# Assignment Problem

## Introduction

The assignment problem is a special type of transportation problem, where the objective is to minimize the cost or time of completing a number of jobs by a number of persons. In other words, when the problem involves the allocation of *n* different facilities to *n* different tasks, it is often termed as an assignment problem. The model's primary usefulness is for planning. The assignment problem also encompasses an important sub-class of so-called shortest- (or longest-) route models. The assignment model is useful in solving problems such as, assignment of machines to jobs, assignment of salesmen to sales territories, travelling salesman problem, etc.

It may be noted that with n facilities and n jobs, there are n! possible assignments. One way of finding an optimal assignment is to write all the n! possible arrangements, evaluate their total cost, and select the assignment with minimum cost. But, due to heavy computational burden this method is not suitable. This chapter concentrates on an efficient method for solving assignment problems that was developed by a **Hungarian mathematician D.Konig.**

## Formulation of an assignment problem

Suppose a company has n persons of different capacities available for performing each different job in the concern, and there are the same number of jobs of different types. One person can be given one and only one job. The objective of this assignment problem is to assign n persons to n jobs, so as to minimize the total assignment cost. The cost matrix for this problem is given below:

| | Job | | | | |
|---|---|---|---|---|---|
| Worker | $j_1$ | $j_2$ | ---- | $j_n$ | $a_i$ |
| $i_1$ | $x_{11}$ $c_{11}$ | $x_{12}$ $c_{12}$ | ---- | $x_{n1}$ $c_{1n}$ | 1 |
| $i_2$ | $x_{21}$ $c_{21}$ | $x_{22}$ $c_{22}$ | ---- | $x_{2n}$ $c_{2n}$ | 1 |
| ---- | ---- | ---- | ---- | ---- | -- |
| $i_n$ | $x_{n1}$ $c_{n1}$ | $x_{n2}$ $c_{n2}$ | ---- | $x_{nn}$ $c_{nn}$ | 1 |
| $b_j$ | 1 | 1 | ---- | 1 | |

The structure of an assignment problem is identical to that of a transportation problem.

To formulate the assignment problem in mathematical programming terms, we define the activity variables as

$$x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is performed by worker } i \\ 0 & \text{otherwise} \end{cases}$$

for i = 1, 2, ..., n and j = 1, 2, ..., n

In the above table, $c_{ij}$ is the cost of performing jth job by ith

worker. The optimization model is

Minimize $c_{11}x_{11} + c_{12}x_{12} + \text{-------} + c_{nn}x_{nn}$

subject to

$x_{i1} + x_{i2} + \dots\dots\dots + x_{in} = 1 \qquad i = 1, 2, \dots\dots, n$

$x_{1j} + x_{2j} + \dots\dots\dots + x_{nj} = 1 \qquad j = 1, 2, \dots\dots, n$

$x_{ij} = 0$ or 1

*In Σ Sigma notation*

minimize $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

subject to

$\sum_{i=1}^{n} x_{ij} = 1$ for i = 1, 2, ....., n

$\sum_{j=1}^{n} x_{ij} = 1$ for j = 1, 2, ....., n

$x_{ij} = 0$ or 1 for all i and j

An assignment problem can be solved by transportation methods, but due to high degree of degeneracy the usual computational techniques of a transportation problem become very inefficient. Therefore, a special method is available for solving such type of problems in a more efficient way.

- Number of jobs is equal to the number of machines or persons.
- Each man or machine is assigned only one job.
- Each man or machine is independently capable of handling any job to be done.
- Assigning criteria is clearly specified (minimizing cost or maximizing profit).

## Hungarian Method

It is an efficient method for solving assignment problems . This method is based on the following principle:

- If a constant is added to, or subtracted from, every element of a row and/or a column of the given cost matrix of an assignment problem, the resulting assignment problem has the same optimal solution as the original problem.

## Algorithm

The objective of this section is to examine a computational method - an algorithm - for deriving solutions to the assignment problems. The following steps summarize the approach:

### *Steps*

1. Identify the minimum element in each row and subtract it from every element of that row.
2. Identify the minimum element in each column and subtract it from every element of that column.
3. Make the assignments for the reduced matrix obtained from **steps 1 and 2** in the following way:
    i. For each row or column with a single zero value cell that has not be assigned or eliminated, box ☐that zero value as an assigned cell.
    ii. For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
    iii. If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, then you are at liberty to choose the cell arbitrarily for assignment.
    iv. The above process may be continued until every zero cell is either assigned ☐or crossed (X).
4. An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.
5. Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:

i. Mark all the rows that do not have assignments.
ii. Mark all the columns (not already marked) which have zeros in the marked rows.
iii. Mark all the rows (not already marked) that have assignments in marked columns.
iv. Repeat steps 5 (i) to (iii) until no more rows or columns can be marked.
v. Draw straight lines through all unmarked rows and marked columns.

You can also draw the minimum number of lines by inspection.

6. Select the smallest element from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.
7. Go to step 3 and repeat the procedure until you arrive at an optimal assignment.

For the time being we assume that number of jobs is equal to number of machines or persons. Later in the chapter, we will remove this restrictive assumption and consider a special case where no. of facilities and tasks are not equal.

*Example 1*

The Funny Toys Company has four men available for work on four separate jobs. Only one man can work on any one job. The cost of assigning each man to each job is given in the following table. The objective is to assign men to jobs in such a way that the total cost of assignment is minimum.

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 20 | 25 | 22 | 28 |
| B | 15 | 18 | 23 | 17 |
| C | 19 | 17 | 21 | 24 |
| D | 25 | 23 | 24 | 24 |

*Solution.*

*Step 1*

Identify the minimum element in each row and **subtract** it from every element of that row. The result is shown in the following table.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 5 | 2 | 8 |
| B | 0 | 3 | 8 | 2 |
| C | 2 | 0 | 4 | 7 |
| D | 2 | 0 | 1 | 1 |

*Step 2*

Identify the minimum element in each column and subtract it from every element of that column.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 5 | 1 | 7 |
| B | 0 | 3 | 7 | 1 |
| C | 2 | 0 | 3 | 6 |
| D | 2 | 0 | 0 | 0 |

### *Step 3*

Make the assignments for the reduced matrix obtained from **steps 1 and 2** in the following way:

a. For each row or column with a single zero value cell that has not be assigned or eliminated, box ☐ that zero value as an assigned cell.
b. For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
c. If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, choose the cell arbitrarily for assignment.
d. The above process may be continued until every zero cell is either assigned ☐or crossed (X).

### *Step 4*

An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.

*Table*

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | [0] | 5 | 1 | 7 |
| B | X | 3 | 7 | 1 |
| C | 2 | [0] | 3 | 6 |
| D | 2 | X | [0] | X |

### *Step 5*

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:

i. Mark all the rows that do not have assignments.
ii. Mark all the columns (not already marked) which have zeros in the marked rows.
iii. Mark all the rows (not already marked) that have assignments in marked columns.
iv. Repeat steps 5 (ii) and (iii) until no more rows or columns can be marked.
v. Draw straight lines through all unmarked rows and marked columns.

You can also draw the minimum number of lines by inspection.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 5 | 1 | 7 |
| B | 0 | 3 | 7 | 1 |
| C | 2 | 0 | 3 | 6 |
| D | 2 | 0 | 0 | 0 |

*Step 6*

Select the smallest element (i.e., 1) from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 4 | 0 | 6 |
| B | 0 | 2 | 6 | 0 |
| C | 3 | 0 | 3 | 6 |
| D | 3 | 0 | 0 | 0 |

Now again make the assignments for the reduced matrix.

*Final Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 4 | 0 | 6 |
| B | 0 | 2 | 6 | 0 |
| C | 3 | 0 | 3 | 6 |
| D | 3 | 0 | 0 | 0 |

Since the number of assignments is equal to the number of rows (& columns), this is the optimal solution.

The total cost of assignment = A1 + B4 + C2 + D3

Substituting values from original table:
20 + 17 + 17 + 24 = Rs. 78.

*Example 2*

The Winner Publishing Company employs typists on hourly basis. There are five typists for service and their charges and speeds are different. According to an earlier understanding only one job is given to one typist and the typist is paid for full hour even if he works for a fraction of an hour. Find the least cost allocation for the following data:

| Typist | Rate per hour (Rs.) | No. of pages Typed / |
|---|---|---|
| A | 5 | 12 |
| B | 6 | 14 |
| C | 3 | 8 |
| D | 4 | 10 |
| E | 4 | 11 |

| Job | No. of pages |
|---|---|
| P | 199 |
| Q | 175 |
| R | 145 |
| S | 198 |
| T | 178 |

*Solution.*

The following matrix gives the cost incurred if the ith typist (i = A, B, C, D & E) executes the jth job (j = P, Q, R, S & T):

| Job | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 85 | 75 | 65 | 125 | 75 |
| B | 90 | 78 | 66 | 132 | 78 |

| | | | | | |
|---|---|---|---|---|---|
| C | 75 | 66 | 57 | 114 | 69 |
| D | 80 | 72 | 60 | 120 | 72 |
| E | 76 | 64 | 56 | 112 | 68 |

Identify the minimum element in each row and subtract it from every element of that row.

*Table*

| Job | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 20 | 10 | 0 | 60 | 10 |
| B | 24 | 12 | 0 | 66 | 12 |
| C | 18 | 9 | 0 | 57 | 12 |
| D | 20 | 12 | 0 | 60 | 12 |
| E | 20 | 8 | 0 | 56 | 12 |

Identify the minimum element in each column and subtract it from every element of that column.

*Table*

| Job | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 2 | 0 | 4 | 0 |
| B | 6 | 4 | 0 | 10 | 2 |
| C | 0 | 1 | 0 | 1 | 2 |
| D | 2 | 4 | 0 | 4 | 2 |
| E | 2 | 0 | 0 | 0 | 2 |

Make the assignments for the reduced matrix

*Table*

| Job | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 2 | ~~0~~ | 4 | **0** |
| B | 6 | 4 | **0** | 10 | 2 |
| C | **0** | 1 | ~~0~~ | 1 | 2 |
| D | 2 | 4 | ~~0~~ | 4 | 2 |
| E | 2 | **0** | ~~0~~ | ~~0~~ | 2 |

The number of assigned cells is not equal to the number of rows (and columns). Therefore, we draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix

*Table*

| Job | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 2 | ~~0~~ | 4 | **0** |
| B | 6 | 4 | **0** | 10 | 2 |
| C | **0** | 1 | ~~0~~ | 1 | 2 |
| D | 2 | 4 | ~~0~~ | 4 | 2 |
| E | 2 | **0** | ~~0~~ | ~~0~~ | 2 |

Repeating the usual process as explained in the previous example, we get the following matrix:

*Table*

| Jo | | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 2 | 2 | 4 | **0** |
| B | 4 | 2 | **0** | 8 | ~~0~~ |
| C | **0** | 1 | 2 | 1 | 2 |
| D | ~~0~~ | 2 | ~~0~~ | 2 | ~~0~~ |
| E | 2 | **0** | 2 | ~~0~~ | 2 |

The number of assigned cells is not equal to the number of rows (and columns). Therefore, we draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix

*Table*

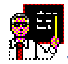
| | Job | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 2 | 2 | 4 | 0 |
| B | 4 | 2 | 0 | 8 | 0 |
| C | 0 | 1 | 2 | 1 | 2 |
| D | 0 | 2 | 0 | 2 | 0 |
| E | 2 | 0 | 2 | 0 | 2 |

*Final Table*

| | Jo | | | | |
|---|---|---|---|---|---|
| Typist | P | Q | R | S | T |
| A | 2 | 1 | 2 | 3 | 0 |
| B | 4 | 1 | 0 | 7 | 0 |
| C | 0 | 0 | 2 | 0 | 2 |
| D | 0 | 1 | 0 | 1 | 0 |
| E | 3 | 0 | 3 | 0 | 3 |

Since the number of assignments is equal to the number of rows (& columns), this is the optimal solution.

Substituting the values from original table: 75 + 66 + 114 + 80 + 64 = Rs. 399.



## Some Special Cases

- Unbalanced Assignment Problem
- Maximization Problem
- Multiple Optimal Solutions

## 1. Unbalanced Assignment Problem

In the previous section, the number of persons and the number of jobs were assumed to be the same. In this section, we remove this assumption and consider a situation where the number of persons is not equal to the number of jobs. In all such cases, fictitious rows and/or columns are added in the matrix to make it a square matrix. Then, we apply the usual Hungarian Method to this resulting balanced assignment problem. We provide the following example to illustrate the solution of an unbalanced assignment problem.

*Example*

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 20 | 25 | 22 | 28 |
| B | 15 | 18 | 23 | 17 |
| C | 19 | 17 | 21 | 24 |

*Solution*

Since the number of persons is less than the number of jobs, we introduce a dummy person (D) with zero values. The revised assignment problem is given below:

*Table*

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 20 | 25 | 22 | 28 |
| B | 15 | 18 | 23 | 17 |

| | | | | |
|---|---|---|---|---|
| C | 19 | 17 | 21 | 24 |
| D (dummy) | 0 | 0 | 0 | 0 |

Now use the Hungarian method to obtain the optimal solution yourself. Ans. = 20 + 17 + 17 + 0 = 54.

## 2. Maximization In An Assignment Problem

There are problems where certain facilities have to be assigned to a number of jobs, so as to maximize the overall performance of the assignment. The Hungarian Method can also solve such problems, as it is easy to obtain an equivalent minimization problem by converting every number in the matrix to an opportunity loss. The conversion is accomplished by subtracting all the elements of the given matrix from the highest element. It turns out that minimizing opportunity loss produces the same assignment solution as the original maximization problem.

*Example*

At the head office of www.universalteacher.com there are five registration counters. Five persons are available for service.

| | Person | | | | |
|---|---|---|---|---|---|
| Counter | A | B | C | D | E |
| 1 | 30 | 37 | 40 | 28 | 40 |
| 2 | 40 | 24 | 27 | 21 | 36 |
| 3 | 40 | 32 | 33 | 30 | 35 |
| 4 | 25 | 38 | 40 | 36 | 36 |
| 5 | 29 | 62 | 41 | 34 | 39 |

How should the counters be assigned to persons so as to **maximize the profit**?

*Solution.*

Here, the highest value is **62**. So we subtract each value from 62. The conversion is shown in the following table.

*Table*

| Person | | | | | |
|---|---|---|---|---|---|
| Counter | A | B | C | D | E |
| 1 | 32 | 25 | 22 | 34 | 22 |
| 2 | 22 | 38 | 35 | 41 | 26 |
| 3 | 22 | 30 | 29 | 32 | 27 |
| 4 | 37 | 24 | 22 | 26 | 26 |
| 5 | 33 | 0 | 21 | 28 | 23 |

Now the above problem can be easily solved by Hungarian method. After applying steps 1 to 3 of the Hungarian method, we get the following matrix.

*Table*

| Person | | | | | |
|---|---|---|---|---|---|
| Counter | A | B | C | D | E |
| 1 | 10 | 3 | [0] | 8 | ~~0~~ |
| 2 | [0] | 16 | 13 | 15 | 4 |
| 3 | ~~0~~ | 8 | 7 | 6 | 5 |
| 4 | 15 | 2 | ~~0~~ | [0] | 4 |
| 5 | 33 | [0] | 21 | 24 | 23 |

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix.

*Table*

| Person | | | | | |
|---|---|---|---|---|---|
| Counter | A | B | C | D | E |
| 1 | 10 | 3 | 0 | 8 | 0 |
| 2 | 0 | 16 | 13 | 15 | 4 |
| 3 | 0 | 8 | 7 | 6 | 5 |
| 4 | 15 | 2 | 0 | 0 | 4 |
| 5 | 33 | 0 | 21 | 24 | 23 |

Select the smallest element from all the uncovered elements, i.e., 4. Subtract this element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment. Repeating step 3, we obtain a solution which is shown in the following table.

*Table*

| Person | | | | | |
|---|---|---|---|---|---|
| Counter | A | B | C | D | E |
| 1 | 14 | 3 | 0 | 8 | 0 |
| 2 | 0 | 12 | 9 | 11 | 0 |
| 3 | 0 | 4 | 3 | 2 | 1 |
| 4 | 19 | 2 | 0 | 0 | 4 |
| 5 | 37 | 0 | 21 | 24 | 23 |

The total cost of assignment = 1C + 2E + 3A + 4D + 5B

Substituting values from original table: 40 + 36 + 40 + 36 + 62 = 214.

## Multiple Optimal Solutions

Sometimes, it is possible to cross out all the zeros in the reduced matrix in two or more ways. If you can choose a zero cell arbitrarily, then there will be multiple optimum solutions with the same total pay-off for assignments made. In such a case, the management may select that set of optimal assignments, which is more suited to their requirement.

 *Example*

The Spicy Spoon restaurant has four payment counters. There are four persons available for service. The cost of assigning each person to each counter is given in the following table.

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 1 | 8 | 15 | 22 |
| B | 13 | 18 | 23 | 28 |
| C | 13 | 18 | 23 | 28 |
| D | 19 | 23 | 27 | 31 |

Assign one person to one counter to minimize the total cost.

*Solution.*

After applying steps 1 to 3 of the Hungarian Method, we obtain the following matrix.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | **0** | 3 | 6 | 9 |
| B | ~~0~~ | 1 | 2 | 3 |
| C | ~~0~~ | 1 | 2 | 3 |
| D | ~~0~~ | **0** | ~~0~~ | ~~0~~ |

Now by applying the usual procedure, we get the following matrix.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |

| | | | | |
|---|---|---|---|---|
| A | **[0]** | 2 | 5 | 8 |
| B | ~~0~~ | **[0]** | 1 | 2 |
| C | ~~0~~ | ~~0~~ | 1 | 2 |
| D | 1 | ~~0~~ | **[0]** | ~~0~~ |

The resulting matrix suggest the alternative optimal solutions as shown in the following tables.

*Table*

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | **[0]** | 2 | 4 | 7 |
| B | ~~0~~ | **[0]** | ~~0~~ | 1 |
| C | ~~0~~ | ~~0~~ | **[0]** | 1 |
| D | 2 | 1 | ~~0~~ | **[0]** |

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | **[0]** | 2 | 4 | 7 |
| B | ~~0~~ | ~~0~~ | **[0]** | 1 |
| C | ~~0~~ | **[0]** | ~~0~~ | 1 |
| D | 2 | 1 | ~~0~~ | **[0]** |

The persons B & C may be assigned either to job 2 or 3. The two alternative assignments are:

A1 + B2 + C3 + D4
1 + 18 + 23 + 31 = 73

A1 + B3 + C2+ D4
1 + 23 + 18 + 31 = 73

## Travelling Salesman Problem

This humorously named problem refers to the following situation:
A travelling salesman, named Rover plans to visit each of n cities. He wishes to visit each city once and only once, arriving back to city from where he started. The distance between City i and City j is $c_{ij}$. What is the shortest tour Rover can take?

If there are n cities, there are (n - 1)! possible ways for his tour. For example, if the number of cities to be visited is 5, then there are 4! different combinations. Such type of problems can be solved by the assignment method.
Let $c_{ij}$ be the distance (or cost or time) between City i to City j and

$x_{ij}$ = { 1 if a tour includes travelling from city i to city j $\neq$ j)
(for i 0 otherwise

The following example will help you in understanding the travelling salesman problem.


*Example*

A travelling salesman, named Rolling Stone plans to visit five cities 1, 2, 3, 4 & 5. The travel time (in hours) between these cities is shown below:

| | | To | | | |
|---|---|---|---|---|---|
| From | 1 | 2 | 3 | 4 | 5 |
| 1 | ∝ | 5 | 8 | 4 | 5 |
| 2 | 5 | ∝ | 7 | 4 | 5 |
| 3 | 8 | 7 | ∝ | 8 | 6 |
| 4 | 4 | 4 | 8 | ∝ | 8 |
| 5 | 5 | 5 | 6 | 8 | ∝ |

How should Mr. Rolling Stone schedule his touring plan in order to **minimize** the total travel
**time**, if he visits each city once a week?

*Solution*

After applying steps 1 to 3 of the Hungarian method, we get the following assignments.

*Table*

| | | To | | | |
|---|---|---|---|---|---|
| From | 1 | 2 | 3 | 4 | 5 |
| 1 | ∝ | 1 | 3 | | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | ∝ | 2 | ~~0~~ | 1 |
| 3 | 2 | 1 | ∝ | 2 | [0] |
| 4 | [0] | ~~0~~ | 3 | ∝ | 4 |
| 5 | ~~0~~ | [0] | ~~0~~ | 3 | ∝ |

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix.

*Table*

| | To | | | | |
|---|---|---|---|---|---|
| From | 1 | 2 | 3 | 4 | 5 |
| 1 | ∝ | 1 | 3 | [0] | 1 |
| 2 | 1 | ∝ | 2 | ~~0~~ | 1 |
| 3 | 2 | 1 | ∝ | 2 | [0] |
| 4 | [0] | ~~0~~ | 3 | ∝ | 4 |
| 5 | ~~0~~ | [0] | ~~0~~ | 3 | ∝ |

Select the smallest element from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment. Repeating step 3 on the reduced matrix, we get the following assignments.

*Table*

| | T | | | | |
|---|---|---|---|---|---|
| From | 1 | 2 | 3 | 4 | 5 |
| 1 | ∝ | ~~0~~ | 2 | | 1 |
| 2 | | ∝ | 1 | ~~0~~ | 1 |
| 3 | 1 | ~~0~~ | ∝ | 2 | [0] |
| 4 | ~~0~~ | | 3 | ∝ | 5 |
| 5 | ~~0~~ | ~~0~~ | | 4 | ∝ |

The above solution suggests that the salesman should go from city 1 to city 4, city 4 to city 2, and then city 2 to 1 (original starting point). The above solution is not a solution to the travelling salesman problem as he visits city 1 twice.

The next best solution can be obtained by bringing the minimum non-zero element, i.e., 1 into the solution. Please note that the value 1 occurs at four places. We will consider all the cases separately until the acceptable solution is obtained. To make the assignment in the cell (2, 3), delete the row & the column containing this cell so that no other assignment can be made in the second row and third column.

Now, make the assignments in the usual manner as shown in the following table.

*Table*

| | To | | | | |
|---|---|---|---|---|---|
| From | 1 | 2 | 3 | 4 | 5 |
| 1 | ∝ | 0 | 2 | 0 | 1 |
| 2 | 0 | ∝ | 1 | 0 | 1 |
| 3 | 1 | 0 | ∝ | 2 | 0 |
| 4 | 0 | 0 | 3 | ∝ | 5 |
| 5 | 0 | 0 | 0 | 4 | ∝ |

He starts from city 1 and goes to city 4; from city 4 to city 2; from city 2 to city 3; from city 3 to city 5; from city 5 to city 1.

Substituting values from original table: 4 + 7 + 6+ 4 + 5 = 26 hours.