OPERATION RESEARCH
SEQUENCING PROBLEM
BCA, KHARAGPUR COLLEGE

ALGORITHM TO SOLVE JOB SEQUENCING PROBLEM

Job sequencing is the arrangement of the task that is to be performed or processed in a machine in that particular order. Job sequencing problem has become the major problem in the computer field. A finite set of n jobs where each job consists of a chain of operations and a finite set of m machines where each machine can handle at most one assignment at a time. Each assignment needs to be evaluated during an uninterrupted period of a given length on a given machine and our Purpose is to find a inventory, that is, an allocation of the operations to time intervals to machines that has minimal length.

Solving a problem or getting a job done by a computer can be quite a demanding task. This is because it requires considerable thought, careful planning, and attention to details. It can also be challenging and exciting with possibilities for creative satisfaction. This is because when we are solving a problem with a computer, we are essentially teaching a dumb servant how to do a job. We have already seen that the description of how to do work is initially prepared in form of an algorithm and we did give a definition! But that definition is to be used with some care. We should always remember the capabilities and limitations of the

executing representative, the computer. Components of any model can be of

• Decision variables (What we can change to upgrade the system, i.e., model output)

• Parameters (Model input values, i.e., values that cannot be modified)

• Objective function (E.g. profit, quality to be maximized, costs, time to be minimized)

• Constraints (Determine which decision variable values are allowed)

Three basic types of decision variables are described as follows

• Sequence (Permutation of the jobs)

• Schedule (Allocation of the jobs in a more complex environment)

• Scheduling policy (Determines the next job, given the present state of the system)

Other variables are regarded as dynamic data, or as auxiliary variables. Some of the auxiliary variables are specified as follows.

- Completion time (C_{ij})

- Lateness (L_j)

- Tardiness (T_j)

- Makespan (M_j)

There are three primary objectives (goals) which apply to all inventory problems. They are

- Job completions should not be "late" (about due dates).

- The course of the time during which a job stays in the system should be

"minimum" (about flow time)

- It is greedy to fully utilize the capacities of work centers (about work center utilization).

A schedule is a job sequence determined for every machine of the processing system. Scheduling is a decision-making process of allocating limited resources to activities over time. The schedule can be characterized by starting or completion times of all operations of the jobs. The objective is to construct a schedule that minimizes a given objective function. The scheduling task is shown in figure 1.
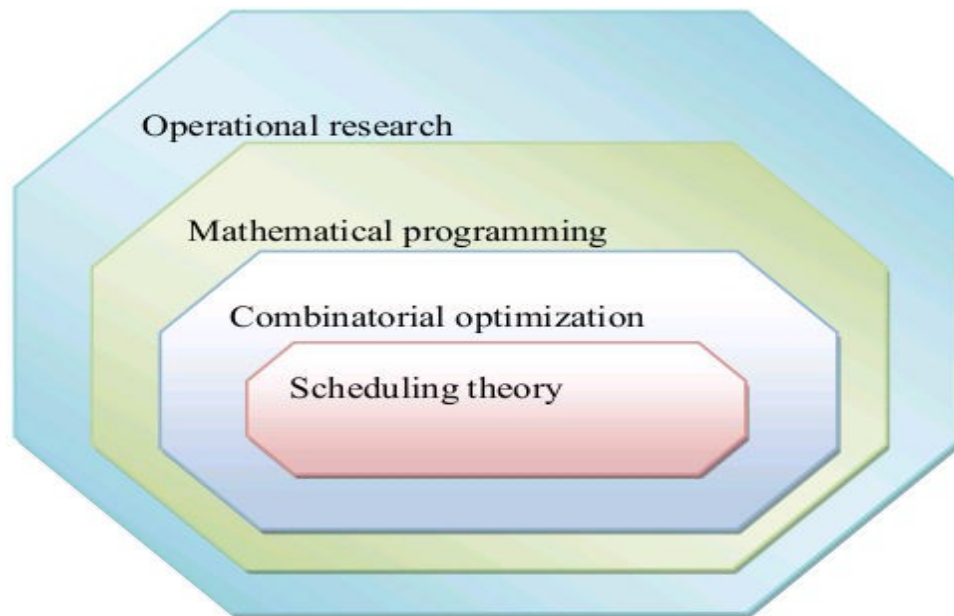

Fig 1: Scheduling Task

Standard scheduling requirements are specified as follows.

- A job cannot be processed by two or more machines at a time,

- A machine cannot process two or more jobs at the same time.

Depending on the type of scheduling system, specific constraints should be satisfied (jobs may be released at different times, there may be allowed preemption of jobs by other jobs, etc.).The various scheduling issues are described as follows

- Scheduling accord with the timing of operations

- The assignment is the allocation and prioritization of demand

- Significant issues are the type of scheduling, backword or forward

- The criteria for priorities

There are two types of scheduling. They are

- Forward scheduling

- Backward scheduling

Forward scheduling starts as soon as the requirements are known. It produces a feasible schedule though it may not meet due dates. It frequently results in buildup of work in process inventory.
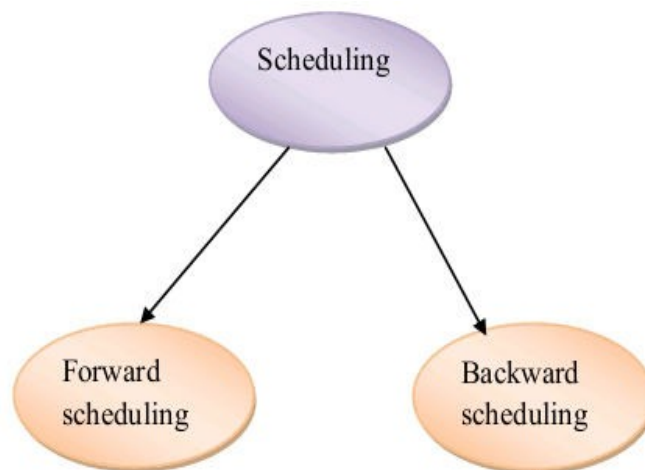
Fig 2: Types of scheduling

Backward scheduling begins with the due date and schedules the final operation first. Schedule is produced by working conversely though the processes Resources may not be available to accomplish the schedule. The forward and the backward scheduling is shown in the figure 2.

In a typical job scheduling conditions, there is a centralized scheduling stage with a set of on-demand technologies to support its functions. The IT operations and significance scheduling teams interact with the scheduler through a client tier or a Web interface. Conventionally, job scheduling solutions performed their work using platform specific promotors, including Microsoft Windows, UNIX, Linux, and mainframe operating systems. Job scheduling solutions schedule, define and manage application jobs, which are the individual steps needed to deliver the services. There has also been an effort to integrate and schedule jobs using the latest business intelligence, data warehouse, transform, and extract, load (ETL) applications. These solutions require integration of database activities.

The solutions process data from multiple applications across the enterprise to deliver the business insight required to make the right strategic decisions at the right time. Administering dependency logic and automating job step sequencing are critical to the delivery of relevant and timely information for strategic decision making. Job scheduling solutions can also manage backup, file delivery, and storage processes. More recently, job schedulers have been integrated with virtualization solutions and certain cloud services that have the capability to schedule jobs in a virtualized environment.
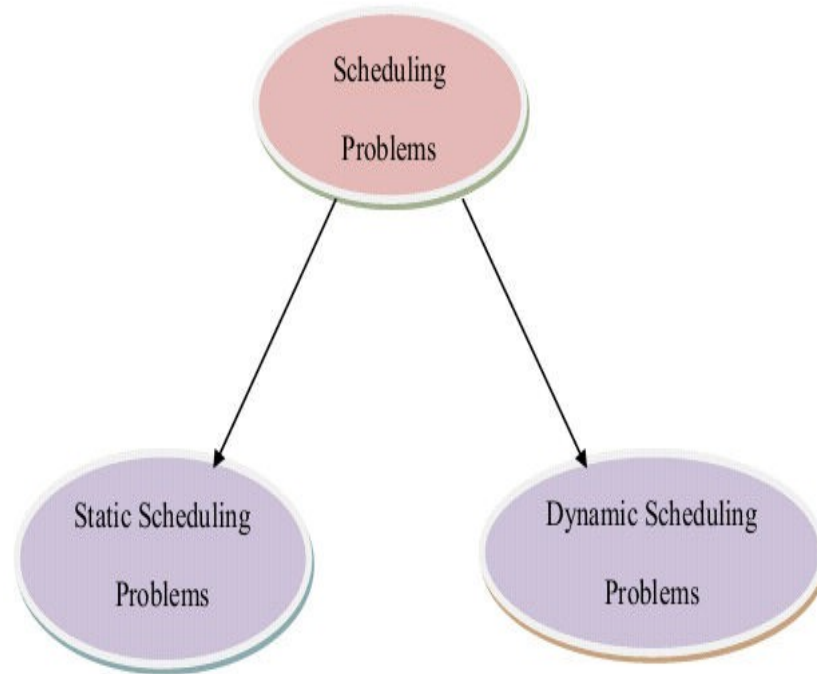
Fig 3: Types of scheduling problems

There are two fundamental kinds of scheduling problems. They are given as follows

- Static scheduling problem

- Dynamic scheduling problem

The static scheduling problem deals with the problem which consists of a fixed set of jobs which are to be all completed. In this static scheduling the number of jobs does not change. The Dynamic scheduling problem deals with scheduling of a continuous situation, which means that, new jobs are continually added to the production environment. The types of scheduling problems are shown in figure 3.

The static scheduling problem contains a fixed set of jobs which are to be completed. The typical basic assumptions in this kind of problem can be stated as follows;

- Complete set of jobs arrive simultaneously, and all respective work centers are available at that time.

Majority of the static scheduling problems use a criterion called "minimum make span " which can be defined as the minimum total time to process all subject jobs. This standard is related with the scheduling objective which is about "flow time".
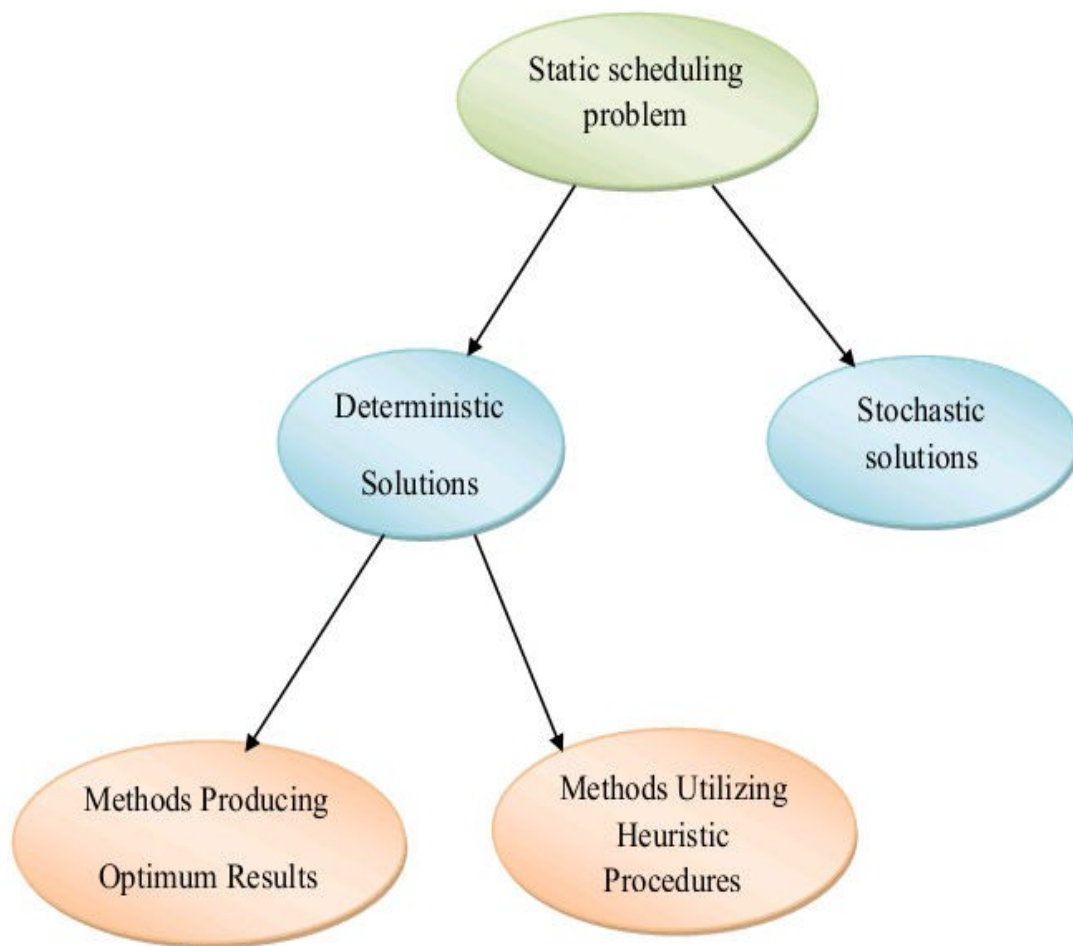

Fig 4: Static scheduling approach

The static scheduling approach makes use of deterministic solutions and stochastic solutions. The deterministic solutions use known and non varying

process times and it uses methods producing optimum results and methods utilizing heuristic procedures. The

static scheduling approach is shown in the figure 4. Methods producing Optimum results are only applicable to relatively small problems. Methods utilizing heuristic procedures are also called as dispatching or sequencing rules by which scheduling decisions are made sequentially. Stochastic solutions use process times which are subject to random variations. The dynamic scheduling approach is shown in figure 5.
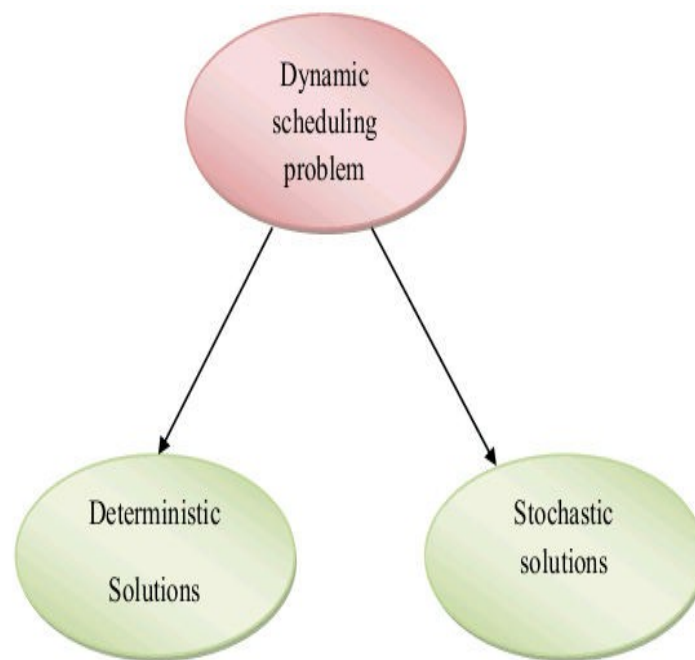
Fig 5: Dynamic scheduling approach

Analytical approaches like deterministic solutions, stochastic solutions had been developed which are "based on queuing models that provide expected steady state conditions for certain kinds of situations and time distributions. Typically, the criteria used in such queuing cases are; Average Flow Time, Average Work-in-process or number of jobs in the system and Work/machine center utilization. To find a solution to dynamic scheduling problems, usually the approach is, to use

different "dispatching rules" at the work centers.

The common priority sequencing rules are given as follows
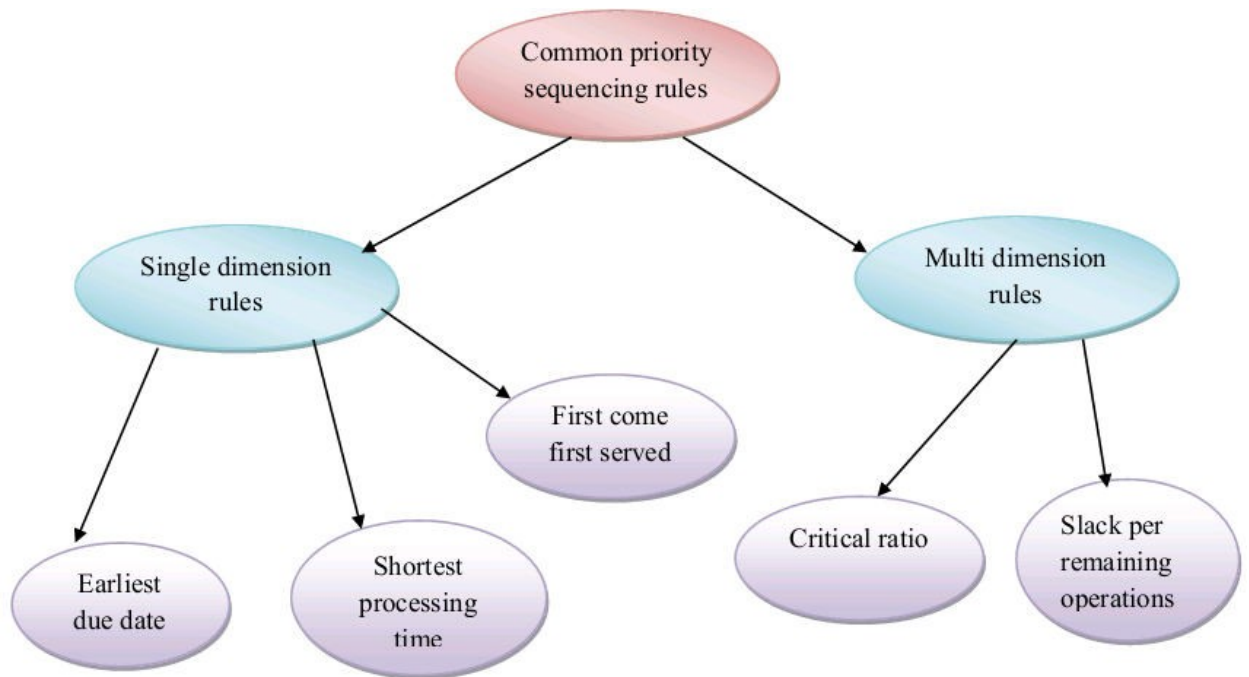
Fig 6: Sequencing rules

Common priority sequencing rules are single dimension rules and multi dimension rules. Single dimension rules base the priority on single aspect of the job, such as, the due date, arrival time, or the processing time at the current work place. Multi dimension rules incorporate information about the remaining work centers at which the job must be processed, in addition to the processing time at the present work station or the due date considered by single dimension rules.

The single dimension rules are

- Earliest due date

There may be different ways to set the due dates of jobs. For example, a due date might have been calculated by computerized methods as in MRP or might have been simply determined by the customer.

The job with the earliest due date is the job which is to be scheduled next.

- Shortest processing time

It is the king of the priority sequencing rules. The job which has the shortest processing time is scheduled next.

- First come first served

The job which arrives the work station earlier than the others has the highest priority and is to be scheduled next.

The Multi dimension rules are

- Critical ratio

The critical ratio is calculated by dividing "the time remaining until the due date of the job" by the "remaining total shop time for the job".

- Slack per remaining operations

Slack is the difference between "the time remaining until the due date of the job" and "remaining total shop time for the job" including that of the operation being scheduled.

The job's priority is determined by dividing the slack by the number of the operations

that remain including the operation which is being scheduled.

Scheduling deals with the problems of optimal arrangement, sequencing and timetabling. Sequencing is similar to that of scheduling. Job Sequencing is the arrangement of the tasks required to be carried out sequentially. The job sequencing problem can be solved by means of the new modified heuristic technique called time deviation method. The modified time deviation method can be used for processing n jobs through two machines, n jobs through three machines, n jobs through m machines.

4.1   MODIFIED TIME DEVIATION METHOD

Time deviation method is used to obtain the optimal sequence of the jobs. In this method time duration table is calculated for each job in the row wise and the column wise.

- Maximum time duration of the row minus the time duration of the cell gives the row deviation of the cell in the time duration table.

$$p_{ij} = r_i - t_{ij} \qquad\qquad (1)$$

Where r_i is the maximum time of the ith row, p_{ij} is the row time deviation of the $(i, j)^{th}$ cell and t_{ij} be the time required for processing i^{th} job on the jth machine.

- Maximum time duration of the column minus the time duration of the cell gives the column deviation of the cell in the time duration table.

$$c_{ij} = \frac{t_{ij}}{s_i} - \qquad\qquad (2)$$

Where s_i is the maximum time of the ith column, c_{ij} is the column time deviation of

the $(i, j)^{th}$ cell and t_{ij} be the time required for processing i^{th} job on the jth machine.

The time deviation method used for solving job sequencing problem is explained with the examples.

4.2   SEQUENCING N JOBS IN TWO MACHINES

The N jobs can be sequenced in two machines by the following steps.

- Calculate the time deviation table for the sequencing problem.

- The cell which has both the time deviation vectors as zero for machine 1, perform that job first.

- If more than one cell has both vectors as zero then calculate sum deviation of the corresponding columns. Cell which has largest sum deviation is performed first and so on.

- Similar steps are followed for machine 2 but here the jobs are performed lastly.

- Again calculate the reduced time deviation table for all non assigned jobs and continue the steps mentioned above.

- Stop the process if we get sequence involving all jobs.

- Arrange the obtained sequence in reverse order to get minimum total elapsed time.

Sequencing N jobs in two machines can be explained by the following example.

Example: Processing of N jobs in two machines is given as follows

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7	J8	J9
M1	2	5	4	9	6	8	7	5	4
M2	6	8	7	4	3	9	3	8	11

The time deviation table will be first calculated for the given problem according to the equations (1) and (2). The table is as follows

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7	J8	J9
M1	(7,4)	(4,3)	(5,3)	(0,0)	(3,0)	(1,1)	(2,0)	(4,3)	(5,7)
M2	(5,0)	(3,0)	(4,0)	(7,5)	(8,3)	(2,0)	(8,4)	(3,0)	(0,0)

The jobs J4 of machine 1 and J9 of machine 2 has both vectors zero and is assigned as follows

J4								J9

The reduced time duration table for the remaining jobs is as follows

Jobs/Machines	J1	J2	J3	J5	J6	J7	J8
M1	2	5	4	6	8	7	5
M2	6	8	7	3	9	3	8

The time deviation table is calculated for this sequence as follows

Jobs/Machines	J1	J2	J3	J5	J6	J7	J8
M1	(6,4)	(3,3)	(4,3)	(2,0)	(0,1)	(1,0)	(3,3)
M2	(3,0)	(1,0)	(2,0)	(6,3)	(0,0)	(6,4)	(1,0)

Job J6 in machine 2 has both the vectors as zero and so it is assigned to the sequence as

J4						J6	J9

The time duration table for the jobs J1, J2, J3, J5, J7, and J8 is specified and the jobs J4, J6, J9 will not be specified in this table since it is assigned to the machines as shown above.

Jobs/Machines	J1	J2	J3	J5	J7	J8
M1	2	5	4	6	7	5
M2	6	8	7	3	3	8

The time deviation table for the above jobs is calculated as follows

Jobs/Machines	J1	J2	J3	J5	J7	J8
M1	(5,4)	(2,3)	(3,3)	(1,0)	(0,0)	(2,3)
M2	(2,0)	(0,0)	(1,0)	(5,3)	(5,4)	(0,0)

The jobs J7 in machine 1, J2 and J8 in machine 2 have both vectors zero and it is assigned as follows

J4	J7				J2	J8	J6	J9

The time duration table for the remaining jobs J1, J3, J5 is given as follows

Jobs/Machines	J1	J3	J5
M1	2	4	6
M2	6	7	3

The time deviation table for the above jobs is calculated as follows

Jobs/Machines	J1	J3	J5
M1	(4,4)	(2,3)	(0,0)
M2	(1,0)	(0,0)	(4,3)

Here J5 in machine 1 and J3 in machine 2 contain both the vectors as zero and so it is assigned and the remaining J1 is also assigned as follows

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J4	J7	J5	J1	J3	J2	J8	J6	J9

This sequence is then arranged in the reverse order from last obtained job to the machine 1 jobs and machine 2 jobs. It is represented as follows

J1	J5	J7	J4	J9	J6	J8	J2	J3

The above sequence is the required sequence for the given problem. The minimum total elapsed time is calculated for this job sequence. The total elapsed time is given as Total Elapsed Time = Time between starting the first job in the optimum sequence on machine 1 and completing the last job in the optimum sequence on machine M.

Job sequence	Machine 1		Machine 2	
	Time-in	Time-out	Time-in	Time-out
1	0	2	2	8
5	2	8	8	11
7	8	15	11	14
4	15	24	14	18
9	24	28	18	29
6	28	36	29	38
8	36	41	38	46
2	41	46	46	54
3	46	50	54	61

The minimum total elapsed time thus obtained is 61 hours.

4.3   SEQUENCING N JOBS IN THREE MACHINES

The N jobs can be sequenced in three machines by the following steps

- Calculate the time deviation table for the sequencing problem.

- The cell which has both the time deviation vectors as zero for machine 1, perform that job first. If both the time deviation vectors are zero in machine 3 then perform that job in the last and if the vectors are zero in machine 2 then find the sum of deviation vectors separately for above and below of the zero cell. Compare both the deviations.

- Perform that particular job first if the sum of deviation vectors above the cell is less than the other one. If both are same then perform the job either in first or last.

- Again calculate the reduced time deviation table for all non assigned jobs and continue the steps mentioned above.

- Stop the process if we get sequence involving all jobs.

- Arrange the obtained sequence in reverse order to get minimum total elapsed time.

Example: N jobs to be processed in three machines are given as follows

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	11	5	6	12

The time deviation table will be first calculated for the given problem according to the equations (1) and (2). The table is as follows

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7
M1	(6, 3)	(1, 0)	(2, 0)	(5, 7)	0, 0	(1, 0)	(2, 5)
M2	(1, 2)	(2, 5)	(3, 5)	(0, 6)	(4, 8)	(1, 4)	(2, 9)
M3	(6, 0)	(5, 1)	(7, 2)	(1, 0)	(7, 4)	(6, 2)	(0, 0)

Jobs J5 in machine 1 and J7 in machine 3 have both the vectors zero and so they are assigned to job sequence as follows

J5						J7

The time duration table for the jobs other than J5 and J7 are given as follows

Jobs/Machines	J1	J2	J3	J4	J6
M1	3	8	7	4	8
M2	4	3	2	5	4
M3	6	7	5	11	6

The time deviation table for the above jobs is specified as follows

Jobs/Machines	J1	J2	J3	J4	J6
M1	(5, 3)	(0, 0)	(1, 0)	(4, 7)	(0, 0)
M2	(1, 2)	(2, 4)	(3, 5)	(0, 6)	(1, 4)
M3	(5, 0)	(4, 1)	(6, 2)	(0, 0)	(5, 2)

The jobs J2, J6 in machine 1 and J4 in machine 3 have both the vectors zero and so they are arranged as follows

J5	J6	J2			J4	J7

The time duration table for the remaining jobs is given as follows

Jobs/Machines	J1	J3
M1	3	7
M2	4	2
M3	6	5

The time deviation table for the above two jobs are given as follows

Jobs/Machines	J1	J3
M1	(4, 3)	(0, 0)
M2	(0, 2)	(2, 5)
M3	(0, 0)	(1, 2)

The job J3 in machine 1 and the job J1 in machine 3 are then assigned to the sequence as follows

J5	J6	J2	J3	J1	J4	J7

This sequence is then arranged in the reverse order from last obtained job to the machine 1 jobs and machine 2 jobs. It is represented as follows

J1	J3	J2	J6	J5	J7	J4

The minimum total elapsed time is calculated for the obtained sequence as follows

Job sequence	Machine 1		Machine 2		Machine 3	
	Time-in	Time-out	Time-in	Time-out	Time-in	Time-out
1	0	3	3	7	7	13
3	3	10	7	9	13	18
2	10	18	9	12	18	25
6	18	26	12	16	25	31
5	26	35	16	17	31	36
7	35	42	17	20	36	48
4	42	46	20	25	48	59

Thus the minimum total elapsed time for the given sequence is 59 hours.

4.4   SEQUENCING N JOBS IN M MACHINES

The N jobs can be sequenced in M machines by the following steps

- The M machine problem is first converted in to the two machine problem by either of the following conditions

Minimum of M1 ≥ Maximum of (M2, M3 … Mm-1)   (3)

Minimum of Mm ≥ Maximum of (M2, M3 … Mm-1)   (4)

Or both hold.

- The required sequence is then obtained by applying the time deviation method as done in two machine problem previously.

Example: Sequencing of N jobs in M machines can be given by the following problem

Jobs/Machines	J1	J2	J3	J4
M1	7	6	5	8
M2	5	6	4	3
M3	2	4	5	3
M4	3	5	6	2
M5	9	10	8	6

The M machine problem is converted into two machine problem by the conditions (3) and (4). The converted two machine problem is given as follows

Jobs/Machines	J1	J2	J3	J4
G	17	21	20	16
H	19	25	23	14

The required job sequence is then obtained by the procedure that is normally applied for the two machine problem as explained by the above example.

4.5 Conclusion

Thus the new modified heuristic technique called time deviation method is used to obtain the required job sequence and the minimum total elapsed time is also calculated for this sequence of jobs by the usual procedure as specified in the above examples. Processing of N jobs in 2 machines, Processing of N jobs in 3 machines, Processing of N jobs in M machines were also explained by the examples.