Now

$$\vec{\nabla} \times \vec{E}_1 = -\frac{\partial \vec{B}_1}{\partial t} = -\mu_1 \frac{\partial \vec{H}_1}{\partial t}.$$

So taking $\vec{H}_1 = \vec{H}_{10} e^{-j\omega t}$ we can write

$$\vec{H}_1 = \frac{\vec{\nabla} \times \vec{E}_1}{j\omega\mu}.$$

This indicates that inside a perfect conductor, where $\vec{E}_1 = 0$, $\vec{H}_1$ is also zero. So both the tangential and normal components of $\vec{E}_1$ and $\vec{H}_1$ are zero, i.e.,

$$E_{1n} = E_{1t} = 0 \quad \text{and} \quad H_{1n} = H_{1t} = 0.$$

So in this case, the boundary conditions become

$$\epsilon_2 E_{2n} = \sigma, \quad B_{2n} = B_{1n} = 0$$

$$\frac{1}{\mu_2} \hat{n} \times \vec{B}_2 = \vec{K}, \quad E_{2t} = E_{1t} = 0. \tag{16.2-7}$$

16.3 Reflection and Refraction of Electromagnetic Waves at the Interface of Two Dielectric Media

Reflection and refraction of electromagnetic waves (light) at a plane interface of two different media are familiar phenomena. Various aspects of these phenomena including ordinary laws of reflection and refraction, amplitude relations, phase changes, polarization etc. can be well-described by electromagnetic theory.

A. Ordinary laws of reflection and refraction

Suppose that the xy-plane forms the interface ($z = 0$) between two linear homogeneous dielectric (nonconducting) media characterised by permittivities ϵ_1 and ϵ_2, and permeabilities μ_1 and μ_2, respectively as shown in Fig 16.3-1. Let a plane electromagnetic wave be incident on the interface at the point O. The wave is partly reflected and partly refracted. Let us represent the electric fields associated with these waves by

$$\begin{aligned} \vec{E} &= \vec{E}_0 e^{j(\vec{k}\cdot\vec{r}-\omega t)}, \\ \vec{E}_1 &= \vec{E}_{10} e^{j(\vec{k}_1\cdot\vec{r}-\omega_1 t)}, \\ \text{and} \quad \vec{E}_2 &= \vec{E}_{20} e^{j(\vec{k}_2\cdot\vec{r}-\omega_2 t)}, \end{aligned} \tag{16.3-1}$$

where $\vec{E}_0$, $\vec{E}_{10}$ and $\vec{E}_{20}$ are the amplitudes, $\vec{k}$, $\vec{k}_1$ and $\vec{k}_2$ are the propagation vectors, ω, ω_1 and ω_2 are the frequencies of the incident, reflected and refracted waves respectively.

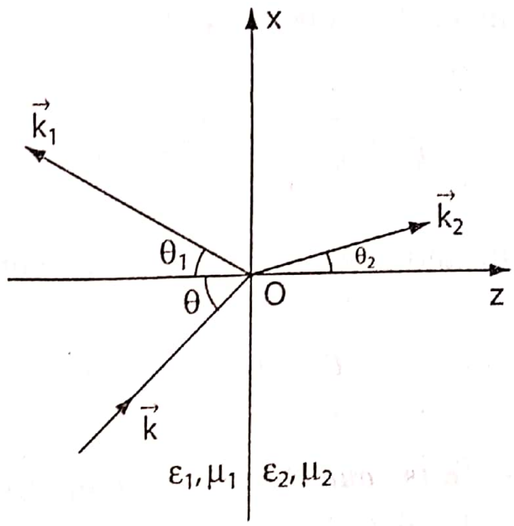

Fig 16.3-1: Reflection and refraction of EM waves.

Let us now apply the boundary condition that the tangential components of electric field are continuous across the interface $z = 0$. Thus,

$$\left(\vec{E}_0\right)_t \cdot e^{j(\vec{k}\cdot\vec{r}-\omega t)} + \left(\vec{E}_{10}\right)_t \cdot e^{j(\vec{k}_1\cdot\vec{r}-\omega_1 t)} = \left(\vec{E}_{20}\right)_t \cdot e^{j(\vec{k}_2\cdot\vec{r}-\omega_2 t)}, \qquad (16.3\text{-}2)$$

where the subscript t is used to denote tangential components. The condition (16.3-2) must be valid for all values of time and it immediately follows that

$$\omega = \omega_1 = \omega_2. \qquad (16.3\text{-}3)$$

Thus, we find that *the frequency of electromagnetic wave does not change on reflection and refraction.*

The condition (16.3-2) must also hold at all points on the interface ($z = 0$). So we must have

$$\vec{k}\cdot\vec{r} = \vec{k}_1\cdot\vec{r} = \vec{k}_2\cdot\vec{r} \quad \text{at } z = 0$$

Therefore,

$$k_x = k_{1x} = k_{2x} \quad \text{and} \quad k_y = k_{1y} = k_{2y}. \qquad (16.3\text{-}4)$$

If the incident beam is assumed to be in the xz-plane then $k_y = 0$. Consequently $k_{1y} = k_{2y} = 0$ and both $\vec{k}_1$ and $\vec{k}_2$ lie in the xz-plane.

Since z-axis is normal to the interface we may conclude that *the incident, reflected, refracted waves and normal to the interface at the point of incidence all lie in the same plane.*

From Fig 16.3-1,

$$k_x = k\sin\theta, \quad k_{1x} = k_1\sin\theta_1 \quad \text{and} \quad k_{2x} = k_2\sin\theta_2. \qquad (16.3\text{-}5)$$

Therefore, from the condition $k_x = k_{1x}$ we get

$$k\sin\theta = k_1\sin\theta_1. \qquad (16.3\text{-}6)$$

Since the vectors $\vec{k}$ and $\vec{k}_1$ lie in the same medium we can write for the phase velocity in medium 1 as

$$\frac{\omega}{k} = \frac{\omega_1}{k_1} = \frac{1}{\sqrt{\mu_1 \epsilon_1}}. \tag{16.3-7}$$

As $\omega = \omega_1$, therefore, $k = k_1$ and from Eq. (16.3-6) we get

$$\theta = \theta_1. \tag{16.3-8}$$

That is, *the angle of incidence is equal to the angle of reflection.*

Again from Eqs. (16.3-4) and (16.3-5),

$$k_{1x} = k_{2x}$$

$$\text{or} \quad k_1 \sin\theta_1 = k_2 \sin\theta_2$$

$$\text{or} \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \frac{\omega/k_1}{\omega/k_2} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1}, \tag{16.3-9}$$

where $v_1 = \omega/k_1$ and $v_2 = \omega/k_2$ represent the phase velocities of the waves in media 1 and 2, respectively; n_1 and n_2 are the refractive indices of the media 1 and 2, respectively. Obviously,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{16.3-10}$$

which is the well-known *Snell's law of refraction.*

B. Fresnel's equations

The ordinary laws of reflection and refraction as discussed above tells nothing about the fractions of incident wave-amplitude reflected or refracted. The equations which relate the amplitudes of the reflected and refracted waves with that of the incident wave are known as *Fresnel's equations.*

We know that $\vec{E}$-vector in a plane electromagnetic wave is perpendicular to the direction of propagation of the wave. However, it may lie in arbitrary directions in the plane normal to the direction of propagation. It is convenient to consider two extreme cases separately, one in which the $\vec{E}$-vector of the incident wave is parallel to the plane of incidence (the plane defined by $\vec{k}$ and $\hat{n}$); this is called *p polarization* (p for parallel). The other in which the $\vec{E}$-vector is perpendicular to the plane of incidence is called *s polarization* (s for a German word *senkrecht* meaning perpendicular). The general case of arbitrary polarization can then be obtained by a suitable linear combination of the two extreme results.

Case IV. $\vec{E}$ is perpendicular to the plane of incidence (s-polarized wave).

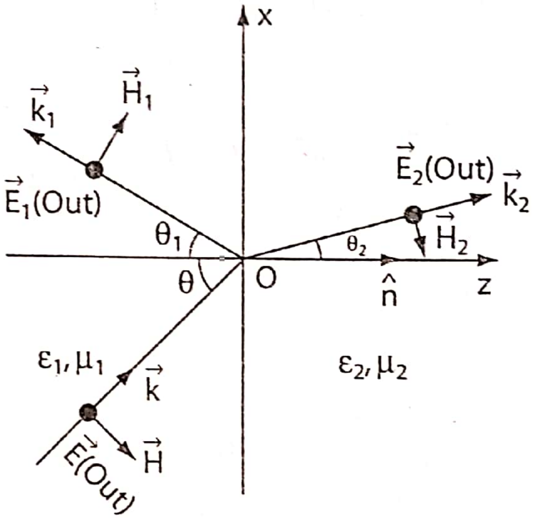

Fig 16.3-2: Reflection and refraction with $\vec{E}$-vector normal to the plane of incidence.

Suppose that the $\vec{E}$-vector of the incident wave lies perpendicular to the plane of incidence (xz-plane) as shown in Fig 16.3-2. As the media on two sides of the interface $z = 0$ are isotropic the $\vec{E}$-vectors of reflected and refracted waves will also be perpendicular to the plane of incidence. In Fig 16.3-2 we assume that all the $\vec{E}$-vectors are directed normally out of the plane of the paper. Magnetic vectors $\vec{H}$, $\vec{H}_1$ and $\vec{H}_2$ are in the plane of incidence and their directions are chosen so as to give a positive flow of energy in the direction of the respective propagation vectors. Let us represent the incident, reflected and refracted waves by

$$\vec{E} = \vec{E}_0 e^{j(\vec{k}\cdot\vec{r}-\omega t)}, \quad \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu_1} \tag{16.3-11}$$

$$\vec{E}_1 = \vec{E}_{10} e^{j(\vec{k}_1\cdot\vec{r}-\omega t)}, \quad \vec{H}_1 = \frac{\vec{k}_1\times\vec{E}_1}{\omega\mu_1} \tag{16.3-12}$$

$$\vec{E}_2 = \vec{E}_{20} e^{j(\vec{k}_2\cdot\vec{r}-\omega t)}, \quad \vec{H}_1 = \frac{\vec{k}_2\times\vec{E}_2}{\omega\mu_2} \tag{16.3-13}$$

Since the electric vectors are all parallel to the interface $z = 0$, the continuity of the tangential components of $\vec{E}$-field gives

$$E_0 e^{j(\vec{k}\cdot\vec{r}-\omega t)} + E_{10} e^{j(\vec{k}_1\cdot\vec{r}-\omega t)} = E_{20} e^{j(\vec{k}_2\cdot\vec{r}-\omega t)}.$$

This condition must be valid for all values of t and for all points on the interface. This leads to the cancellations of the exponential factors and gives rise to ordinary laws of reflection and refraction. Thus,

$$E_0 + E_{10} = E_{20} \tag{16.3-14}$$

The continuity of the tangential components of $\vec{H}$-vectors across the interface $z = 0$ requires that

$$-H_0 \cos\theta + H_{10} \cos\theta_1 = -H_{20} \cos\theta_2. \tag{16.3-15}$$

Now

$$\vec{H} = \frac{\vec{k} \times \vec{E}}{\omega\mu_1} = \sqrt{\frac{\epsilon_1}{\mu_1}} \cdot \frac{\vec{k} \times \vec{E}}{k} \quad \text{[using Eq. (16.3-7)]}$$

$$\therefore \quad H_0 = \sqrt{\frac{\epsilon_1}{\mu_1}} E_0$$

Similarly,

$$H_{10} = \sqrt{\frac{\epsilon_1}{\mu_1}} E_{10} \quad \text{and} \quad H_{20} = \sqrt{\frac{\epsilon_2}{\mu_2}} E_{20}.$$

Therefore, Eq. (16.3-15) may be rewritten as

$$(-E_0 \cos\theta + E_{10} \cos\theta)\sqrt{\frac{\epsilon_1}{\mu_1}} = -E_{20} \cos\theta_2 \sqrt{\frac{\epsilon_2}{\mu_2}}, \tag{16.3-16}$$

where we use $\theta_1 = \theta$.

Solving Eqs. (16.3-14) and (16.3-16) we get

$$r_\perp = \frac{E_{10}}{E_0} = \frac{\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta - \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2}{\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2} \tag{16.3-17}$$

and

$$t_\perp = \frac{E_{20}}{E_0} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta}{\sqrt{\frac{\epsilon_1}{\mu_1}} \cdot \cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2}, \tag{16.3-18}$$

where $r_\perp$ is called *amplitude reflection coefficient* and $t_\perp$, the *amplitude transmission coefficient*. The subscript $\perp$ is used to denote that we are dealing with the case in which $\vec{E}$ is perpendicular to the plane of incidence. Equations (16.3-17) and (16.3-18) are known as *Fresnel's equations*.

Note that these equations give the amplitudes of the reflected and transmitted waves relative to the amplitude of the incident wave. Most dielectrics are essentially nonmagnetic and we can use $\mu_1 = \mu_2 \approx \mu_0$ and $n_1 = \sqrt{\epsilon_1/\epsilon_0}$ as the refractive index of medium 1 and $n_2 = \sqrt{\epsilon_2/\epsilon_0}$ as the refractive index of medium 2. In this case, Fresnel's Eqs. (16.3-17) and (16.3-18) take the following common from:

$$r_\perp = \frac{E_{10}}{E_0} = \frac{n_1 \cos\theta - n_2 \cos\theta_2}{n_1 \cos\theta + n_2 \cos\theta_2} \tag{16.3-19}$$

and

$$t_\perp = \frac{E_{20}}{E_0} = \frac{2n_1 \cos\theta}{n_1 \cos\theta + n_2 \cos\theta_2}. \tag{16.3-20}$$

Using Snell's law

$$\frac{\sin\theta}{\sin\theta_2} = \frac{n_2}{n_1},$$

the Fresnel's equations can be rewritten as

$$r_\perp = \frac{E_{10}}{E_0} = -\frac{\sin(\theta - \theta_2)}{\sin(\theta + \theta_2)} \tag{16.3-21}$$

and

$$t_\perp = \frac{E_{20}}{E_0} = \frac{2\cos\theta \sin\theta_2}{\sin(\theta + \theta_2)} \tag{16.3-22}$$

Case V. $\vec{E}$ *is parallel to the plane of incidence (p-polarized wave).*

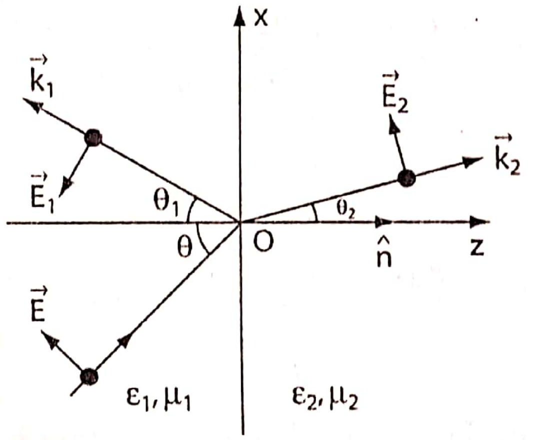

Fig 16.3-3: Reflection and refraction with $\vec{E}$-vector parallel to the plane of incidence.

Now we consider the case when $\vec{E}$-vector of the incident light is parallel to the plane of incidence (xz-plane) as shown in Fig 16.3-3. Since the media on the two sides of the interface $z = 0$ are isotropic the $\vec{E}$-vectors of reflected and refracted waves will also be in the plane of incidence. Corresponding magnetic vectors $\vec{H}$, $\vec{H}_1$ and $\vec{H}_2$ are perpendicular to the plane of incidence. Their directions are chosen so as to give a positive flow of energy in the direction of respective propagation vectors. Thus, in Fig 16.3-3 the vectors $\vec{H}$, $\vec{H}_1$ and $\vec{H}_2$ are all directed normally out of the plane of the paper and parallel to the interface $z = 0$.

The incident, reflected and refracted waves may be represented by the Eqs. (16.3-11)–(16.3-13) as before. Now the continuity of the tangential components of $\vec{E}$-fields gives

$$E_0 \cos\theta - E_{10} \cos\theta = E_{20} \cos\theta_2, \tag{16.3-23}$$

where we use $\theta_1 = \theta$.

The continuity of the tangential components of $\vec{H}$-fields gives

$$H_0 + H_{10} = H_{20}. \tag{16.3-24}$$

As before,

$$H_0 = \sqrt{\frac{\epsilon_1}{\mu_1}}E_0, \quad H_{10} = \sqrt{\frac{\epsilon_1}{\mu_1}}E_{10} \quad \text{and} \quad H_{20} = \sqrt{\frac{\epsilon_2}{\mu_2}}E_{20}.$$

Therefore, Eq. (16.3-24) becomes

$$(E_0 + E_{10})\sqrt{\frac{\epsilon_1}{\mu_1}} = E_{20}\sqrt{\frac{\epsilon_2}{\mu_2}} \tag{16.3-25}$$

Solving Eqs. (16.3-23) and (16.3-25) we get

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta - \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2}{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2} \tag{16.3-26}$$

and

$$t_{\parallel} = \frac{E_{20}}{E_0} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta}{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2}. \tag{16.3-27}$$

These two equations are known as *Fresnel's equation* for the case of polarization in the plane of incidence.

Most dielectrics are essentially nonmagnetic and we can use $\mu_1 = \mu_2 \approx \mu_0$,

$$n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}} \quad \text{and} \quad n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}.$$

In this case, Fresnel's Eqs. (16.3-26) and (16.3-27) take the following common form:

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{n_2\cos\theta - n_1\cos\theta_2}{n_2\cos\theta + n_1\cos\theta_2} \tag{16.3-28}$$

and

$$t_{\parallel} = \frac{E_{20}}{E_0} = \frac{2n_1\cos\theta}{n_2\cos\theta + n_1\cos\theta_2} \tag{16.3-29}$$

Using Snell's law

$$\frac{\sin\theta}{\sin\theta_2} = \frac{n_2}{n_1},$$

Fresnel's equation can be rewritten as

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{\sin 2\theta - \sin 2\theta_2}{\sin 2\theta + \sin 2\theta_2} = \frac{2\cos(\theta+\theta_2)\cdot\sin(\theta-\theta_2)}{2\sin(\theta+\theta_2)\cdot\cos(\theta-\theta_2)}$$

$$= \frac{\tan(\theta-\theta_2)}{\tan(\theta+\theta_2)} \quad (16.3\text{-}30)$$

$$\text{and} \quad t_{\parallel} = \frac{E_{20}}{E_0} = \frac{4\sin\theta_2\cdot\cos\theta}{\sin 2\theta + \sin 2\theta_2} = \frac{2\sin\theta_2\cdot\cos\theta}{\sin(\theta+\theta_2)\cos(\theta-\theta_2)}. \quad (16.3\text{-}31)$$

16.4 Physical Implications of Fresnel's Equations

Various important aspects of electromagnetic waves can be described by using Fresnel's equations. In particular, we are interested in polarizing effects, phase changes, total internal reflection, reflectance and transmittance.

1. Polarization by reflection

From Eq. (16.3-30) we find that $r_{\parallel} = 0$ for $\theta + \theta_2 = \pi/2$. This means that if $\theta + \theta_2 = \pi/2$ then electric field polarized parallel to the plane of incidence is not reflected at all. Equation (16.3-21) shows that under this condition $r_{\perp} \neq 0$, i.e., the electric field polarized normal to the plane of incidence is partly reflected. Thus, an unpolarized light consisting of both types of $\vec{E}$-fields incident at an angle $\theta = \theta_p$ satisfying the condition $\theta_p + \theta_2 = \pi/2$, will be plane polarized normal to the plane of incidence. This angle of incidence θ_p for which $r_{\parallel} = 0$ is known as *Brewster's angle or angle of polarization.* Snell's law in this case, gives

$$\frac{n_2}{n_1} = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\left(\frac{\pi}{2} - \theta_p\right)} = \tan\theta_p. \quad (16.4\text{-}1)$$

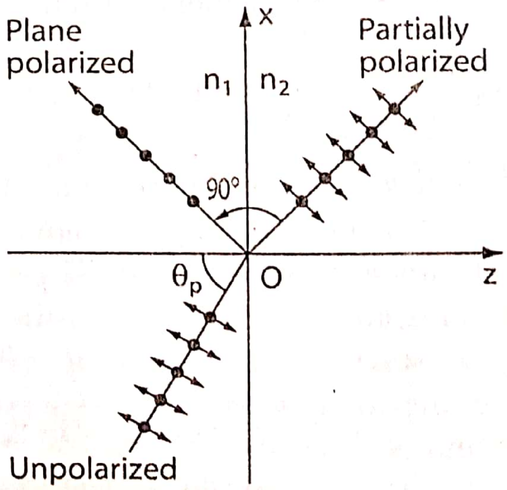

Fig 16.4-1: Brewster's angle.

Thus, *the tangent of the angle of polarization is equal to the refractive index of the reflecting medium relative to the incident medium.* This is known as *Brewster's law.* The law is illustrated in Fig 16.4-1. At the angle of polarization the reflected and refracted rays are 90° apart.

2. Amplitude coefficients and phase change

Fresnel's equations show that the amplitude coefficients depend on the value of the angle of incidence θ. Let us examine this dependence over the entire range of values of θ. For nearly normal incidence θ and θ_2 tend to zero. Then Eqs. (16.3-19), (16.3-20), (16.3-28) and (16.3-29) give

$$[r_\perp]_{\theta=0} = -\left[r_\parallel\right]_{\theta=0} = \frac{n_1 - n_2}{n_1 + n_2} \tag{16.4-2}$$

and

$$\left[t_\parallel\right]_{\theta=0} = [t_\perp]_{\theta=0} = \frac{2n_1}{n_1 + n_2}. \tag{16.4-3}$$

As an example, for air-glass interface $n_1 = 1$ and $n_2 = 1.5$, then

$$[r_\perp]_{\theta=0} = -\left[r_\parallel\right]_{\theta=0} = -0.2$$

and

$$\left[t_\parallel\right]_{\theta=0} = [t_\perp]_{\theta=0} = 0.8.$$

For normal incidence the plane of incidence becomes undefined and any distinction between the parallel and perpendicular components vanishes. So the physical result must be independent of polarization. The difference in signs of $r_\parallel$ and $r_\perp$ arises only because $\vec{E}$ and $\vec{E}_1$ are antiparallel in Fig 16.3-3 when θ goes to zero, where as $\vec{E}$ and $\vec{E}_1$ in Fig 16.3-2 point in the same direction. For grazing incidence θ tends to 90° then

$$r_\parallel = r_\perp = -1 \quad \text{and} \quad t_\parallel = t_\perp = 0. \tag{16.4-4}$$

For $n_2 > n_1$, from Snell's law $\theta_2 < \theta$ and hence from (16.3-21), $r_\perp$ is negative for all values of θ. The significance of this negative sign is that *the component of electric field perpendicular to the plane of incidence suffers a phase change $\Delta\phi_\perp = \pi$ when reflected from a surface backed by an optically denser medium.* In contrast Eq. (16.3-30) shows that for $n_2 > n_1$, i.e., $\theta > \theta_2$, $r_\parallel$ starts out positive at $\theta = 0$ and decrease gradually until it becomes zero at the polarizing angle $\theta = \theta_p$. As θ increases beyond θ_p, $r_\parallel$ becomes progressively more negative and reaches the value -1 at $\theta = 90°$ (Fig 16.4-2). Thus, for $n_2 > n_1$ and the electric field polarized parallel to the plane of incidence $r_\parallel$ is positive (i.e., no phase change, $\Delta\phi_\parallel = 0$) for $\theta < \theta_p$ and $r_\parallel$ is negative (i.e., a phase change of π) for $\theta > \theta_p$ (Fig 16.4-2).

16.4.1 Reflectance and Transmittance

The usual measurable quantities are not the amplitudes of the reflected and transmitted $\vec{E}$-fields, but the intensities of the reflected and transmitted waves. The intensity of a wave defined as the average energy per unit time crossing a unit area imagined in the plane normal to the direction of flow. This is related to the time average Poynting's vector,

$$I = \langle s \rangle = \tfrac{1}{2}\epsilon_1 E_0^2 \cdot v_1, \tag{16.4-11}$$

where $v_1 = 1/\sqrt{\epsilon_1\mu_1}$ is the velocity of the wave. Let the incident beam falls on an area A of the interface. Then the cross-sectional area of the incident, reflected and transmitted beams are respectively, $A\cos\theta$, $A\cos\theta_1$ and $A\cos\theta_2$. So the energy per unit time flowing in the incident beam is $IA\cos\theta$ and it is the power incident on the area A of the interface. Similarly, the power being reflected from area A is $I_1 A\cos\theta_1$ and the power being transmitted through A is $I_2 A\cos\theta_2$. We now define the reflectance (or the reflection coefficient) R of the surface as the fraction of incident power that is reflected and the *transmittance* (or the transmission coefficient) T as the fraction of incident power that is transmitted. Thus,

$$R = \frac{I_1 A\cos\theta_1}{IA\cos\theta} = \frac{I_1}{I} = \left(\frac{E_{10}}{E_0}\right)^2 \tag{16.4-12}$$

[∵ Incident and reflected waves are in the same medium]

and

$$T = \frac{I_2 A\cos\theta_2}{IA\cos\theta} = \frac{I_2\cos\theta_2}{I\cos\theta} = \left(\frac{E_{20}}{E_0}\right)^2 \cdot \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \cdot \frac{\cos\theta_2}{\cos\theta}. \tag{16.4-13}$$

For nonmagnetic dielectric medium,

$$T = \left(\frac{E_{02}}{E_0}\right)^2 \cdot \frac{n_2\cos\theta_2}{n_1\cos\theta}. \tag{16.4-14}$$

The principle of conservation of energy requires that the rate of energy flowing into area A must be equal to the rate of energy being reflected from it plus the rate of energy being transmitted through A, provided there is no loss at the interface. This requires that

$$R + T = 1. \tag{16.4-15}$$

For the case of waves with $\vec{E}$-vector parallel to the plane of incidence, we can write for R and T, using Eqs. (16.3-28) and (16.3-29) as

$$R_{\parallel} = \left(\frac{n_2\cos\theta - n_1\cos\theta_2}{n_2\cos\theta + n_1\cos\theta_2}\right)^2 \tag{16.4-16}$$

and

$$T_{\parallel} = \left(\frac{2n_1\cos\theta}{n_2\cos\theta + n_1\cos\theta_2}\right)^2 \cdot \frac{n_2\cos\theta_2}{n_1\cos\theta}. \tag{16.4-17}$$

They are plotted as a function of the angle of incidence in Fig 16.4-6. For waves polarized normal to the plane of incidence one can write, using Eqs. (16.3-19) and (16.3-20),

$$R_{\perp} = \left(\frac{E_{10}}{E_0}\right)^2 = \left(\frac{n_1 \cos\theta - n_2 \cos\theta_2}{n_1 \cos\theta + n_2 \cos\theta_2}\right)^2 \tag{16.4-18}$$

and

$$T_{\perp} = \left(\frac{E_{02}}{E_0}\right)^2 \cdot \frac{n_2 \cos\theta_2}{n_1 \cos\theta} = \left(\frac{2n_1 \cos\theta}{n_1 \cos\theta + n_2 \cos\theta_2}\right)^2 \cdot \frac{n_2 \cos\theta_2}{n_1 \cos\theta} \tag{16.4-19}$$

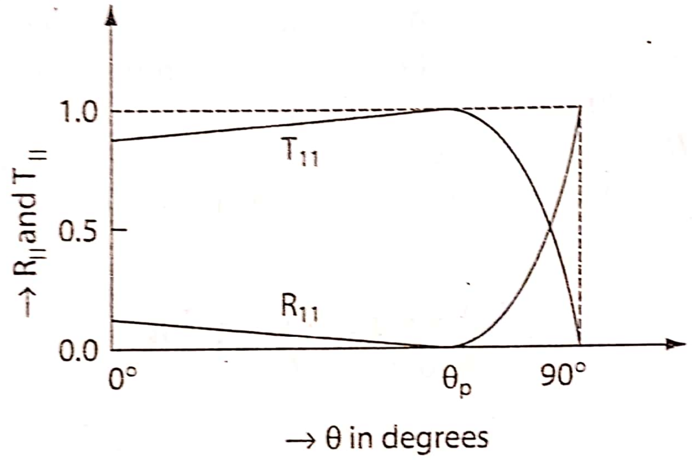

Fig 16.4-6: Reflectance and tramittance as a function of the angle of incidence for $\vec{E}$-field polarized parallel to the plane of incidence.

A plot of $R_{\perp}$ and $T_{\perp}$ as a function of the angle of incidence is shown in Fig 16.4-7.

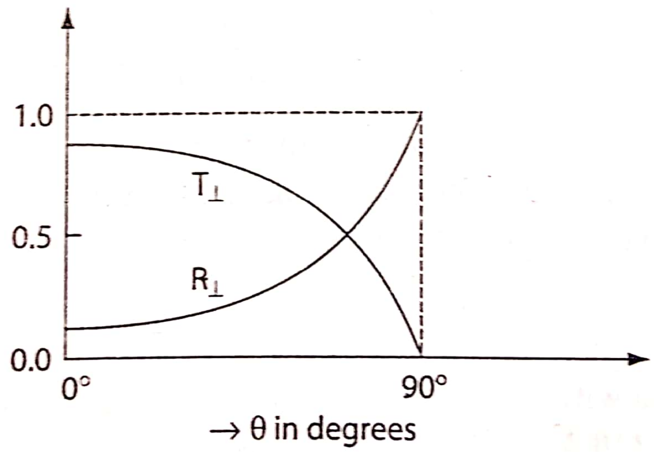

Fig 16.4-7: Reflectance and transmittance as a function of angle of incidence for $\vec{E}$-field polarized normal to the plane of incidence.

Note that the value of reflectance is high near grazing incidence. This explains why a glass, polished metal and smooth wet surfaces appear shining when looked at them tangentially.

For normal incidence which is a situation of great practical importance

$$\theta = \theta_2 = 0$$

$$\text{and} \quad R = R_{\perp} = R_{\parallel} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$\text{and} \quad T = T_{\perp} = T_{\parallel} = \frac{4n_1 n_2}{(n_1 + n_2)^2}.$$

Obviously, $R + T = 1$, which is in accordance with the principle of conservation of energy. For an air-glass surface $n_2/n_1 = 1.5$ and $R = 1/25$. This means that 4% of light incident normally on an air-glass interface is reflected back. For $n_1 > n_2$ and angle of incidence greater than θ_c we note from Eq. (16.4-9) that reflectance

$$R_{\perp} = |r_{\perp}|^2 = r_{\perp} \cdot r_{\perp}^* = 1.$$

Similarly, we can show that in this case, $R_{\parallel} = 1$. This means that all the energy is reflected when the angle of incidence exceeds the critical angle.

16.5 Reflection and Transmission at a Conducting Surface

Suppose xy-plane forms an interface ($z = 0$) between a dielectric medium (1) characterised by the permittivity ϵ_1 and permeability μ_1 and a conducting medium (2) with permittivity ϵ_2, permeability μ_2 and conductivity σ. Let a plane electromagnetic wave of frequency ω polarized along the x-direction and travelling along z-direction be incident normally on the interface. In Fig 16.5-1 $\vec{E}$, $\vec{H}$ describe the incident wave; $\vec{E}_1$, $\vec{H}_1$ describe the reflected wave travelling along negative z-direction and $\vec{E}_2$, $\vec{H}_2$ describe the transmitted wave.

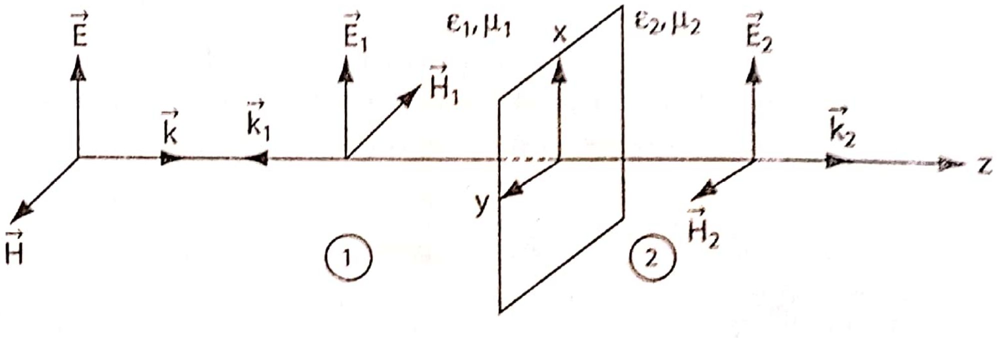

Fig 16.5-1: Reflection and transmission at normal incidence from a conducting surface.

Let us describe the wave fields by:

$$\begin{aligned} &\text{Incident wave:} \quad \vec{E} = \hat{i}E_0 e^{j(kz-\omega t)}, \quad \vec{H} = \hat{j}H_0 e^{j(kz-\omega t)} \\ &\text{Reflected wave:} \quad \vec{E}_1 = \hat{i}E_{10} e^{-j(k_1 z+\omega t)}, \quad \vec{H}_1 = -\hat{j}H_{10} e^{-j(kz+\omega t)} \\ &\text{Transmitted wave:} \quad \vec{E}_2 = \hat{i}E_{20} e^{j(k_2 z-\omega t)}, \quad \vec{H}_2 = \hat{j}H_{20} e^{j(k_2 z-\omega t)}, \end{aligned} \qquad (16.5\text{-}1)$$