NETWORK TECHNIQUES (PERT & CPM)
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PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)
7.1: An Introduction

A project defines a combination of interrelated activities that must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logic sequences in the sense that some activities cannot start until others are completed .An activity in a project is usually viewed as a job requiring time and resources for its completion .In general, a project is a one – time effort; that is, the same sequence of activities may not be repeated in the future.

In the past ,the scheduling of a project (over time) was done with little specifies the start and finish for each activity on a horizontal time scale . Its disadvantage is that the interdependency between the different activities(which mainly controls the progress of the project)cannot be determined from the bar chart .The growing complexities of today's project have demanded more systematic and more effective planning techniques with the objective of optimizing the efficiency of executing the project .Efficiency here implies effecting almost reduction in the time required to complete the project while accounting for the economic feasibility of using available resources .
Project management has evolved as a new field with the development of two analytic techniques for planning, scheduling, and controlling of projects.

7.2 Background:
These are the critical path method (CPM) and the project evaluation and review technique (PERT). The two techniques were developed by two different groups almost simultaneously (1956- 1958) CPM was first developed by E.Idu Pout de Nemours and company as an application to construction projects and was later extended to a more advanced status by Mauchly associates . PERT, on the other hand was developed for the U.S. Navy by a consulting firm for scheduling the research and development activities for the Polaris missile program.

PERT and CPM are basically time –oriented methods in the sense that they both lead to the determination of a time schedule. Although the two methods were developed independently, they are strikingly similar. Perhaps the most important difference is that originally the time estimates for the activities were assumed deterministic in CPM and probabilistic in PERT. Today , PERT and CPM actually comprise one technique and the differences , if any, are only historical . Consequently, both techniques will be referred to as "project scheduling" techniques.

Project scheduling by PERT-CPM consist of three basic phases

1. Planning
2. Scheduling and
3. Controlling.

7.3 Planning Phase

The planning phase is initiated by breaking down the project into distinct activities . The time estimates for these activities are then determined and a network (or arrow)diagram is constructed with each of its arcs(arrows) representing an activity. The entire arrow diagram gives a graphic representation of the interdependencies between the activities of the project. The construction of the arrow diagram as a planning phase has the advantage of studying the different jobs in detail .perhaps suggesting improvement before the project is actually executed . More important will be its use to develop a schedule for the project.

7.4 Scheduling phase

The ultimate objective of the Scheduling phase is to construct a time chart showing the start and finish times for each activity as well as its relationship to other activities in the project. In addition, the Schedule must pinpoint the critical (in view of time) activities that require special attention if the project is to be completed on time. For the non critical activities the schedule must show the amount of slack or float time that can be used advantageously when such activities are delayed or when limited resources are to be used effectively.

7.5 Controlling

The final phase in project management is project control .This includes the use of the arrow diagram and the time chart for making periodic progress reports. The network may thus be updated and analyzed and, if necessary, a new schedule is determined for the remaining portion of the project.

7.6 Project Management

Project management is concerned with the overall planning and co-ordination of a project from conception to completion aimed at meeting the stated requirements and ensuring completion on time, within cost and to required quality standards. Project management is normally reserved for focused, non- repetitive, time-limited activities with some degree of risk and that are beyond the usual scope of operational activities for which the organization is responsible. A project is a temporary endeavour involving a connected sequence of activities and a range of resources, which is designed to achieve a specific and unique outcome and which operates within time, cost and quality constraints and which is often used to introduce change.

7.7 Characteristic of a project

- A unique, one-time operational activity or effort
- Requires the completion of a large number of interrelated activities
- Established to achieve specific objective
- Resources, such as time and/or money, are limited
- Typically has its own management structure
- Need leadership
- The application of a collection of tools and techniques to direct the use of diverse resources towards the accomplishment of a unique, complex, one time task within time, cost and quality constraints.
- Used the techniques of operational research to plan the optimum use of resources.
- One of these techniques was the use of networks to represent a system of related activities

7.8 Project Management Process

- Project planning
- Project scheduling
- Project control
- Project team -made up of individuals from various areas and departments within a company
- Matrix organization -a team structure with members from functional areas, depending on skills required

- Project Manager -most important member of project team Scope statement -a document that provides an understanding, justification, and expected result of a project
- Statement of work -written description of objectives of a project
- Organizational Breakdown Structure -a chart that shows which organizational units are responsible for work items
- Responsibility Assignment Matrix -shows who is responsible for work in a project

7.9 Work breakdown structure

- A method of breaking down a project into individual elements (components, subcomponents, activities and tasks) in a hierarchical structure which can be scheduled and cost
- It defines tasks that can be completed independently of other tasks, facilitating resource allocation, assignment of responsibilities and measurement and control of the project
- It is foundation of project planning
- It is developed before identification of dependencies and estimation of activity durations
- It can be used to identity the tasks in the CPM and PERT

7.10 Project Planning

- Resource Availability and/or Limits
  - Due date, late penalties, early completion incentives
  - Budget
- Activity Information
  - Identify all required activities
  - Estimate the resources required (time) to complete each activity
  - Immediate predecessor(s) to each activity needed to create interrelationships

7.11 Project Scheduling and Control Techniques

1. Critical Path Method (CPM)
2. Program Evaluation and Review Technique (PERT)

7.12 Project Network

- Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects

- Use of nodes and arrows

  Arrows → An arrow leads from tail to head directionally Indicate ACTIVITY, a time consuming effort that is required to perform a part of the work.

  Nodes A node is represented by a circle Indicate **EVENT,** a point in time where one or more activities start and/or finish

- Activity
  - A task or a certain amount of work required in the project
  - Requires time to complete
  - Represented by an arrow
- Dummy Activity
  - Indicates only precedence relationships
  - Does not require any time of effort
- Event
  - Signals the beginning or ending of an activity
  - Designates a point in time
  - Represented by a circle (node)
- Network
  - Shows the sequential relationships among activities using nodes and arrows
- Activity-on-node (AON)

  nodes represent activities, and arrows show precedence relationships
- Activity-on-arrow (AOA)

  arrows represent activities and nodes are events for points in time

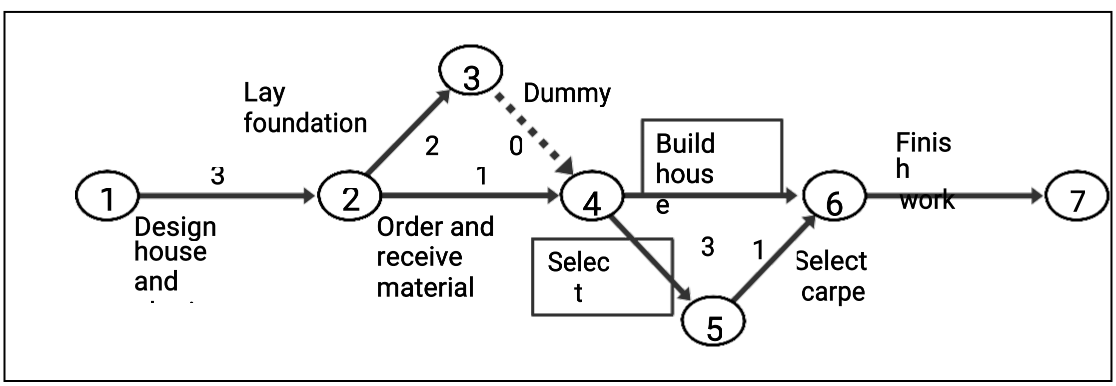


Fig 7.1 : AOA Project Network for House

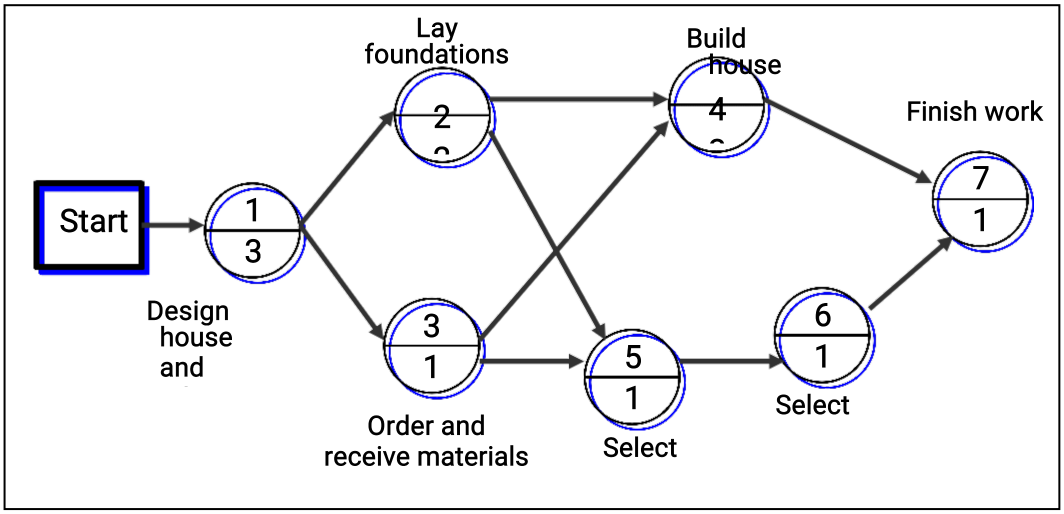


Fig7.2: AON Project Network for House Situations in network diagram

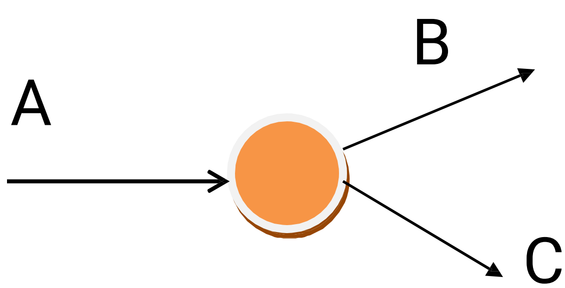


Fig 7.3: A must finish before either B or C can start

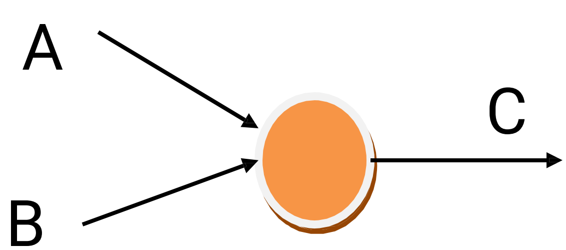


Fig 7.4: both A and B must finish before C can start

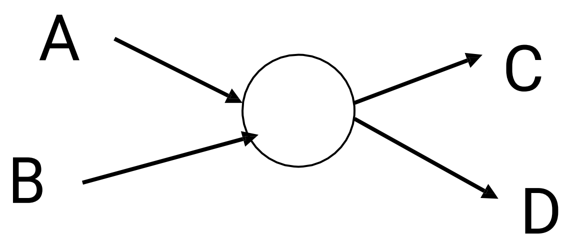


Fig 7.5: both A and C must finish before either of B or D can start

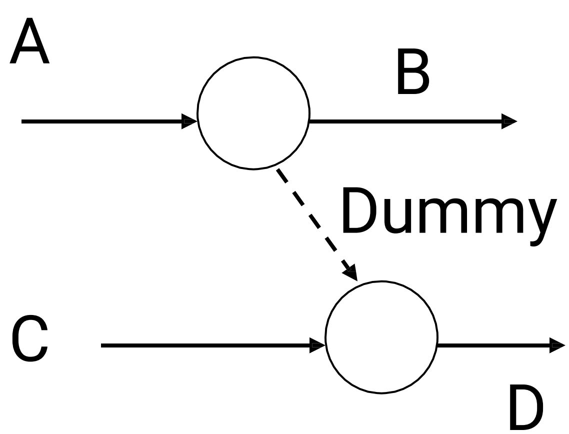


Fig 7.6: A must finish before B can start
both A and C must finish before D can start

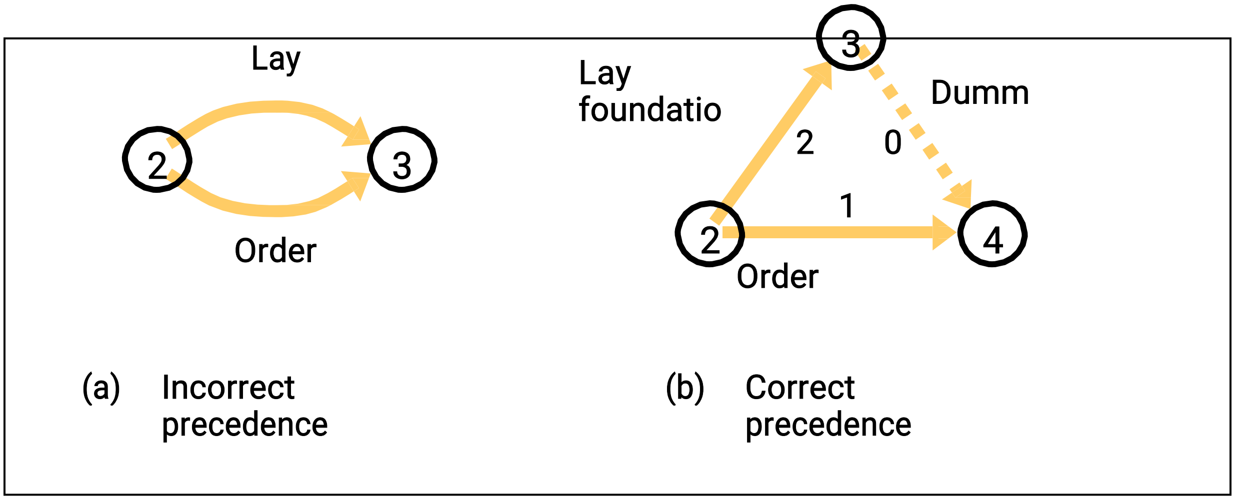


Fig 7.7: Concurrent Activities

7.12 Uncertainty in project scheduling

During project execution, however, a real-life project will never execute exactly as it was planned due to uncertainty. It can be ambiguity resulting from subjective estimates that are prone to human errors or it can be variability arising from unexpected events or risks. The main reason that the Project Evaluation and Review Technique (PERT) may provide inaccurate information about the project completion time is due to this schedule uncertainty. This inaccuracy is large enough to render such estimates as not helpful.

One possibility to maximize solution robustness is to include safety in the baseline schedule in order to absorb the anticipated disruptions. This is called proactive scheduling. A pure proactive scheduling is a utopia, incorporating safety in a baseline schedule that allows to cope with every possible disruption would lead to a baseline schedule with a very large make-span. A second approach, reactive scheduling, consists of defining a procedure to react to disruptions that cannot be absorbed by the baseline schedule.

CPM – CRITICAL PATH METHOD

7.13 An Introduction:

In 1957, Dupont developed a project management method designed to address the challenge of shutting down chemical plants for maintenance and then restarting the plants once the maintenance had been completed. Given the complexity of the process, they developed the Critical Path Method (CPM) for managing such projects.

CPM provides the following benefits:

- Provides a graphical view of the project
- Predicts the time required to complete the project.
- Shows which activities are critical to maintaining the schedule and which are not.

CPM models the events and activities of a project as a network. Activities are depicted as nodes on the network and events that signify the beginning or ending of activities are depicted as arcs or lines between the nodes.

7.14 Work Breakdown Structures (WBS)

The development of a project plan is predicated on having a clear, and detailed understanding of both the tasks involved, the estimated length of time each task will take, the dependencies between those tasks, and the sequence in which those tasks have to be performed. Additionally, resource availability must be determined in order to assign each task or group of tasks to the appropriate worker.

One of the methods used to develop the list of tasks is to create what is known as a Work Breakdown Structure (WBS).

- A definition

A Work Breakdown Structure (WBS) is a hierarchic decomposition or breakdown of a project or major activity into successively levels, where each level is a finer breakdown of the preceding one. In final form a WBS is very similar in structure and layout to a document outline. Each item at a specific level of a WBS is numbered consecutively (e.g. 10, 20, 30, 40, 50). Each item at the next level is numbered within the number of its parent item (e.g. 10.1, 10.2, 10,3, 10.4)

The WBS may be drawn in diagrammatic form (if automated tools are available) or in a chart form resembling an outline.

The WBS begins with a single overall task representing the totality of work to be performed on the project. This becomes the name of the project plan WBS

Using a methodology or system life cycle steps as a guide, the project is divided into its major steps. In our case we will use the Unified Process phases. Each of these phases must be broken down into their next level of detail, and each of those, into still finer level of detail, until a manageable task size is arrived at. The first level of Work Breakdown Structure for the unified process is illustrated in Table 6.1

Tasks at each successively finer level of detail are numbered to reflect the task from which they were derived. Thus the first level of tasks would be numbered 1.0, 2.0, 3.0, and so forth. Each of their sub-tasks would have a two part number. The first part reflecting the parent task and the second part being the sub-task number itself e.g. 1.1, 1.2, 1.3, etc. As each of these is in turn decomposed or broken down into their component tasks, the components receive a number comprised of its parent's number plus a unique number of its own.

- Another Definition:

A manageable task is one where the expected results can be easily identified, success, failure, or completion of the task can easily be ascertained, the time to complete the task can easily be estimated, and the resource requirements of the task can easily be determined.

WBS Number	Task Description
1.0	Inception
1.1	Draft Project Plan
2.0	Elaboration
2.1	Plan User Interviews
2.2	Schedule User Interviews
3.0	Construction
4.0	Transition

Table 7.1 First level of Work Breakdown Structure for the life cycle

7.15 Steps in CPM Project Planning:

1. Specify the individual activities.
2. Determine the sequence of those activities.
3. Draw a network diagram.
4. Estimate the completion time for each activity.

5. Identify the critical path (the longest path through the network)
6. Update the CPM diagram as the project progresses.

1. Specify the individual activities.

From the Work Breakdown Structure, a listing can be made of all the activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.

2. Determine the sequence of those activities.

Some activities are dependent upon the completion of others. A listing of the immediate predecessors of each activity is useful for constructing the CPM network diagram.

3. Draw a network diagram.

Once the activities and their sequencing have been defined, the CPM diagram can be drawn. CPM originally was developed as an activity on node (AON) network, but some project planners prefer to specify the activities on the arcs.

4. Estimate the completion time for each activity.

The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number can be used for an activity's time estimate.

5. Identify the critical path

The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the following four parameters for each activity:

- ES – earliest start time: the earliest time at which an activity can begin given that its predecessor activities must be completed first.

- EF – earliest finish time, equal to the earliest start time for the activity plus the time required to complete the activity.
- LF – latest finish time: the latest time at which an activity can be completed without delaying the project.
- LS – latest start time, equal to the latest finish time minus the time required to complete the activity.

The slack or float time for an activity is the time between the earliest and latest start time, or between the earliest and latest finish time. Slack is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project.

The critical path is the path through the project network in which none of the activities have slack, that is, the path for which LS=ES and LF=EF for all activities in the path. A delay in the critical path delays the project. Similarly, to accelerate the project it is necessary to reduce the total time required for the activities in the critical path.

6. Update CPM diagram

As the project progresses, the actual task completion times will be known and the diagram can be updated to include this information. A new critical path may emerge, and structural changes may be made in the network if project requirements change.

7.16 CPM limitations

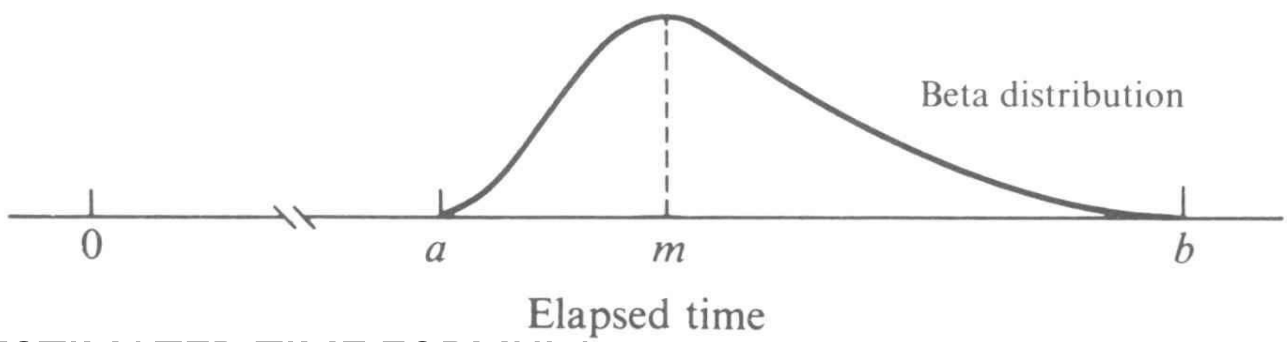
CPM was developed for complex but fairly routine projects with minimal uncertainty in project completion times. For less routine projects there is more uncertainty in the completion times, and this uncertainty limits the usefulness of the deterministic CPM model. An alternative to CPM is the PERT (Program Evaluation and Review Technique) project planning model, which allows a range of durations to be specified for each activity.

7.17 Time Estimates

Networks provide a planned approach to project management. To be effective, networks require a clear definition of all the tasks that make up the project and pertinent time estimates. If the project manager cannot clarify the necessary tasks and the resource requirements, then no matter how sophisticated the network, it will not compensate for these shortcomings. A number of claims have been made about the benefits of project management techniques but others have argued that in part, the benefits are due to managers having to know and clarify the tasks rather than the diagram which follows (which may by then be self -evident).

The objectives of network analysis are to locate the activities that must be kept to time, manage activities to make the most effective use of resources and look for ways of reducing the total project time. For any but the smallest projects, this analysis is likely to be done using a computer package, but your understanding of the output will only develop if you have some experience of the basic steps of analysis.

ACTIVITY DURATION ESTIMATION – BETA DISTRIBUTION



ESTIMATED TIME FORMULA

$$TE = \frac{A + 4(B) + C}{6}$$

WHERE:

A = MOST OPTIMISTIC TIME B

= MOST LIKELY TIME

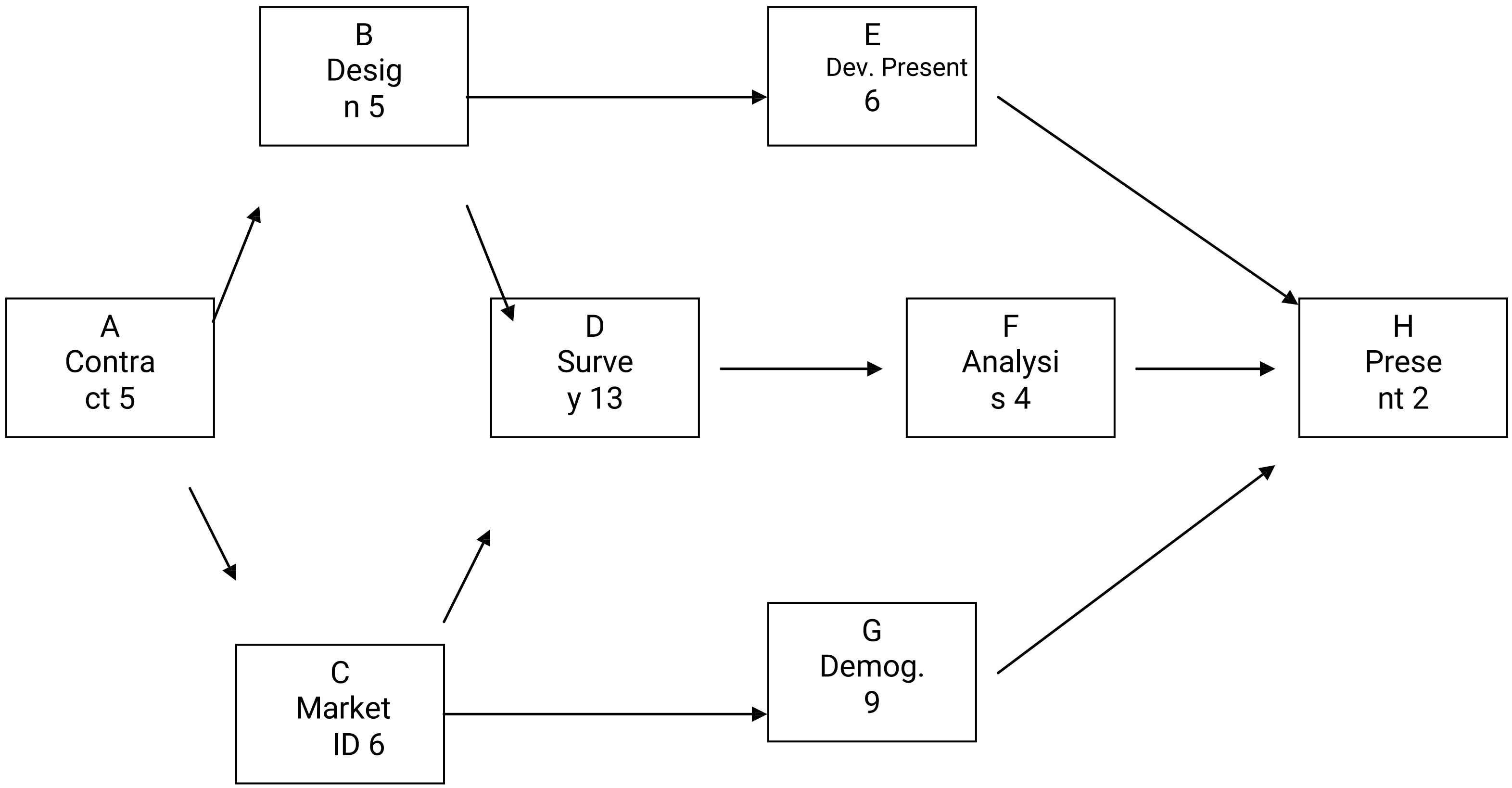
C = MOST PESSIMISTIC TIME

CONSTRUCTING THE CRITICAL PATH

INFORMATION FOR PROJECT DELTA

Activity	Description	Predecessors	Estimated Duration
A	Contract signing	None	5
B	Questionnaire design	A	5
C	Target market ID	A	6
D	Survey sample	B, C	13
E	Develop presentation	B	6
F	Analyze results	D	4
G	Demographic analysis	C	9
H	Presentation to client	E, F, G	2

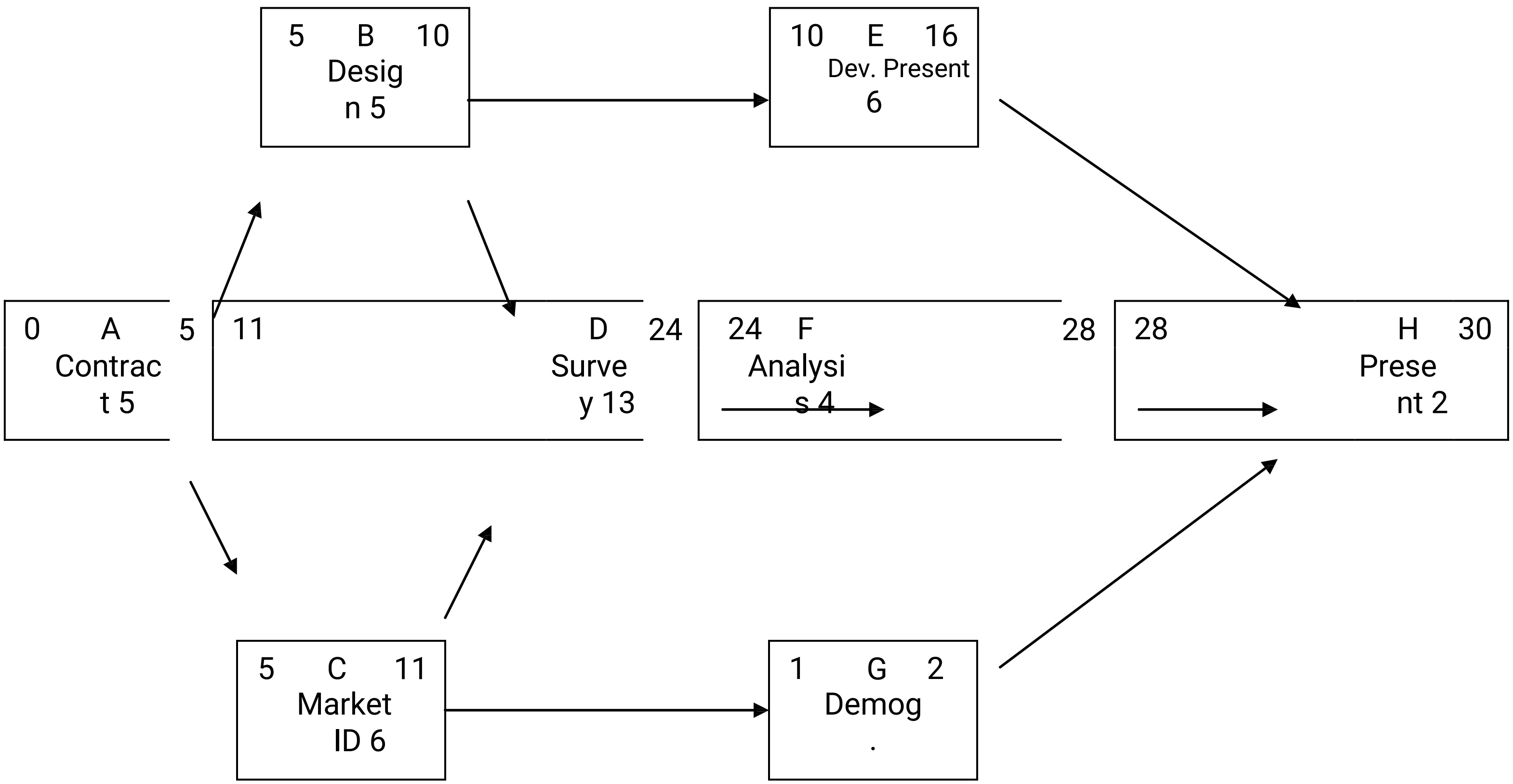
Partial Project Activity Network with Task Durations



RULES WHEN USING THE FORWARD PASS

1. Add all activity times along each path as we move through the network (ES + Dur = EF),
2. Carry the EF time to the activity nodes immediately succeeding the recently completed node. That EF becomes the ES of the next node, unless the succeeding node is a merge point.
3. At a merge point, the largest preceding EF becomes the ES for that node.

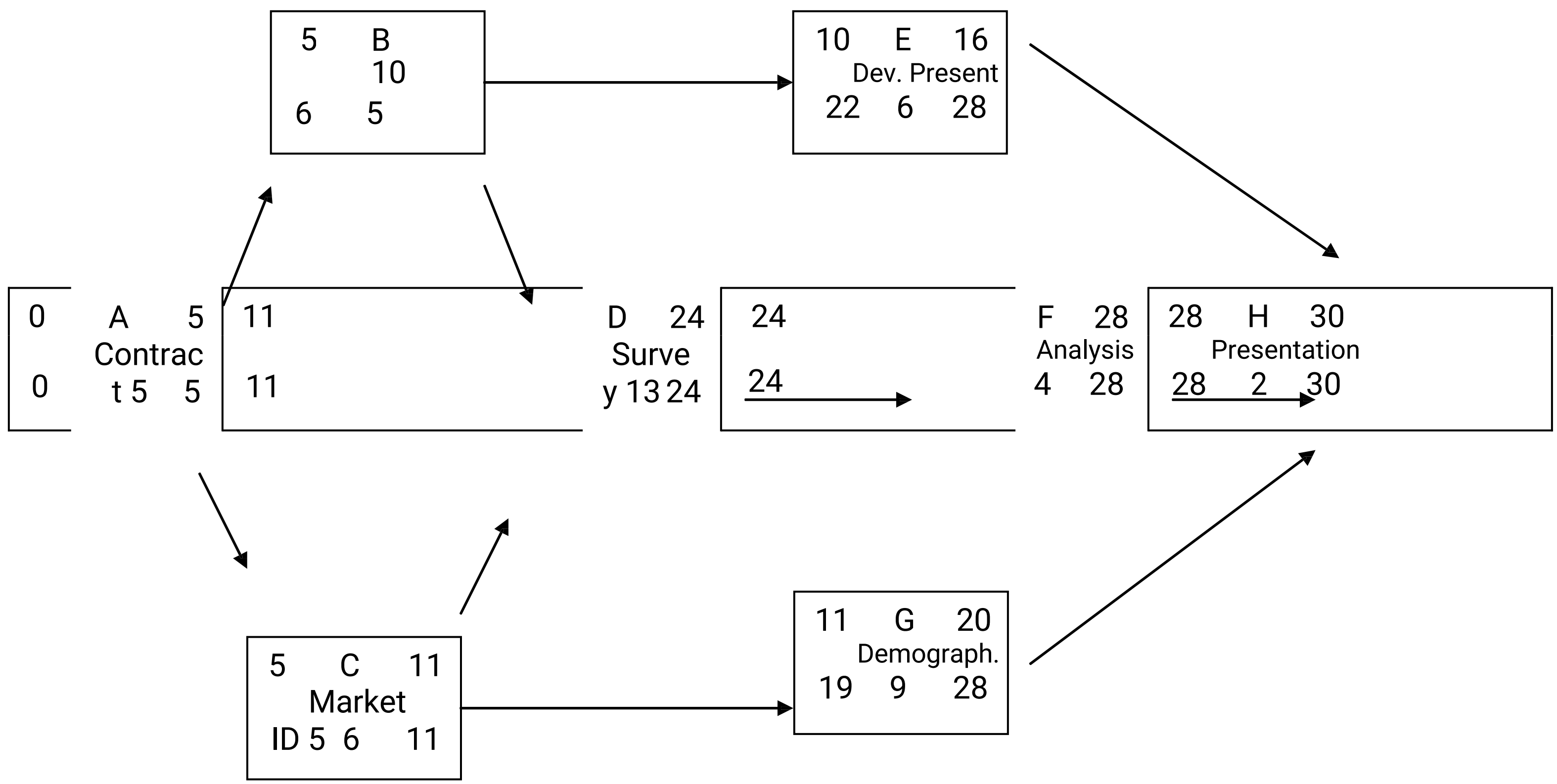
ACTIVITY NETWORK WITH FORWARD PASS



RULES FOR USING THE BACKWARD PASS

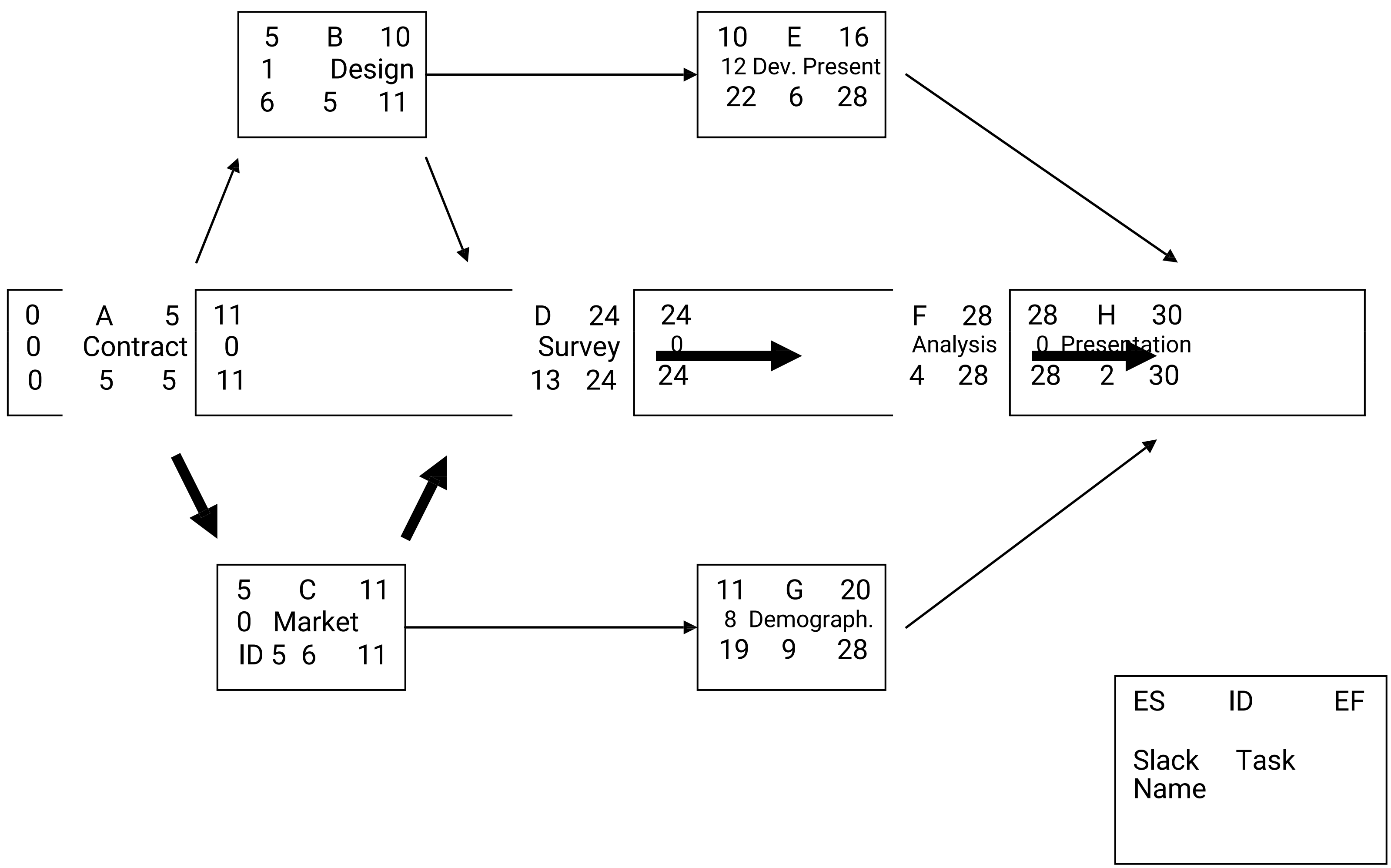
1. Subtract activity times along each path as you move through the network (LF – Dur = LS),
2. Carry back the LS time to the activity nodes immediately preceding the successor node. That LS becomes the LF of the next node, unless the preceding node is a burst point.
3. In the case of a burst point, the smallest succeeding LS becomes the LF for that node.

ACTIVITY NETWORK WITH BACKWARD PASS

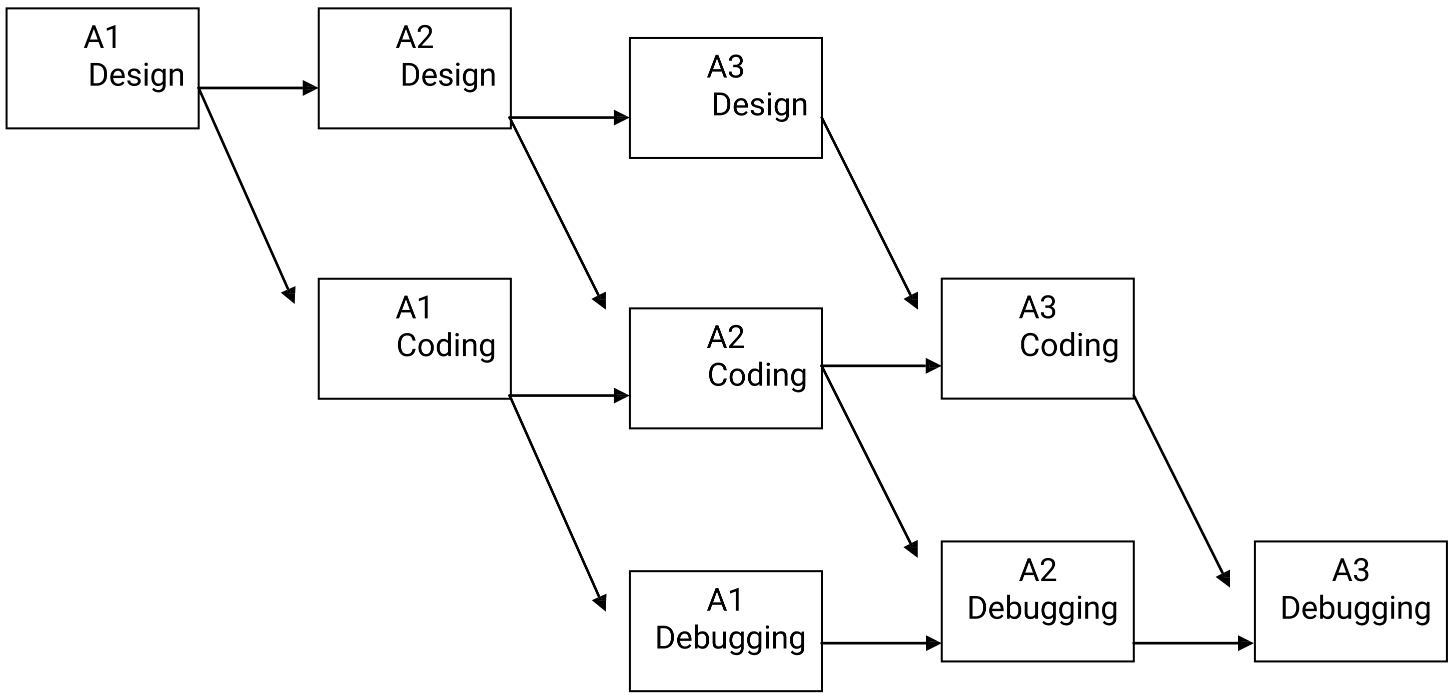


COMPLETED ACTIVITY NETWORK WITH CRITICAL PATH AND ACTIVITY SLACK TIMES IDENTIFIED

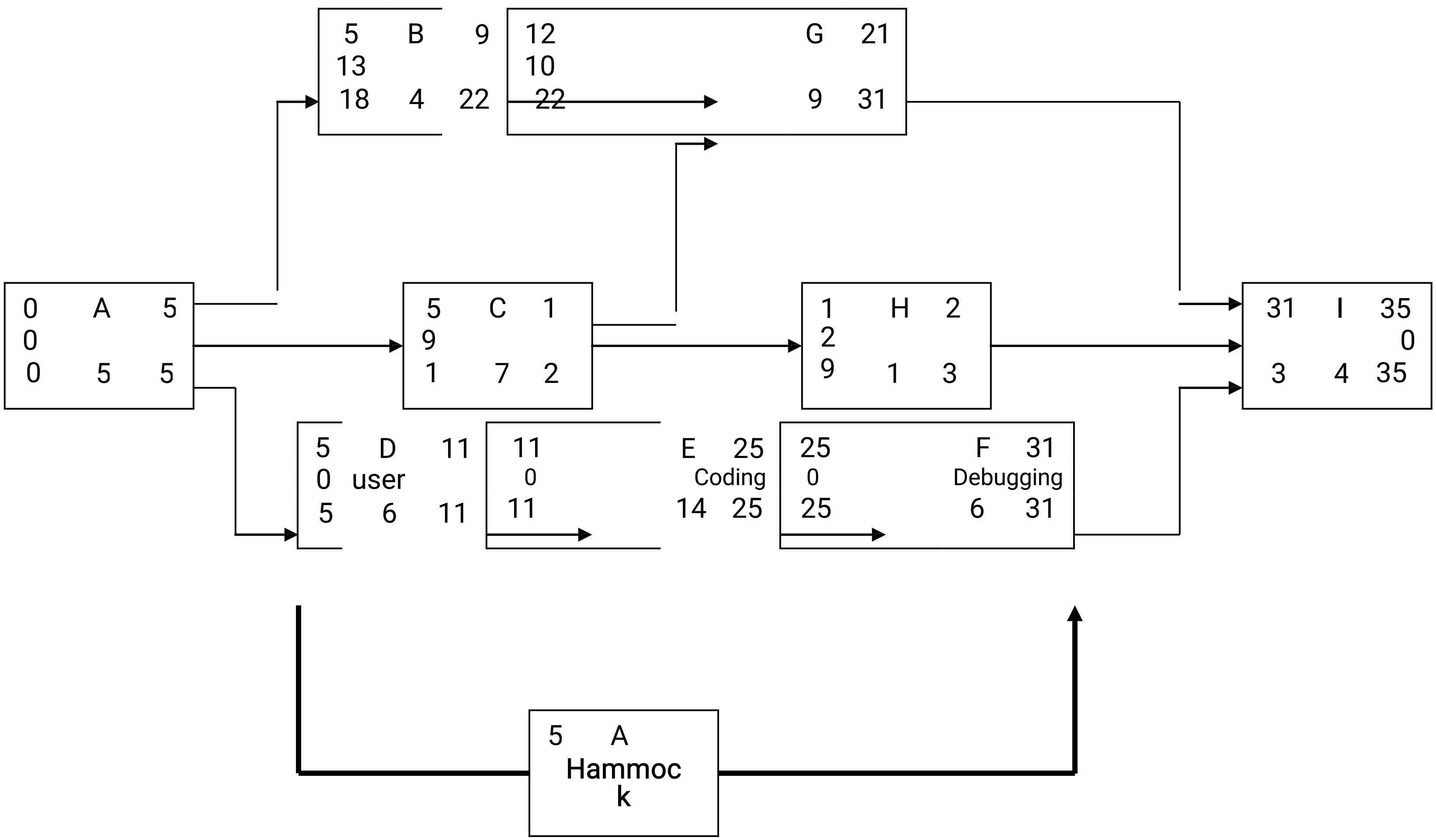
Critical Path is indicated in bold



ACTIVITY NETWORK DEMONSTRATING LADDERING TECHNIQUE



NETWORK DEMONSTRATING HAMMOCK ACTIVITY



STEPS TO REDUCE THE CRITICAL PATH

1. Eliminate Tasks On The Critical Path
2. Replan Serial Paths To Be Parallel
3. Overlap Sequential Tasks
4. Shorten The Duration On Critical Path Activities
5. Shorten Early Tasks
6. Shorten Longest Tasks
7. Shorten Easiest Tasks
8. Shorten Tasks That Cost The Least To Speed Up

7.18 Floats or Slack : Measures of float

We have seen that if the difference between the maximum time available and the duration of the activity, the total float, is 0 then the activity is critical. There are two other important measures of float, **free float** and **independent float**.

Free float

Free float is the time that an activity could be delayed without affecting any of the activities that follow.

Free float = EST for j - EST for i - duration of activity

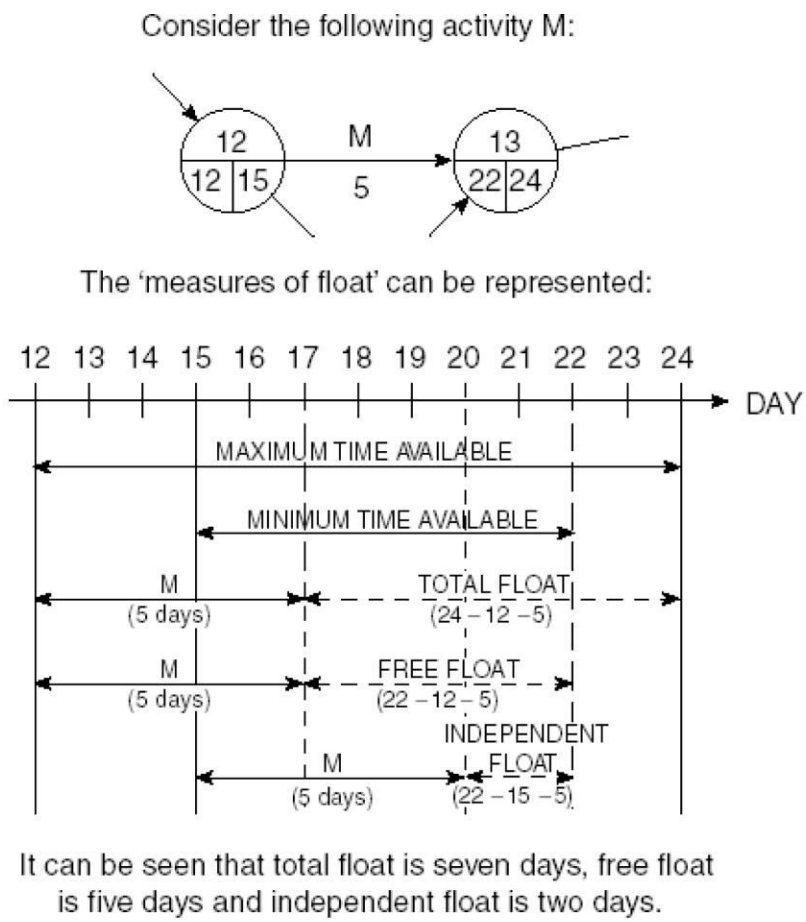
However, free float does assume that previous activities run to time.

Independent float

The **independent float** gives the time that an activity could be delayed if all the previous activities are completed as late as possible and all the following activities are to start as early as possible.

Independent float = EST for j - LST for i - duration of activity

The determination of total, free and independent float is illustrated in Figure 20.10.



Example:7.1 Tasks A,B,C,…..H,I constitute a project. The precedence relationship are

A < D; A < E; B < F; D < F; C < G ; C < H; F < I; G < I.

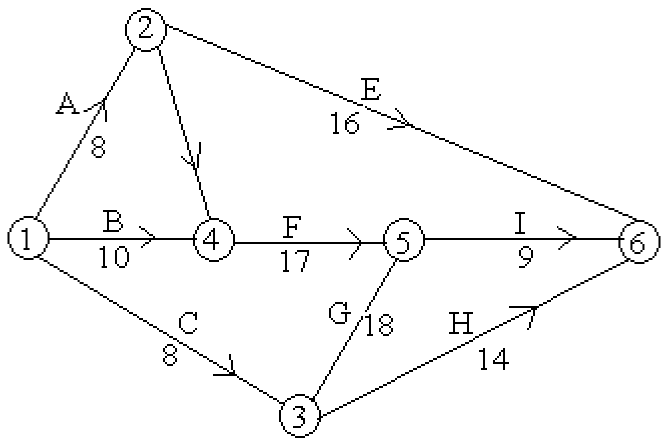
Draw a network to represent the project and find the minimum time of completion of the project when time , in days, of each task is as follows:

Table7.2

Task	:	A	B	C	D	E	F	G	H	I
Time	:	8	10	8	10	16	17	18	14	9

Also identify the critical path.

Sol:



Fig(7.1)

The given procedure order reveals that there are no predecessors to activities A,B and C and they all start from the initial node. Similarly, there are no successor to activities E,H and I and hence they all merge into the end node of project. The network obtained is shown in fig(7.1)

The nodes of the network have been numbered by using the Fulkerson's rule. The activity description and times are written along the activity arrows. To determine the minimum project completion time, let event 1 occur at zero time. The earliest occurrence time (E) and the latest occurrence time (L) of each event is then computed.

$E_1 = 0,$

$E_2 = E_1 + t_{12} = 0 + 8 = 8,$

$E_3 = E_1 + t_{13} = 0 + 8 = 8,$

$E_4 = Max.[0 + 10 , 8+10] = 18,$

$E_5 = Max.[18 + 17 , 8+18] = 35,$

$E_6 = Max.[8 + 16 , 35+ 9, 8+14] = 44.$

Similarly,

$L_6 = E_6 = 44,$

$L_5 = L_6 - t_{56} = 44 - 9 = 35,$ $L_4 =$

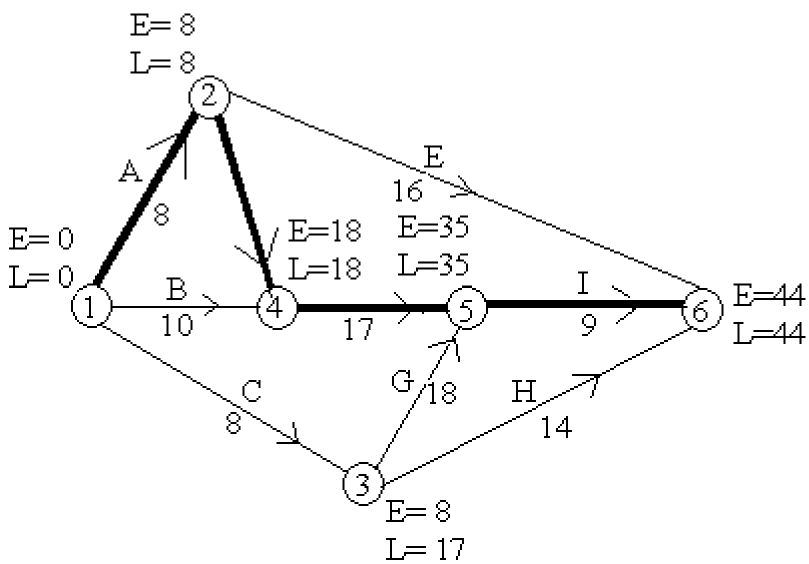
$L_5 - t_{45} = 35 - 17 = 18,$

$L_3 = Min.[44 - 14 , 35-18] = 17,$

L_2 = Min.[44 - 16 , 18-10] = 8,

L_1 = Min.[8 - 8 , 17- 8, 18-10] = 0.

The E and L values for each event have been written along the nodes in Fig(7.2)



FIG(7.2)

The critical path is now determine by any of the following method.
Method 1. The network analysis table is complied as below.

Table7.3

Activity	Duration	Start time Earliest Latest		Finish time Earliest Latest		Total float
1-2	8	0	0	8	8	0
1-3	8	0	9	8	17	9
1-4	10	0	8	10	18	8
2-4	10	8	8	18	18	0
2-6	16	8	28	24	44	20
3-5	18	8	17	26	35	9
3-6	14	8	30	22	44	22
4-5	17	18	18	35	35	0
5-6	9	35	35	44	44	0

Activities1-2,2-4,4-5 and 5-6 having zero float are the critical activities and 1-2-4-5-6 is the critical path.

Method 2. For identifying the critical path, the following condition are checked. If an activity satisfies all the three conditions, it is critical.
(i). E=L for the tail event.
(ii) E= L for the head event.
(iii) Ej-Ei = Lj-Li = tij.
Activities 1-2,2-4,4-5 and 5-6 satisfy these conditions. Other activities do not fulfil all the three conditions. The critical path is , therefore, 1-2-4-5-6.

Method 3. The various paths and their duration are :

Table7.4

Path	Duration(days)
1-2-6	24
1-2-4-5-6	44
1-4-5-6	36
1-3-5-6	35
1-3-6	22

Path1-2-4-5-6, the longest in time involving 44 days, is the critical path. It represented by bold lines in fig(7.2).
E= 8
L= 8

Example 7.2: A project schedule has the following characteristics:

Table7.5

Activity	Time(weeks)	Activity	Times(weeks)
1-2	4	5-6	4
1-3	1	5-7	8
2-4	1	6-8	1
3-4	1	7-8	2
3-5	6	8-10	5

4-9	5	9-10	7

(i). Construct the network.
(ii). Compute E and L for each , and
(iii). Find the critical path.

Solution: The given data result in a network shown in fig(7.5). The figures along the arrows represent the activity times.

The earliest occurrence time(E) and the latest occurrence time (L) of each event are now computed by employing forward and backward pass calculations.
In forward pass computations,

$E_1=0,$

$E_2= E_1+t_{12} = 0+ 4 = 4,$

$E_3= E_1+t_{13} = 0+ 1 = 1,$

$E_4= \text{Max } [E_i + t_{i4}] = \text{Max.}[4+1,1+1] = 5,$
$i=2,3$

$E_5= E_3+t_{35} = 1+ 6 = 7,$

$E_6= E_5+t_{56} = 7+ 4 =11,$

$E_7= E_5+t_{57} = 7+ 8 = 15,$

$E_8 = \text{Max } [E_i + t_{i8}] = \text{Max.}[11+1,15+1] = 17, i=6,7$

$E_9= E_4+t_{49} = 5+ 5 = 10,$ and

$E_{10} = \text{Max } [E_i + t_{i10}] = \text{Max.}[17+5,10+7] = 22,$
$i=8,9$

E values are represented in **fig 7.2.**
In backward pass computations,

$L_{10}=E_{10}=22,$

$L_9 = L_{10} - t_{9,10} = 22-7 = 15,$

$L_8 = L_{10} - t_{8,10} = 22-5 = 17,$

$L_7 = L_8 - t_{78} = 17 -2 = 15,$

$L_6 = L_8 - t_{68} = 17-1 = 16,$

$L_5 = \text{Min } [L_j - t_{5j}] = \text{Min}[16-4 , 15-8] = 7, j=6,7$

$L_4 = L_9 - t_{49} = 15-5 = 10,$

L_3 =Min [Lj – t_{3j}] = Min[10 - 4 , 7 - 6] = 1, j=4,5

L_2 = L_4 – t_{24} = 10-1 = 9,

L_1 =Min [Lj – t_{1j}] = Min [9 - 4 , 1 - 1] = 0, j=2,3

L values are also represent in fig(7.2).

Table 7.6

Activity	Duration (weeks)	Start time Earliest Latest		Finish time Earliest Latest		Total float
1-2	4	0	5	4	9	5
1-3	1	0	0	1	1	0
2-4	1	4	9	5	10	5
3-4	1	1	9	2	10	8
3-5	6	1	1	7	7	0
4-9	5	5	10	10	15	5
5-6	4	7	12	11	16	5
5-7	8	7	7	15	15	0
6-8	1	11	16	12	17	5
7-8	2	15	15	17	17	0
8-10	5	17	17	22	22	0
9-10	7	10	15	17	22	5

Path 1-3-5-7-8-10 with project duration 22 weeks is the critical path.

Example7.3:The utility data for a network are given below. Determine the total, free, independent and interfering floats and identify the critical path.

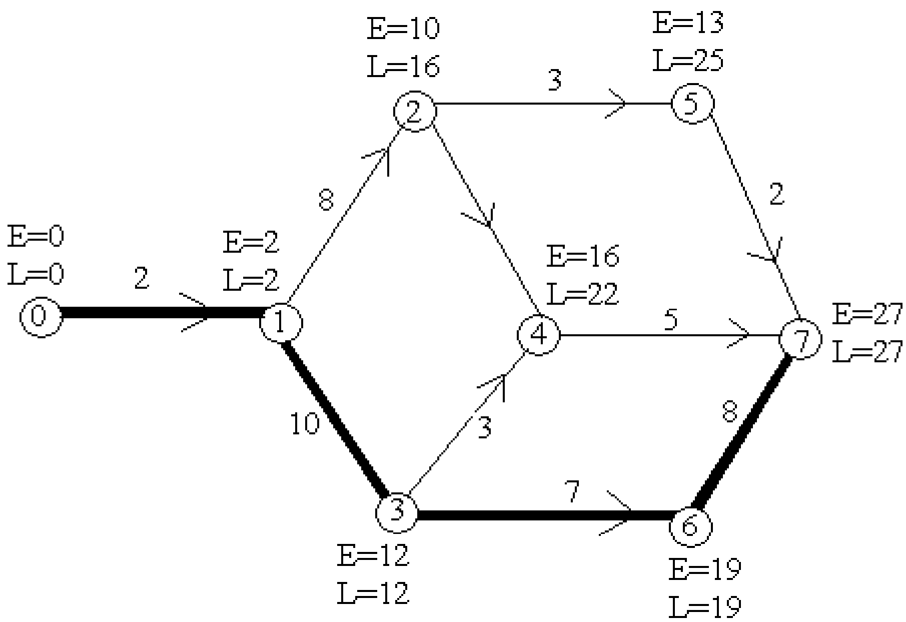
Table7.7

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration	2	8	10	6	3	3	7	5	2	8

Sol: The network diagram for the given project data is shown in fig(1.3). Activity durations are written along the activity arrows.

The earliest start and latest finish times of the activities are computed by employing the forward pass and backward pass calculation as explained in previous example. These times are represented in network around the respective nodes.

The network analysis table is now constructed.



Fig(7.3)

Table 7.8

Activity	Duration	Start Time Earliest Latest		Finish time Earliest Latest		Float Total Free Independent Interfering			
1	2	3	4	5	6	7	8	9	10
0-1	2	0	0	2	2	0	0	0	0
1-2	8	2	8	10	16	6	0	0	6
1-3	10	2	2	12	12	0	0	0	0
2-4	6	10	16	16	22	6	0	-6≈0	6
2-5	3	10	22	13	25	12	0	-6≈0	12
3-4	3	12	19	15	22	7	1	1	6
3-6	7	12	12	19	19	0	0	0	0
4-7	5	16	22	21	27	6	6	0	0
5-7	2	13	25	15	27	12	12	0	0
6-7	8	19	19	27	27	0	0	0	0

Total float is the positive difference between latest and earliest finish times or latest and earliest start time. For activity 1-2 ,

Total float (T.F)=16-10 = 8-2 = 6.

Similarly , for activity , say 2-5 ,

Total float = 25 -13=22-10 =12 and so on.

Total float calculations are depicted in column 7 of table 1.28.

Free float activity i-j = T.F .- head event slack

= T .F. – (L - E) of event j.

Thus free float of activity 0-1

= 0-(L – E) of event 1,

= 0 – (2 - 2)= 0,

Free float of activity 1-2 = 6-(16-10) = 6-6 = 0, etc.

Free float of various activities are calculated in column 8 of the network analysis table.

Independent float of activity i-j= F.F – tail event slack = F.F – (L-E) of event i.

Thus independent float of activity 0-1 = 0 – (0-0)=0,

Independent float of activity 1-2 = 0 – (2 -2) = 0,

Independent float of activity 2-4 = 0 – (16 -10)= -6 ≈0 and so on.

Independent float of various activities are calculated in column 9 of the table .If independent float of an activity is negative , it is taken a zero.

Interfering float of activity i-j = Max[L.F . time of i-j – E.S. time of j-k, 0] Thus interfering float of activity 0-1

= Max. [L.F. time of 0-1 – E.S. time of 1-2 or 1-3 , 0] = Max. [2 -2 , 0]= 0, Interfering float of activity 1-2 = Max. [L.F. time of 1-2 – E.S. time of 2-4 or 2-5 , 0]

= Max. [16 -10,0] = 6 , etc

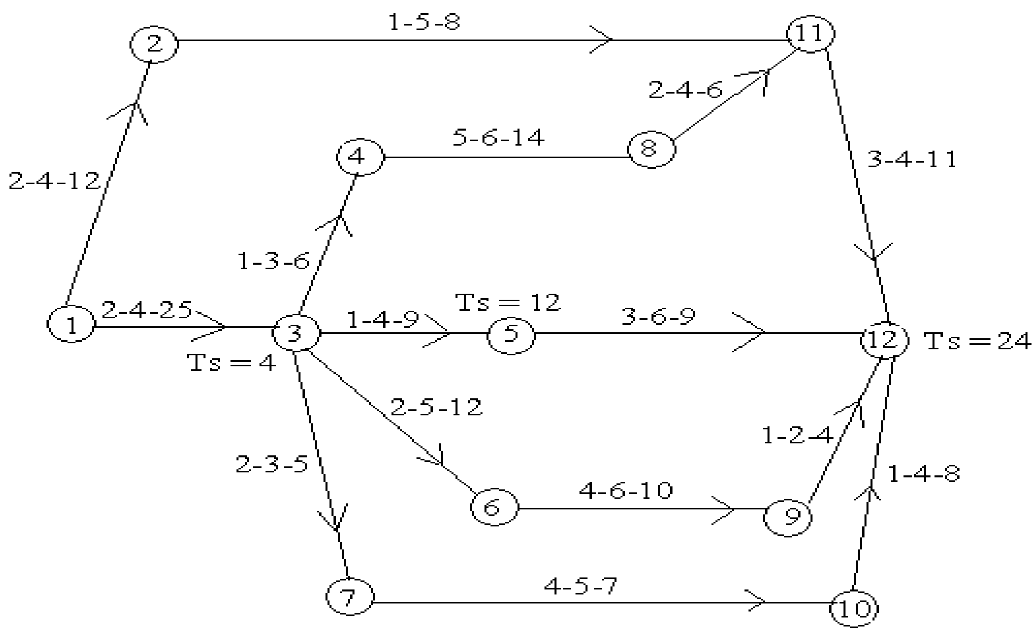
More conveniently , interfering float of an activity = T.F. – F.F.

Thus , interfering float of activity 2-5 = 12-0=12 , etc.

Alternating , interfering float of an activity 2-5 = 25 -13=12, etc.

Interfering floats of various activities are calculated in column 10 of table 7.8.

Example7.4: A PERT network is shown if fig 1.4. The activity times in days are given along the arrows. The scheduled times for some important events are given along the nodes. Determine the critical path and probabilities of meeting the scheduled dates for the specified events. Tabulate the result and determine slack for each event.



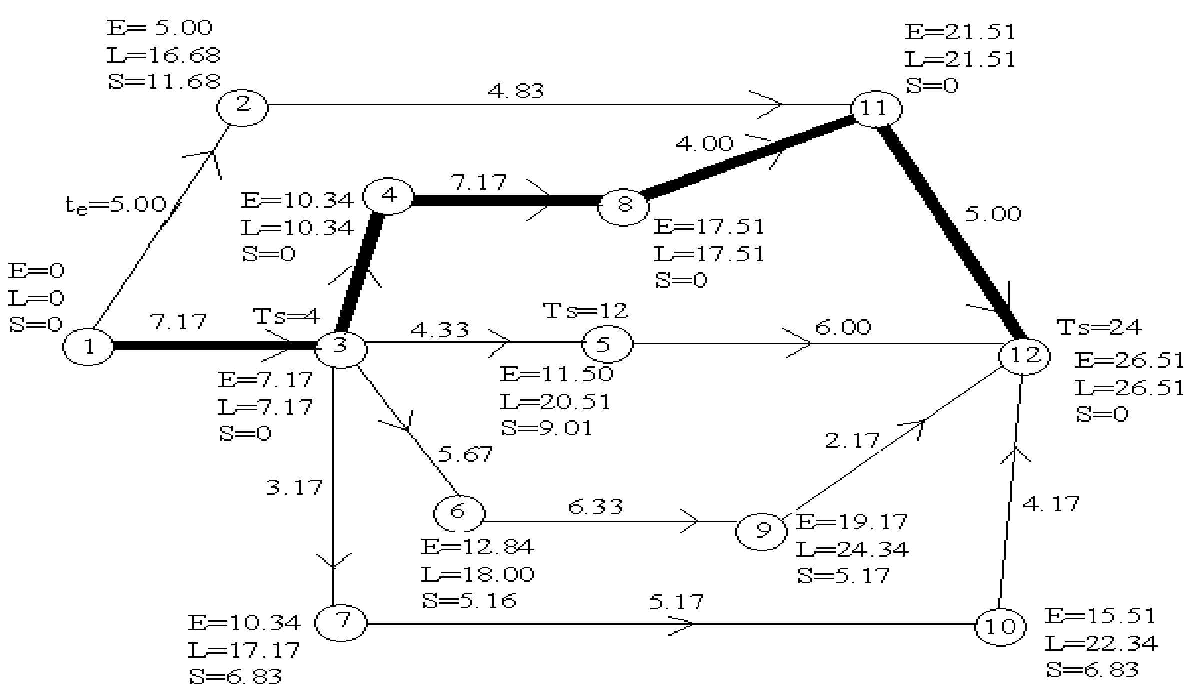
Fig(7.4)

Solution:

Table 7.9

Activity	t_0	t_m	t_p	t_e	σ^2
1-2	2	4	12	5.00	2.78
1-3	2	4	25	7.17	14.69
2-1	1	1	5	84.83	1.36
3-4	1	3	6	3.17	0.69
3-5	1	4	9	4.33	1.78
3-6	2	5	12	5.67	2.78
3-7	2	3	5	3.17	0.25
4-8	5	6	14	7.17	2.25
5-2	3	6	9	6.00	1.00
6-9	4	6	10	6.33	1.00
7-10	4	5	7	5.17	0.25
8-11	2	4	6	4.00	0.44
9-12	1	2	4	2.17	0.25
10-12	1	4	8	4.17	1.36
11-12	3	4	11	5.00	1.78

The arrow diagram for the given data is shown in fig 7.4. The expected activity times are shown along the arrows. The earliest and latest occurrence times as well as the slack of the event are also written along the nodes.



Fig(7.5)

Table 7.10 represent the network analysis . Floats for the activities in question are calculated in the last column. Critical path is 1-3-4-8-11-12 and the project completion is 26.51 days:

Table 7.10

Activity	Duration	Start Time Earliest	Latest	Finish time Earliest	Latest	Total float
1-2	5.00	0	11.68	5.00	16.68	11.68
1-3	7.17	0	0	7.17	7.17	0
2-11	4.83	5.00	16.68	9.83	21.51	11.68
3-4	3.17	7.17	7.17	10.34	10.34	0
3-5	4.33	7.17	16.18	11.50	20.51	9.01
3-6	5.67	7.17	12.33	12.84	18.00	5.16
3-7	3.17	7.17	14.00	10.34	17.17	6.83
4-8	7.17	10.34	10.34	17.51	17.51	0
5-12	6.00	11.50	20.51	17.50	26.51	9.01
6-9	6.33	12.84	18.01	19.17	24.34	5.17
7-10	5.17	10.34	17.17	15.51	22.34	6.83
8-11	4.00	17.51	17.51	21.51	21.51	0
9-12	2.17	19.17	24.34	21.34	26.51	5.17
10-12	4.17	15.51	22.34	19.68	26.51	6.83
11-12	5.00	21.51	21.51	26.51	26.51	0

Probability of completing the project in the scheduled completion time of 24 days (since Ts(12)=24):

$$Z = \frac{24-26.51}{\sqrt{14.69 + 0.69 + 2.25 + 0.44 + 1.78}} = \frac{-2.55}{\sqrt{19.85}} = -0.5634.$$

∴ p(Ts ≤ 24) = 1- value of probability for Z = 0.5634=1- 0.7146 = 29.54 %.

Probability that event 3 will occur on the scheduled date:

Ts (3) = 4,

E = L= 7.17

$$\therefore \quad Z = \frac{4 - 7.17}{\sqrt{14.69}} = -0.8271$$

∴ p = 1- value of probability for Z = 0.8271 = 1- 0.7956 = 20.44%

Probability of meeting scheduled date for event 5:

The earliest occurrence time of event 5 is 11.50, while the scheduled time is 12 .Event 5 is not on critical path and hence its occurrence can be delayed by 9 days.

Variance of path 1-3-5=14.69 + 1.78 = 16.47.

$T_s = 12,$

$E = 11.50$

$\therefore \quad Z = \frac{12 - 11.50}{\sqrt{16.47}} = 0.123$

$\therefore$ Probability = 54.89%

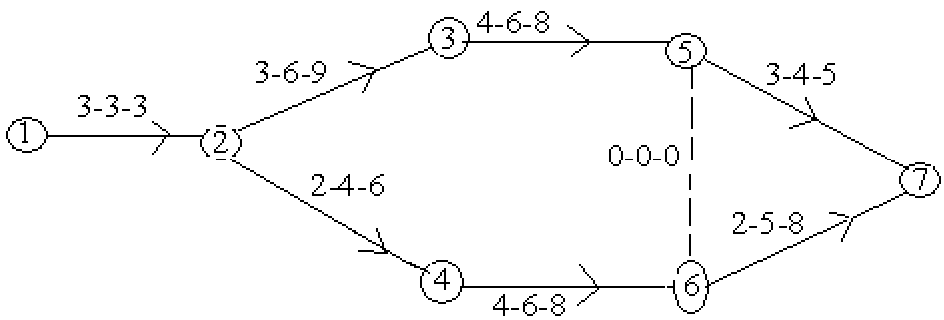
$T_s = 12$

$L = 20.51.$

$\therefore \quad Z = \frac{12 - 20.51}{\sqrt{16.47}} = -2.1$

$\therefore$ Probability = 1 – 0.982 = 0.018 = 1.8 %

Example7.5: In the PERT network shown in fig 7.6 , the activity time estimates(in weeks) are given along the arrows. If the scheduled completion time is 23 weeks, calculate the latest possible occurrence times of the events. Calculate the slack for each event and identify the critical path. What is the probability that project will be completed on the scheduled date ?.



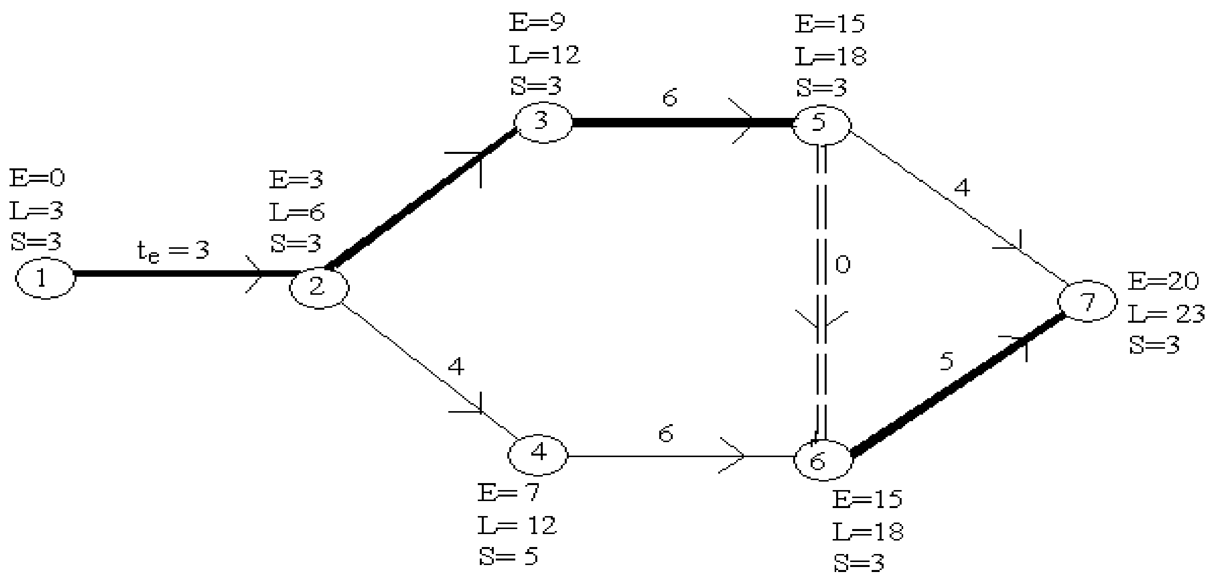
Fig(7.6)

Sol: The expected time of the activities and their variances are computed below.

Activity	t_0	t_m	t_p	t_e	σ^2
1-2	3	3	3	3	0
2-3	3	6	9	6	1
2-4	2	4	6	4	4/9
3-5	4	6	8	6	4/9
4-6	4	6	8	6	4/9
5-6	0	0	0	0	0
5-7	3	4	5	4	1/9
6-7	2	5	8	5	1

Table7.11

The earliest occurrence times of the events have been computed on the network of fig 7.6 , taking the earliest time of event 1 as zero. The earliest occurrence time of event 7 is 20. But the scheduled completion time of the project is 23 weeks and hence the latest occurrence times of the events have been computed taking L(7)=23. Slacks for the event have been shown along the nodes. Path 1-2-3-5-6-7 is the critical path.



Fig(7.7)

Probability of completing the project on schedule date :

T=23 weeks, E = 20 weeks.

Varience of the critical path = 0=1+4/9 = 0+1=22/9=2.444

$$\therefore \quad z = \frac{23-20}{\sqrt{2.444}}$$

Z=1.92.

∴ Probability = 97.26%

Example 7.6: The following table gives data on normal time and cost and crash time and cost for a project.

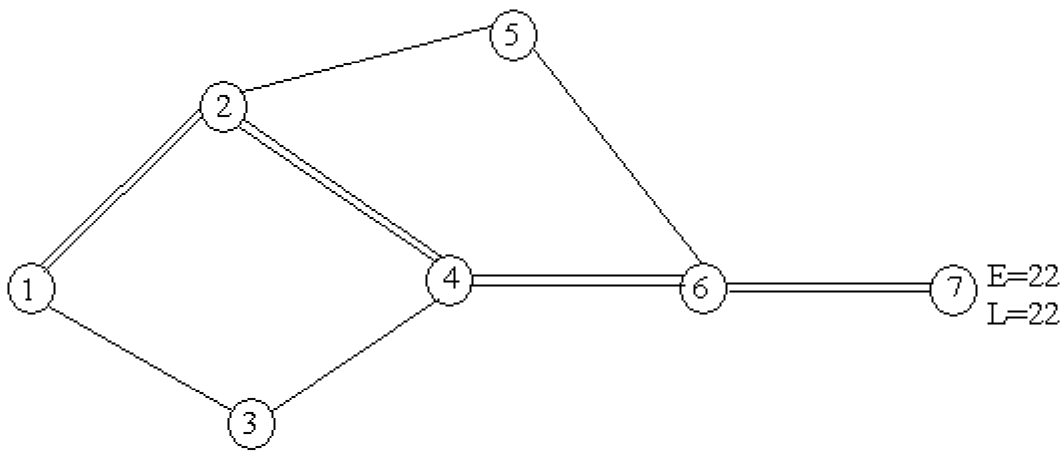
Activity	Normal		Crash	
	Time (days) Cost(Rs.)		Time(Days) (Rs.)	Cost
1-2	6	60	4	100
1-3	4	60	2	200
2-4	5	50	3	150
2-5	3	45	1	65
3-4	6	90	4	200
4-6	8	80	4	300
5-6	4	40	2	100
6-7	3	45	2	80
		470		

The indirect cost per day is Rs .10.
(i). Draw the network for project.
(ii). Find the critical path.
(iii). Determine minimum total time and corresponding cost.

Solution: First the cost slope for each activity and normal direct cost of the project is calculated. This is shown in the table below.

Activity	1-2	1-3	2-4	2-5	3-4	4-6	5-6	6-7
Cost slope (Rs./day)	20	70	50	10	55	55	30	35

(i). Next, the network is draw and critical path is determined. This is shown in fig 1.8.



Fig(7.8)

(ii). The critical path is 1-2-4-6-7.
(iii). Normal duration = 22 days.
Normal cost = Rs.(470+22*10)= Rs.690.
Now represent the network on time-scaled diagram. This is shown in fig (7.8).
E=22
L=22