2.1 Errors in Measurement and Least Count

To get some overview of error, least count and significant figures, let us have some examples.

Example 2.1 *Let us use a centimeter scale (on which only centimeter scales are there) to measure a length AB.*

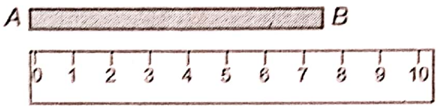

Fig. 2.1

From the figure, we can see that length AB is more than 7 cm and less than 8 cm. In this case, ***Least Count (LC)*** *of this centimeter scale is 1 cm, as it can measure accurately upto centimeters only. If we note down the length* (l) *of line AB as* $l = 7$ *cm then maximum uncertainty or maximum possible error in* l *can be 1 cm (= LC), because this scale can measure accurately only upto 1 cm.*

Example 2.2 *Let us now use a millimeter scale (on which millimeter marks are there). This is also our normal meter scale which we use in our routine life.*

From the figure, we can see that length AB is more than 3.3 cm and less than 3.4 cm. If we note down the length,

$$l = AB = 3.4 \text{ cm}$$

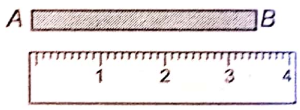

Fig. 2.2

Then, this measurement has two significant figures 3 and 4 in which 3 is absolutely correct and 4 is reasonably correct (doubtful). Least count of this scale is 0.1 cm because this scale can measure accurately only upto 0.1 cm. Further, maximum uncertainty or maximum possible error in l *can also be 0.1 cm.*

INTRODUCTORY EXERCISE 2.1

1. If we measure a length $l = 6.24$ cm with the help of a vernier callipers, then
   (a) What is least count of vernier callipers ?
   (b) How many significant figures are there in the measured length ?
   (c) Which digits are absolutely correct and which is/are doubtful ?
2. If we measure a length $l = 3.267$ cm with the help of a screw gauge, then
   (a) What is maximum uncertainty or maximum possible error in l ?
   (b) How many significant figures are there in the measured length ?
   (c) Which digits are absolutely correct and which is/are doubtful ?

2.2 Significant Figures

From example 2.2, we can conclude that:

"In a measured quantity, significant figures are the digits which are absolutely correct plus the first uncertain digit".

Rules for Counting Significant Figures

Rule 1 All non-zero digits are significant. For example, 126.28 has five significant figures.

Rule 2 The zeros appearing between two non-zero digits are significant. For example, 6.025 has four significant figures.

Rule 3 Trailing zeros after decimal places are significant. Measurement $l = 6.400$ cm has four significant figures. Let us take an example in its support.

Table 2.1

Measurement	Accuracy	l lies between (in cm)	Significant figures	Remarks
$l = 6.4$ cm	0.1 cm	6.3 – 6.5	Two	
$l = 6.40$ cm	0.01 cm	6.39 – 6.41	Three	closer
$l = 6.400$ cm	0.001 cm	6.399 – 6.401	Four	more closer

Thus, the significant figures depend on the accuracy of measurement. More the number of significant figures, more accurate is the measurement.

Rule 4 The powers of ten are not counted as significant figures. For example, 1.4×10^{-7} has only two significant figures 1 and 4.

Rule 5 If a measurement is less than one, then all zeros occurring to the left of last non-zero digit are not significant. For example, 0.0042 has two significant figures 4 and 2.

Rule 6 Change in units of measurement of a quantity does not change the number of significant figures. Suppose a measurement was done using mm scale and we get $l = 72$ mm (two significant figures).

We can write this measurement in other units also (without changing the number of significant figures) :

7.2 cm → Two significant figures.
0.072 m → Two significant figures.
0.000072 km → Two significant figures.
7.2×10^{7} nm → Two significant figures

Rule 7 The terminal or trailing zeros in a number without a decimal point are not significant. This also sometimes arises due to change of unit.

For example, 264 m = 26400 cm = 264000 mm

All have only three significant figures 2, 6 and 4. All trailing zeros are not significant.

Zeroes at the end of a number are significant only if they are behind a decimal point as in Rule-3. Otherwise, it is impossible to tell if they are significant. For example, in the number 8200, it is not clear if the zeros are significant or not. The number of significant digits in 8200 is at least two, but could be three or four. To avoid uncertainty, use scientific notation to place significant zeros behind a decimal point

8.200×10^{3} has four significant digits.

8.20×10^3 has three significant digits.

8.2×10^3 has two significant digits.

Therefore, if it is not expressed in scientific notations, then write least number of significant digits. Hence, in the number 8200, take significant digits as two.

Rule 8 Exact measurements have infinite number of significant figures. For example,

10 bananas in a basket

46 students in a class

speed of light in vacuum = 299,792,458 m/s (exact)

$\pi = \frac{22}{7}$ (exact)

All these measurements have infinite number of significant figures.

Example 2.3 **Table 2.2**

Measured value	Number of significant figures	Rule number
12376 cm	5	1
6024.7 cm	5	2
0.071 cm	2	5
4100 cm	2	7
2.40 cm	3	3
1.60×10^{14} km	3	4

INTRODUCTORY EXERCISE 2.2

1. Count total number of significant figures in the following measurements:
   (a) 4.080 cm (b) 0.079 m (c) 950
   (d) 10.00 cm (e) 4.07080 (f) 7.090×10^5

2.3 Rounding Off a Digit

Following are the rules for rounding off a measurement :

Rule 1 If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

For example, $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.

Rule 2 If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is increased by 1.

For example, $x = 14.252$ is rounded off to $x = 14.3$ to three significant digits.

Rule 3 If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit it left unchanged if it is even.

For example, $x = 6.250$ or $x = 6.25$ becomes $x = 6.2$ after rounding off to two significant digits.

Rule 4 If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one if it is odd.

For example, $x = 6.350$ or $x = 6.35$ becomes $x = 6.4$ after rounding off to two significant digits.

Example 2.4

Table 2.3

Measured value	After rounding off to three significant digits	Rule
7.364	7.36	1
7.367	7.37	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4

INTRODUCTORY EXERCISE 2.3

1. Round off the following numbers to three significant figures :
   (a) 24572 (b) 24.937 (c) 36.350 (d) 42.450×10^9
2. Round 742396 to four, three and two significant digits.

2.4 Algebraic Operations with Significant Figures

The final result shall have significant figures corresponding to their number in the least accurate variable involved. To understand this, let us consider a chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus, the strength of the chain cannot be more than the strength of the weakest link in the chain.

Addition and Subtraction

Suppose, in the measured values to be added or subtracted the least number of digits after the decimal is n. Then, in the sum or difference also, the number of digits after the decimal should be n.

Example 2.5 $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

Example 2.6 $12.63 - 10.2 = 2.43 \approx 2.4$

Multiplication or Division

Suppose in the measured values to be multiplied or divided the least number of significant digits be n. Then in the product or quotient, the number of significant digits should also be n.

Example 2.7 $1.2 \times 36.72 = 44.064 \approx 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore, the answer is 44.

Example 2.8 $\dfrac{1101\ \text{ms}^{-1}}{10.2\ \text{ms}^{-1}} = 107.94117647 \approx 108$

Example 2.9 *Find, volume of a cube of side $a = 1.4 \times 10^{-2}$ m.*

Solution Volume $V = a^3$

$$= (1.4 \times 10^{-2}) \times (1.4 \times 10^{-2}) \times (1.4 \times 10^{-2}) = 2.744 \times 10^{-6}\ \text{m}^3$$

Since, each value of a has two significant figures. Hence, we will round off the result to two significant figures.

$\therefore$ $$V = 2.7 \times 10^{-6}\ \text{m}^3$$ **Ans.**

Example 2.10 *Radius of a wire is 2.50 mm. The length of the wire is 50.0 cm. If mass of wire was measured as 25 g, then find the density of wire in correct significant figures.*
[Given, $\pi = 3.14$, exact]

Solution Given,

$r = 2.50$ mm (three significant figures)

$= 0.250$ cm (three significant figures)

Note *Change in the units of measurement of a quantity does not change the number of significant figures.*

Further given that,

$l = 50.0$ cm (three significant figures)

$m = 25$ gm (two significant figures)

$\pi = 3.14$ exact (infinite significant figures)

$$\rho = \frac{m}{V} = \frac{m}{\pi r^2 l}$$

$$= \frac{25}{(3.14)(0.250)(0.250)(50.0)}$$

$$= 2.5477\ \text{g/cm}^3$$

But in the measured values, least number of significant figures are two. Hence, we will round off the result to two significant figures.

$\therefore$ $$\rho = 2.5\ \text{g/cm}^3$$ **Ans.**

INTRODUCTORY EXERCISE 2.4

1. Round to the appropriate number of significant digits
   (a) $13.214 + 234.6 + 7.0350 + 6.38$
   (b) $1247 + 134.5 + 450 + 78$
2. Simplify and round to the appropriate number of significant digits
   (a) $16.235 \times 0.217 \times 5$
   (b) 0.00435×4.6

2.5 Error Analysis

We have studied in the above articles that no measurement is perfect. Every instrument can measure upto a certain accuracy called **Least Count (LC)**.

Least Count

The smallest measurement that can be measured accurately by an instrument is called its least count.

Instrument	Its least count
mm scale	1 mm
Vernier callipers	0.1 mm
Screw gauge	0.01 mm
Stop watch	0.1 sec
Temperature thermometer	1°C

Permissible Error due to Least Count

Error in measurement due to the limitation (or least count) of the instrument is called permissible error. Least count of a millimeter scale is 1 mm. Therefore, maximum permissible error in the measurement of a length by a millimeter scale may be 1 mm.

If we measure a length $l = 26$ mm. Then, maximum value of true value may be $(26+1)$ mm $= 27$ mm and minimum value of true value may be $(26-1)$ mm $= 25$ mm.

Thus, we can write it like,

$$l = (26 \pm 1) \text{ mm}$$

If from any other instrument we measure a length $= 24.6$ mm, then the maximum permissible error (or least count) from this instrument is 0.1 mm. So, we can write the measurement like,

$$l = (24.6 \pm 0.1) \text{ mm}$$

Classification of Errors

Errors can be classified in two ways. First classification is based on the cause of error. Systematic error and random errors fall in this group. Second classification is based on the magnitude of errors. Absolute error, mean absolute error and relative (or fractional) error lie on this group. Now, let us discuss them separately.

Systematic Error

Systematic errors are the errors whose causes are known to us. Such errors can therefore be minimised. Following are few causes of these errors :

(a) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

(b) Sometimes errors arise on account of ignoring certain facts. For example in measuring time period of simple pendulum error may creep because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

(c) Change in temperature, pressure, humidity, etc., may also sometimes cause errors in the result. Relevant corrections can be made to minimise their effects.

Random Error

The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons. For example the reading of a sensitive beam balance may change by the vibrations caused in the building due to persons moving in the laboratory or vehicles running nearby. The random errors can be minimized by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus,

$$\boxed{a_{\text{mean}} = \frac{a_1 + a_2 + \ldots + a_n}{n}}$$

Absolute Error

The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

Then by definition, absolute errors in the measured values of the quantity are,

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots \quad \ldots \quad \ldots$$
$$\Delta a_n = a_m - a_n$$

Absolute error may be positive or negative.

Mean Absolute Error

Arithmetic mean of the magnitudes of absolute errors in all the measurements is called the mean absolute error. Thus,

$$\boxed{\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}}$$

The final result of measurement can be written as, $a = a_m \pm \Delta a_{mean}$

This implies that value of a is likely to lie between $a_m + \Delta a_{mean}$ and $a_m - \Delta a_{mean}$.

Relative or Fractional Error

The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error. Thus,

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_m}$$

Relative error expressed in percentage is called as the percentage error, i.e.

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_m} \times 100$$

Example 2.11 *The diameter of a wire as measured by screw gauge was found to be 2.620, 2.625, 2.630, 2.628 and 2.626 cm. Calculate*

(a) mean value of diameter *(b) absolute error in each measurement*
(c) mean absolute error *(d) fractional error*
(e) percentage error *(f) Express the result in terms of percentage error*

Solution (a) Mean value of diameter

$$a_m = \frac{2.620 + 2.625 + 2.630 + 2.628 + 2.626}{5}$$

$$= 2.6258 \text{ cm}$$

$$= 2.626 \text{ cm}$$ (rounding off to three decimal places)

(b) Taking a_m as the true value, the absolute errors in different observations are,

$$\Delta a_1 = 2.626 - 2.620 = +0.006 \text{ cm}$$
$$\Delta a_2 = 2.626 - 2.625 = +0.001 \text{ cm}$$
$$\Delta a_3 = 2.626 - 2.630 = -0.004 \text{ cm}$$
$$\Delta a_4 = 2.626 - 2.628 = -0.002 \text{ cm}$$
$$\Delta a_5 = 2.626 - 2.626 = 0.000 \text{ cm}$$

(c) Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$

$$= \frac{0.006 + 0.001 + 0.004 + 0.002 + 0.000}{5}$$

$$= 0.0026 = 0.003$$ (rounding off to three decimal places)

(d) Fractional error $= \pm \dfrac{\Delta a_{\text{mean}}}{a_m} = \dfrac{\pm 0.003}{2.626} = \pm 0.001$

(e) Percentage error $= \pm 0.001 \times 100 = \pm 0.1\%$

(f) Diameter of wire can be written as,

$$d = 2.626 \pm 0.1\%$$

Combination of Errors

Errors in Sum or Difference

Let $x = a \pm b$

Further, let Δa is the absolute error in the measurement of a, Δb the absolute error in the measurement of b and Δx is the absolute error in the measurement of x.

Then,
$$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b)$$
$$= (a \pm b) \pm (\pm \Delta a \pm \Delta b)$$
$$= x \pm (\pm \Delta a \pm \Delta b)$$

or
$$\Delta x = \pm \Delta a \pm \Delta b$$

The four possible values of Δx are $(\Delta a - \Delta b)$, $(\Delta a + \Delta b)$, $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$.

Therefore, the maximum absolute error in x is,

$$\Delta x = \pm (\Delta a + \Delta b)$$

i.e. the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Example 2.12 *The volumes of two bodies are measured to be $V_1 = (10.2 \pm 0.02)\, cm^3$ and $V_2 = (6.4 \pm 0.01)\, cm^3$. Calculate sum and difference in volumes with error limits.*

Solution $V_1 = (10.2 \pm 0.02)\, \text{cm}^3$

and $$V_2 = (6.4 \pm 0.01)\, \text{cm}^3$$

$$\Delta V = \pm (\Delta V_1 + \Delta V_2)$$

$$= \pm (0.02 + 0.01)\, \text{cm}^3 = \pm 0.03\, \text{cm}^3$$

$$V_1 + V_2 = (10.2 + 6.4)\, \text{cm}^3 = 16.6\, \text{cm}^3$$

and $$V_1 - V_2 = (10.2 - 6.4)\, \text{cm}^3 = 3.8\, \text{cm}^3$$

Hence, sum of volumes $= (16.6 \pm 0.03)\, \text{cm}^3$

and difference of volumes $= (3.8 \pm 0.03)\, \text{cm}^3$

Errors in a Product

Let $x = ab$

Then, $$(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

or $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b} \qquad (\text{as } x = ab)$$

or $$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$.

Hence, maximum possible value of

$$\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

Errors in Division

Let $$x = \frac{a}{b}$$

Then, $$x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$$

or $$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1} \qquad \left(\text{as } x = \frac{a}{b}\right)$$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right)$$

or $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is small quantity, so can be neglected. Therefore,

$$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$. Therefore, the maximum value of

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

or the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

Error in Quantity Raised to Some Power

Let, $x = \frac{a^n}{b^m}$. Then, $\ln(x) = n \ln(a) - m \ln(b)$

Differentiating both sides, we get

$$\frac{dx}{x} = n \cdot \frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm \frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore, maximum value of

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$$

Note *Errors in product and division can also be obtained by taking logarithm on both sides* $\left(\text{in } x = ab \text{ or } x = \frac{a}{b}\right)$ *and then differentiating.*

Example 2.13 *The mass and density of a solid sphere are measured to be* $(12.4 \pm 0.1)\,kg$ *and* $(4.6 \pm 0.2)\,kg/m^3$. *Calculate the volume of the sphere with error limits.*

Solution Here, $m \pm \Delta m = (12.4 \pm 0.1)$ kg

and $$\rho \pm \Delta\rho = (4.6 \pm 0.2)\ \text{kg/m}^3$$

Volume $$V = \frac{m}{\rho} = \frac{12.4}{4.6}$$

$$= 2.69\ \text{m}^3 = 2.7\ \text{m}^3$$ (rounding off to one decimal place)

Now, $$\frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$$

or $$\Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right) \times V$$

$$= \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right) \times 2.7 = \pm 0.14$$

∴ $$V \pm \Delta V = (2.7 \pm 0.14)\ \text{m}^3$$

Example 2.14 *Calculate percentage error in determination of time period of a pendulum*

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l and g are measured with $\pm 1\%$ *and* $\pm 2\%$.

Solution $T = 2\pi\sqrt{\frac{l}{g}}$

or $$T = (2\pi)(l)^{+1/2}(g)^{-1/2}$$

Taking logarithm of both sides, we have

$$\ln(T) = \ln(2\pi) + \frac{1}{2}(\ln l) - \left(\frac{1}{2}\right)\ln(g) \qquad ...(i)$$

Here, 2π is a constant, therefore $\ln(2\pi)$ is also a constant.

Differentiating Eq. (i), we have

$$\frac{1}{T}dT = 0 + \frac{1}{2}\left(\frac{1}{l}\right)(dl) - \frac{1}{2}\left(\frac{1}{g}\right)(dg)$$

or $$\left(\frac{dT}{T}\right)_{max} = \text{maximum value of}\left(\pm\frac{1}{2}\frac{dl}{l}\mp\frac{1}{2}\frac{dg}{g}\right)$$

$$=\frac{1}{2}\left(\frac{dl}{l}\right)+\frac{1}{2}\left(\frac{dg}{g}\right)$$

This can also be written as

$$\left(\frac{\Delta T}{T}\times 100\right)_{max}=\frac{1}{2}\left[\frac{\Delta l}{l}\times 100\right]+\frac{1}{2}\left[\frac{\Delta g}{g}\times 100\right]$$

or percentage error in time period

$$=\pm\left[\frac{1}{2}\text{(percentage error in } l\text{)}+\frac{1}{2}\text{(percentage error in } g\text{)}\right]$$

$$=\pm\left[\frac{1}{2}\times 1+\frac{1}{2}\times 2\right]=\pm 1.5\%$$ **Ans.**

Final Touch Points

Order of Magnitude In physics, a number of times we come across quantities which vary over a wide range. For example, size of universe, mass of sun, radius of a nucleus etc. In this case, we use the powers of ten method. In this method, each number is expressed as $n\times 10^m$, where $1\le n\le 10$ and m is a positive or negative integer. If n is less than or equal to 5, then order of number is 10^m and if n is greater than 5 then order of number is 10^{m+1}.

For example, diameter of the sun is 1.39×10^9 m. Therefore, the diameter of the sun is of the order of 10^9 m as n or $1.39 \le 5$.

Solved Examples

Example 1 *Round off 0.07284 to four, three and two significant digits.*

Solution

0.07284 (four significant digits)

0.0728 (three significant digits)

0.073 (two significant digits)

Example 2 *Round off 231.45 to four, three and two significant digits.*

Solution

231.5 (four significant digits)

231 (three significant digits)

230 (two significant digits)

Example 3 *Three measurements are $a = 483$, $b = 73.67$ and $c = 15.67$. Find the value $\frac{ab}{c}$ to correct significant figures.*

Solution

$$\frac{ab}{c} = \frac{483 \times 73.67}{15.67}$$

$$= 2270.7472$$

$$= 2.27 \times 10^3$$

Ans.

Note *The result is rounded off to least number of significant figures in the given measurement i.e. 3 (in 483).*

Example 4 *Three measurements are, $a = 25.6$, $b = 21.1$ and $c = 2.43$. Find the value $a - b - c$ to correct significant figures.*

Solution

$$a - b - c = 25.6 - 21.1 - 2.43$$

$$= 2.07 = 2.1$$

Ans.

Note *In the measurements, least number of significant digits after the decimal is one (in 25.6 and 21.1). Hence, the result will also be rounded off to one decimal place.*

Example 5 *A thin wire has a length of 21.7 cm and radius 0.46 mm. Calculate the volume of the wire to correct significant figures.*

Solution Given, $l = 21.7$ cm, $r = 0.46$ mm $= 0.046$ cm

Volume of wire $V = \pi r^2 l$

$$= \frac{22}{7}(0.046)^2(21.7)$$

$$= 0.1443 \text{ cm}^3 = 0.14 \text{ cm}^3$$

Note *The result is rounded off to least number of significant figures in the given measurements i.e. 2 (in 0.46 mm).*

Example 6 *The radius of a sphere is measured to be (1.2 ± 0.2) cm. Calculate its volume with error limits.*

Solution Volume, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\left(\frac{22}{7}\right)(1.2)^3$

$$= 7.24 \text{ cm}^3 = 7.2 \text{ cm}^3$$

Further, $\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$

$\therefore$ $$\Delta V = 3\left(\frac{\Delta r}{r}\right)V = \frac{3 \times 0.2 \times 7.2}{1.2}$$

$$= 3.6 \text{ cm}^3$$

$\therefore$ $$V = (7.2 \pm 3.6) \text{ cm}^3$$

Example 7 *Calculate equivalent resistance of two resistors R_1 and R_2 in parallel where, $R_1 = (6 \pm 0.2)$ ohm and $R_2 = (3 \pm 0.1)$ ohm*

Solution In parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{...(i)}$$

or, $$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{(6)(3)}{6+3} = 2 \text{ ohm}$$

Differentiating Eq. (i), we have

$$-\frac{dR}{R^2} = -\frac{dR_1}{R_1^2} - \frac{dR_2}{R_2^2}$$

Therefore, maximum permissible error in equivalent resistance may be

$$\Delta R = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)(R^2)$$

Substituting the values we get,

$$\Delta R = \left[\frac{0.2}{(6)^2} + \frac{0.1}{(3)^2}\right](2)^2$$

$$= 0.07 \text{ ohm}$$

$\therefore$ $$R = (2 \pm 0.07) \text{ ohm}$$ **Ans.**