Example 8.1 A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At $t = 0$, it is connected for charging in series with a resistor $R = 1\ M\Omega$ across a 2V battery (Fig. 8.3). Calculate the magnetic field at a point P, halfway between the centre and the periphery of the plates, after $t = 10^{-3}$ s. (The charge on the capacitor at time t is $q\ (t) = CV\ [1 - \exp\ (-t/\tau)]$, where the time constant τ is equal to CR.)

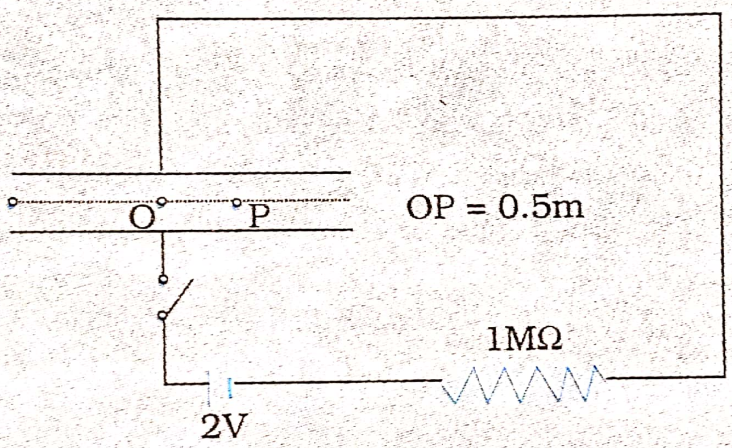

FIGURE 8.3

Solution The time constant of the CR circuit is $\tau = CR = 10^{-3}$ s. Then, we have

$q(t) = CV\ [1 - \exp\ (-t/\tau)]$

$\quad = 2 \times 10^{-9}\ [1- \exp\ (-t/10^{-3})]$

The electric field in between the plates at time t is

$$E = \frac{q(t)}{\varepsilon_0 A} = \frac{q}{\pi \varepsilon_0};\ A = \pi\ (1)^2\ m^2 = \text{area of the plates.}$$

Consider now a circular loop of radius (1/2) m parallel to the plates passing through P. The magnetic field **B** at all points on the loop is

EXAMPLE 8.1

along the loop and of the same value.

The flux Φ_E through this loop is

Φ_E = $E \times$ area of the loop

$$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0}$$

The displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$$

at $t = 10^{-3}$s. Now, applying Ampere-Maxwell law to the loop, we get

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_c + i_d) = \mu_0 (0 + i_d) = 0.5 \times 10^{-6} \mu_0 \exp(-1)$$

or, $B = 0.74 \times 10^{-13}$ T

8.3 Electromagnetic Waves