H is increased in the forward direction, B increases following the curve LPS . Thus different magnetization curves are obtained, depending upon the past treatment of the specimen. When, in the cyclic variation of H from $-H_1$ to $+H_1$, small cyclic changes of H are superimposed at intermediate stages, the hysteresis loop assumes the shape shown in Fig.9.10B.

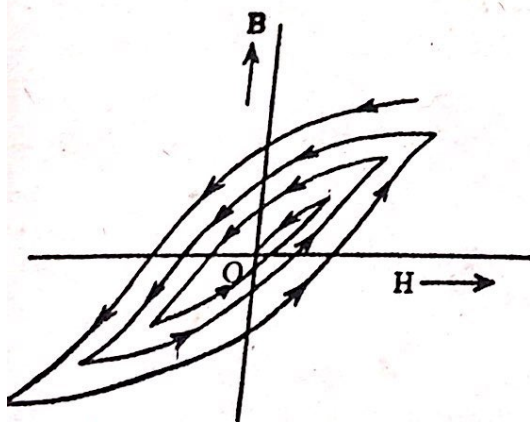


Fig.9.10C. Demagnetization by electrical means

A ferromagnetic material is said to achieve a *cyclic state* when, for the same change in H from a given initial value, the change in B is always the same. This condition can be attained by repeated reversals of H at the given initial value.

Generally, previous treatments leave the specimen with some magnetization. To *demagnetize* the specimen, i.e., to reduce the magnetization to zero in the absence of the magnetic intensity, the specimen is subjected to smaller and smaller cycles of magnetization (Fig. 9.10C). This is achieved by placing the specimen in a solenoid and feeding the solenoid by an alternating current of slowly diminishing amplitude. Before the current amplitude falls to zero, the specimen undergoes a number of cycles of magnetization to wipe out the remnants of the past treatments.

The magnetization curve and the B - H loop of a ferromagnetic substance can be experimentally determined with the help of a ballistic galvanometer (see Sec. 12.8)

Iron and Steel

Hysteresis loops of annealed soft iron and steel after being hardened, are shown schematically in Fig. 9.10D. Soft iron (which is a soft magnetic material) has a smaller loop area than steel (which is a hard magnetic material), and so the hysteresis loss in soft iron is less. Soft iron has smaller remanent induction and coercive force than steel. Thus soft iron is suitable for use in electromagnets and transformer cores, whereas steel is a suitable material for permanent magnets.

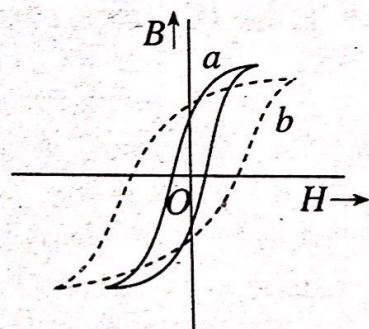


Fig. 9.10D. Hysteresis loops of soft iron (curve a) and steel (curve b)

Energy loss due to hysteresis. According to molecular theory of magnetisation, the molecule of a magnetised or unmagnetised magnetic substance are themselves complete magnets. When a magnetising field is applied, the molecular magnets align themselves in the direction of the field. During this process work is done by the magnetising field in turning the molecular magnets against the mutual attraction forces. This energy required to magnetise a specimen is not completely recovered when the magnetising field is turned off, since the magnetisation does not become zero. The specimen retains some magnetisation because some of the molecular magnets remain aligned in the new formation due to the group forces. To tear them out completely a coercive force in the reverse direction has to be applied. Thus there is a loss of energy in taking a magnet through a cycle of magnetisation. Now we have to calculate this loss of magnetisation. Now we have to calculate this loss of energy.

Consider a magnetic material having n molecular magnets per unit volume. Let m be the magnetic moment of each elementary magnet and θ the angle which its axis makes with the direction of magnetising field H , then magnetic moment per unit volume parallel to the field is

$$M = \Sigma m \cos \theta \quad \dots(1)$$

The magnetic moment per unit volume perpendicular to the field is

$$\Sigma m \sin \theta,$$

and this is equal to zero since there can be no magnetisation perpendicular to H .

Now, the torque due to the field acting on the dipole of moment m when it is inclined at an angle θ to field is

$$\tau = \mu_0 m H \sin \theta \quad \dots(2)$$

and the work done when it moves through a small angle $+d\theta$ is $-\mu_0 m H \sin \theta$. Here minus sign comes in because the work has to be done against the magnetic field in increasing θ by $d\theta$. Hence the work done per unit volume of the material is

$$dW = -\Sigma \mu_0 m H \sin d\theta \quad \dots(3)$$

As θ increases by $d\theta$, the intensity of magnetisation M also increases by dM obtained from equation (1) as

$$\begin{aligned} dM &= d(\Sigma m \cos \theta) \\ &= -\Sigma m \sin \theta d\theta. \end{aligned} \quad \dots(4)$$

Inserting equation (4) into (3), we get the work done per unit volume in turning the elementary magnets through a small angle θ is

$$dW = \mu_0 H dM. \quad \dots(5)$$

Thus work done by the magnetising field per unit volume of the material for completing a cycle is

$$\begin{aligned} W &= \oint \mu_0 H dM = \mu_0 \oint H dM \\ &= \mu_0 \times \text{area of } M-H \text{ loop.} \end{aligned}$$

Since the magnetic induction B is related to magnetisation M by the relation,

$$B = \mu_0 (H + M)$$

or

$$dB = \mu_0 (dH + dM). \quad \dots(6)$$

This implies,

$$dM = \frac{dB}{\mu_0} - dH \quad \dots(7)$$

Thus equation (6) may be written as

$$\begin{aligned} W &= \mu_0 \oint H \left(\frac{dB}{\mu_0} - dH \right) \\ &= \oint H dB - \mu_0 \oint H dH \end{aligned} \quad \dots(8)$$

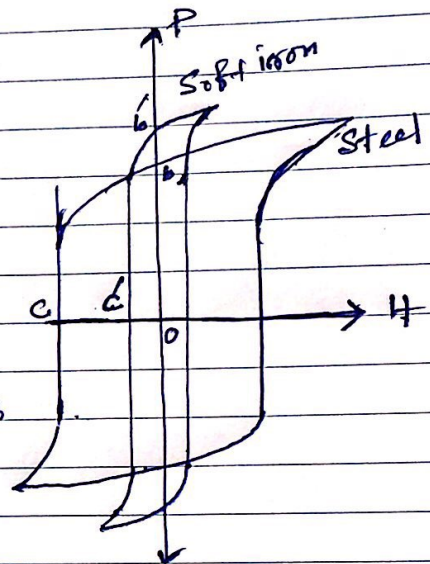
but $\oint H dH = 0$ because the plot of H against H is a straight line and the area enclosed by it is zero. Thus equation (8), gives

$$W = \oint H dB = \text{area of } B-H \text{ loop.}$$

Thus, the work done per unit volume of the material per cycle is equal to area of μ_0 times the area of $M-H$ loop or the area of $B-H$ loop. The work is measured in joule/m³ per cycle and is dissipated in the form of heat.

Uses of Hysteresis Cycle:

The hysteresis curve of a ferromagnetic can be used to study its magnetic properties. The B-H curves for soft-iron & steel are shown in this figure. A study of these curves gives the following information regarding the magnetic properties of soft-iron & steel:



i) **Permeability:** The value of the flux density B is greater for soft iron than that for steel for the same value of H . Therefore, the permeability ($\mu = B/H$) is also greater for soft iron than for steel.

ii) **Retentivity:** When the magnetic field H is reduced to zero, soft iron retains flux density $O b'$ which is greater than the magnetic flux $O b$ retained by steel. Hence the retentivity of soft-iron is greater than that of steel.

iii) **Coercivity:** To demagnetise the soft-iron a coercive force $O c'$ is required which is smaller than the coercive ~~force~~ force, $O c$ required for steel. Hence, the coercivity of soft-iron is less than that of steel.

iv) **Hysteresis loss:** The area of the B-H loop for soft-iron is smaller than that for steel. Hence the hysteresis loss per unit vol^m per cycle is less for soft iron than for steel.