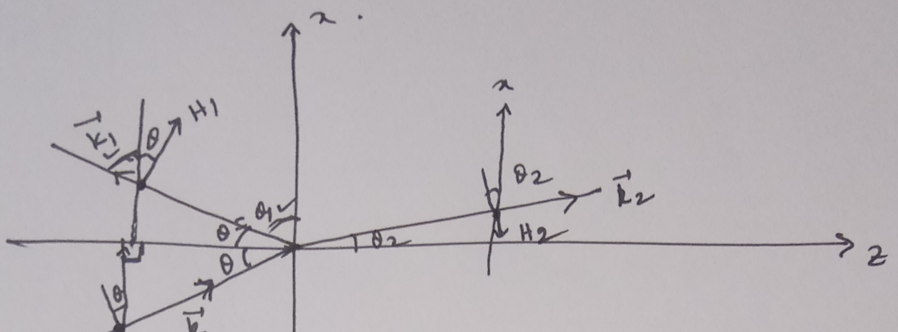
Fresnel's Equations.



$$E_0 + E_{10} = E_{20} \quad \cdots (1)$$

$$-H_0 \cos\theta + H_{10} \cos\theta = -H_{20} \cos\theta_2 \quad \cdots (2)$$

$$H = \frac{\vec{K} \times \vec{E}}{\omega \mu_1} = \frac{\bar{K} \times \bar{E}}{KC \cdot \mu_1} = \frac{K \times E \sqrt{\mu_1 \epsilon_1}}{K \mu_1} = \sqrt{\frac{\epsilon_1}{\mu_1}} \cdot \frac{\vec{K} \times \vec{E}}{K}$$

$$\therefore H_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{K \times E}{K} = \sqrt{\frac{\epsilon_0}{\mu_0}} \cdot E_0 .$$

$$H_{10} = \sqrt{\frac{\epsilon_1}{\mu_1}} E_{10} \quad \& \quad H_{20} = \sqrt{\frac{\epsilon_2}{\mu_2}} \cdot E_{20} .$$

$\therefore$ Equation(2) can be written as,

$$\left(-E_0 \cos\theta + E_{10} \cos\theta\right) \sqrt{\frac{\epsilon_1}{\mu_1}} = -E_{20} \cos\theta_2 \sqrt{\frac{\epsilon_2}{\mu_2}} \quad \cdots (3)$$

Solving equation (1) & (3) we get,

$$\left(-E_0 \cos\theta + E_{10} \cos\theta\right) \sqrt{\frac{\epsilon_1}{\mu_1}} = -\left(E_0 + E_{10}\right) \cos\theta_2 \sqrt{\frac{\epsilon_2}{\mu_2}} .$$

$$\therefore E_{10}\left(\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2\right) = E_0\left(\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta - \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2\right) .$$

$$\therefore r_\perp = \frac{E_{10}}{E_0} = \frac{\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta - \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2}{\left(\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2\right)}$$

$$n_1 = \frac{c}{v} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_0 \epsilon_0}} = \sqrt{\frac{\epsilon_1}{\epsilon_0}} . \qquad \text{if, } \mu_1 = \mu_2 = \mu_0 .$$

$$n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}} .$$

$$\therefore r_\perp = \frac{E_{10}}{E_0} = \frac{\sqrt{\frac{\epsilon_1 \epsilon_0}{\mu_0 \cdot \epsilon_0}} \cos\theta - \sqrt{\frac{\epsilon_2 \epsilon_0}{\mu_0 \cdot \epsilon_0}} \cos\theta_2}{\sqrt{\frac{\epsilon_1 \epsilon_0}{\mu_0 \cdot \epsilon_0}} \cos\theta + \sqrt{\frac{\epsilon_2 \epsilon_0}{\epsilon_0 \cdot \mu_0}} \cos\theta_2}$$

$$= \frac{\left[\sqrt{\frac{\epsilon_1}{\epsilon_0}} \cos\theta - \sqrt{\frac{\epsilon_2}{\epsilon_0}} \cos\theta_2\right] \sqrt{\frac{\epsilon_0}{\mu_0}}}{\left[\sqrt{\frac{\epsilon_1}{\epsilon_0}} \cos\theta + \sqrt{\frac{\epsilon_2}{\epsilon_0}} \cos\theta_2\right] \sqrt{\frac{\epsilon_0}{\mu_0}}} = \frac{n_1 \cos\theta - n_2 \cos\theta_2}{n_1 \cos\theta + n_2 \cos\theta_2} .$$

$$r_{\perp} = \frac{E_{10}}{E_0} = \frac{n_1 \cos\theta - n_2 \cos\theta_2}{n_1 \cos\theta + n_2 \cos\theta_2}$$

Using Snell's law,

$$\frac{\sin\theta}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$= \frac{\cancel{n_1}\left(\cos\theta - \frac{n_2}{n_1}\cos\theta_2\right)}{\cancel{n_1}\left(\cos\theta + \frac{n_2}{n_1}\cos\theta_2\right)}$$

$$= \frac{\cos\theta - \frac{\sin\theta}{\sin\theta_2}\cdot\cos\theta_2}{\cos\theta + \frac{\sin\theta}{\sin\theta_2}\cos\theta_2}$$

$$= \frac{\sin\theta_2\cos\theta - \sin\theta\cos\theta_2}{\sin\theta_2\cos\theta + \sin\theta\cos\theta_2}$$

$$= \frac{\sin(\theta_2 - \theta)}{\sin(\theta_2 + \theta)} = \frac{-\sin(\theta - \theta_2)}{\sin(\theta + \theta_2)}.$$

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{n_2\cos\theta - n_1\cos\theta_2}{n_2\cos\theta + n_1\cos\theta_2}$$

$$= \frac{\cancel{n_1}\left(\frac{n_2}{n_1}\cos\theta - \cos\theta_2\right)}{\cancel{n_1}\left(\frac{n_2}{n_1}\cos\theta + \cos\theta_2\right)}$$

$$= \frac{\frac{\sin\theta}{\sin\theta_2}\cos\theta - \cos\theta_2}{\frac{\sin\theta}{\sin\theta_2}\cos\theta + \cos\theta_2}$$

$$= \frac{\sin\theta\cos\theta - \sin\theta_2\cos\theta_2}{\sin\theta\cos\theta + \sin\theta_2\cos\theta_2}$$

$$= \cancel{\sin(} = \frac{\sin 2\theta - \sin 2\theta_2}{\sin 2\theta + \sin 2\theta_2}.$$

$$= \frac{\cancel{2}\cos(\theta+\theta_2)\cdot\sin(\theta-\theta_2)}{\cancel{2}\cdot\sin(\theta+\theta_2)\cos(\theta-\theta_2)}$$

$$= \frac{\tan(\theta-\theta_2)}{\tan(\theta+\theta_2)}. \quad \ldots (16.3\text{-}30)$$